New Keynesian Model II: (Un)Conventional Monetary Policy

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based on "Optimal (Un)Conventional Monetary Policy" with Markus Brunnermeier

September 12, 2026

Introduction

- (my) Key takeaways from Part I:
  - Uncertainty shocks cause flight to safety
  - Under flexible prices, deflation increases the real value of government debt
  - Sticky prices prevent real debt appreciation $\Longrightarrow$

    flight-to-safety forces capital and total wealth devaluation (demand recession)
  - Conventional monetary policy loses its potency:

    controls debt *growth* but not debt *revaluation*

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- Part II introduces:
  - Distributional aspects into the framework $\implies$ financial sector
  - Unconventional policy instruments $\implies$ balance sheet management and interest rate tiering

Recall: Sticky-Price One Sector Monetary Model

- Households' problem:

$$\max_{c_t^H,\iota_t,\upsilon_t,\theta_t^K} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log(c_t^H) - b(\upsilon_t)\right) dt\right] \quad \text{s.t.}$$

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H}dt + (1-\theta_t^K)dr_t^{\mathcal{B}} + \theta_t^K dr_t^K(\iota_t,\upsilon_t)$$

$$dr_t^{\mathcal{B}} = (i_t - \pi_t)dt$$

$$dr_t^K(\iota_t,\upsilon_t) = \left(\frac{p_t^R a\upsilon_t + \omega_t - \iota_t - \tau_t^K}{q_t^K} + \mu_t^{q^K} + \Phi(\iota_t) - \delta\right)dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$$

$$d\tilde{\sigma}_t^2 = -\psi(\tilde{\sigma}_t^2 - \tilde{\sigma}_{ss}^2)dt + \sigma\tilde{\sigma}_t^2 dZ_t$$

- $+$ NKPC

Introducing Intermediaries

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$$1 = \theta_t^{\mathcal{B}} + \theta_t^{D,I} + \theta_t^{x,I}$$

$$dr_t^{\mathcal{B}} = (i_t - \pi_t) dt \qquad dr_t^D = (i_t^D - \pi_t) dt$$

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- For comparison: $dr_t^{x,H} = r_t^x dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$

Introducing $\mathcal{R}$eserves and Long-term bonds

- Relabel short-term bonds as reserves $\mathcal{R}_t$, only intermediaries can hold them
- Interest rates for mandatory ($\underline{i}_t$) and excess (i_t) reserves:

$$dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) = \left[\underbrace{\frac{\underline{\theta}_t^{\mathcal{R}} \underline{i}_t + (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}}) i_t}{\theta_t^{\mathcal{R}}}}_{i_t^a} - \pi_t \right] dt$$

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- Effective policy instruments: average rate i_t^a and marginal rate $i_t^m = i_t + \lambda_t^{\mathcal{R}}$
- Perpetual long-term bonds L_t with fixed coupon i^L and nominal price P_t^L:

$$dr_t^L = \frac{i^L}{P_t^L}dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right]dt + \sigma_t^{P^L}dZ_t$$

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- Nominal wealth is now $\mathcal{B}_t = \mathcal{R}_t + P_t^L L_t$
- Balance sheet management: buy/sell L_t in exchange for $\mathcal{R}_t$ at market prices

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- Households use deposits for transactions related to capital with velocity ν_t:

$$\theta_t^K = \nu_t \theta_t^{D,H}$$

- Higher velocity $\Longrightarrow$ higher transaction cost $\mathfrak{t}(\nu_t)$

$$dr_t^K(\iota_t, \upsilon_t, \nu_t) = \left[\frac{p_t^R a \upsilon_t + \omega_t - \iota_t - \tau_t^K}{q_t^K} - \mathfrak{t}(\nu_t) + \mu_t^{q^K} + \Phi(\iota_t) - \delta \right] dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$$

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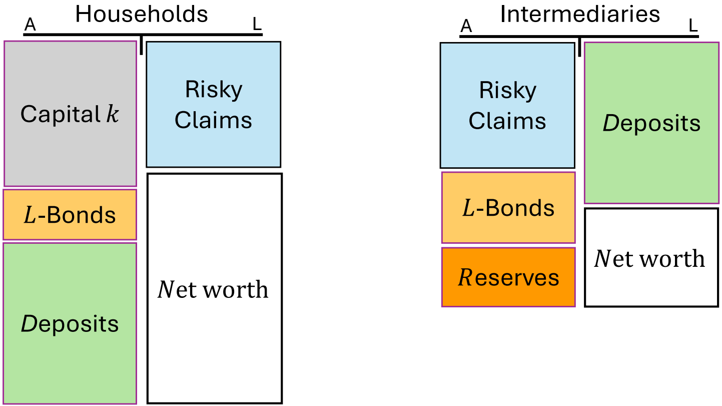
- Introduces deposit convenience yield and prevents perfect aggregate risk sharing

Balance Sheets



Optimal Policy: Roadmap

1. Study a *relaxed* Ramsey problem $\Longrightarrow$ **analytical** characterization of efficient **allocations**
2. Compare efficient outcomes to laissez-faire
3. Consider a *realistic* government and study:
   - Under which conditions efficient allocations are achievable
   - The corresponding **optimal policy mix** (interest rates + balance sheet management)

Constrained Efficiency

- Suppose the government raises taxes over time (flow/dt taxes) and in response to aggregate shocks (loading on dZ_t), but not to idios. shocks (not on $d\tilde{Z}_t$)

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- Behaves as a planner that can directly set:
  - Capital utilization rate υ_t
  - Capital investment rate ι_t
  - Distribution of wealth/consumption across sectors $\eta_t = N_t^I / N_t$,
  - Distribution of wealth across assets $\vartheta_t = \mathcal{B}_t / (\mathcal{P}_t N_t) = (\mathcal{R}_t + P_t^L L_t)/(\mathcal{P}_t N_t)$
  - Distribution of idiosyncratic risk exposure χ_t

Constrained Efficiency

- Planner's objective:

$$\max_{\{\iota_t,\upsilon_t,\vartheta_t,\eta_t,\chi_t\}_{t=0}^{\infty}} \lambda\mathbb{E}\int_0^\infty e^{-\rho t}\log(\tilde{\eta}_t^I\eta_t c_t K_t)dt + (1-\lambda)\mathbb{E}\int_0^\infty e^{-\rho t}\left(\log(\tilde{\eta}_t^H(1-\eta_t)c_t K_t) - b(\upsilon_t)\right)dt$$

$$\text{s.t. } c_t = a\upsilon_t - \iota_t = \rho\frac{q_t^K}{1-\vartheta_t}, \qquad q_t^K = (1+\phi\iota_t)$$

$$\frac{d\tilde{\eta}_t^I}{\tilde{\eta}_t^I} = \chi_t\frac{1-\vartheta_t}{\eta_t}\varphi\tilde{\sigma}_t d\tilde{Z}_t, \qquad \frac{d\tilde{\eta}_t^H}{\tilde{\eta}_t^H} = (1-\chi_t)\frac{1-\vartheta_t}{1-\eta_t}\tilde{\sigma}_t d\tilde{Z}_t$$

Constrained Efficiency

- Planner's objective can be simplified to a static one

$$\max_{\upsilon_t,\iota_t,\eta_t,\chi_t,\vartheta_t} W_t = \overbrace{\log(a\upsilon_t - \iota_t) - (1-\lambda)b(\upsilon_t) + \frac{1}{\rho}\left(\frac{1}{\phi}\log(1+\phi\iota_t) - \delta\right)}^{\text{aggregate efficiency at } t}$$
$$\underbrace{+\,\lambda\log(\eta_t) + (1-\lambda)\log(1-\eta_t) - \frac{\tilde{\sigma}_t^2}{2\rho}\left[\lambda\frac{\chi_t^2}{\eta_t^2}\varphi^2 + (1-\lambda)\frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right](1-\vartheta_t)^2}_{\text{distributional efficiency at } t}$$

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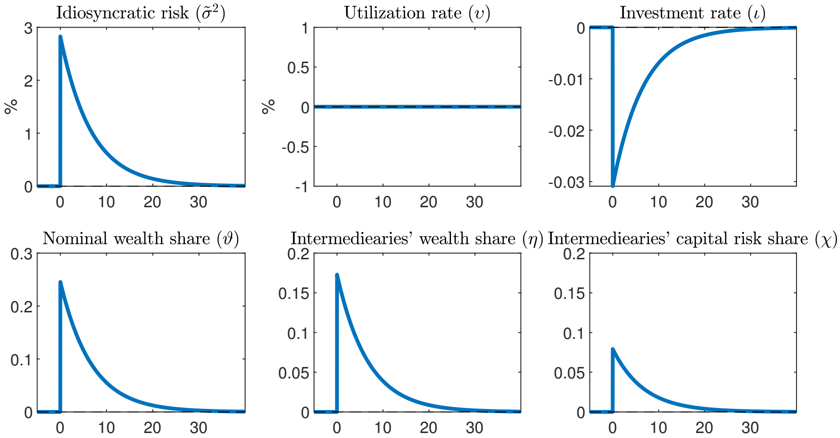
- Constrained efficient allocation $\upsilon^*(\tilde{\sigma})$, $\iota^*(\tilde{\sigma})$, $\eta^*(\tilde{\sigma})$, $\vartheta^*(\tilde{\sigma})$, $\chi^*(\tilde{\sigma})$
- $1 > \chi^*(\tilde{\sigma}) > \eta^*(\tilde{\sigma}) > \lambda$

IRF Planner's Solution after $\tilde{\sigma}_t$-Shock



Laissez-faire

- Goods market clearing + Tobin's q:

$$a\upsilon_t - \iota_t = \rho\left(q_t^K + q_t^{\mathcal{B}}\right) = \rho\left(q_t^K + \frac{\mathcal{R}_t + P_t^L L_t}{\mathcal{P}_t K_t}\right)$$

$$a\upsilon_t = \rho + (1+\rho\phi)\iota_t + \rho\frac{\mathcal{R}_t + P_t^L L_t}{\mathcal{P}_t K_t}$$

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- Output gap $\hat{Y}_t \equiv Y_t / Y_t^* = \frac{a\upsilon_t K_t}{a\upsilon_t^* K_t^*}$:

$$\log \hat{Y}_t = \underbrace{\left(\log \upsilon_t - \log \upsilon_t^*\right)}_{\text{instantaneous}} + \underbrace{\left(\log K_t - \log K_t^*\right)}_{\text{dynamic}}$$

- Sticky prices and no policy response $\implies$ either static or dynamic output gap
- Policy needs to move nominal wealth in response to an aggregate shock (see also part I)

Laissez-faire

- Distributional efficiency requires that $\tilde{\sigma}_t^2 \uparrow \implies \vartheta_t \uparrow$ and $\eta_t \uparrow$ ($\sigma_t^\eta > 0$, $\sigma_t^\vartheta > 0$)

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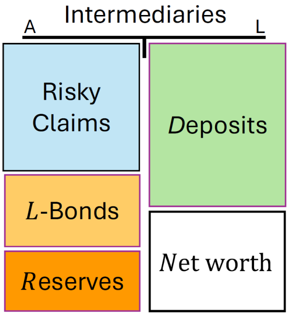

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- Higher $\tilde{\sigma}_t^2 \implies$ higher ϑ_t but lower η_t
- Intermediaries are short in nominal assets and long in capital $\implies$ their net worth share decreases

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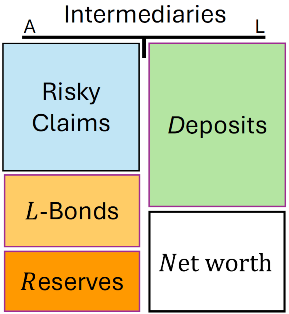

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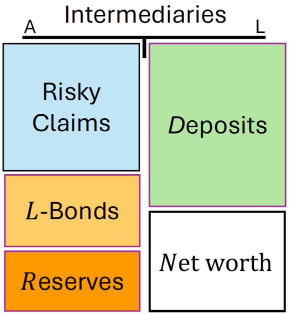

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- Tight link between wealth alloc. across assets (ϑ_t) and agents (η_t) in CE
- Policy needs to redistribute wealth in response to an aggregate shock

Policy

- Realistic government – no dZ_t-taxes
- Focus on interest rate and balance sheet policy
- Fiscal policy operates in the background
- Interest rate controls sensitivity of bond price to aggregate shocks:

$$\sigma_t^{P^L} = \frac{\partial \log P_t^L}{\partial \log \tilde{\sigma}_t^2}\sigma, \qquad P_t^L = \mathbb{E}_t^{Q,I}\left[\int_t^\infty e^{-\int_t^s i_u^m du} i^L ds\right]$$

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- Balance sheet policy controls the share of long-term bonds in nominal wealth:

$$\vartheta_t^L = \frac{P_t^L L_t}{\mathcal{R}_t + P_t^L L_t} = \frac{P_t^L L_t}{\mathcal{B}_t}$$

- Total nominal wealth sensitivity: $\sigma_t^{\mathcal{B}} = \vartheta_t^L \sigma_t^{P^L}$

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- *Preparatory* role of balance sheet management
- What about debt growth channel?

Interest Rate Tiering

- Single interest rate policy (no agg. risk):

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$$\mu_t^{\mathcal{B}} = i_t - s_t$$

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Passive Balance Sheet Management

- Can we also implement distributional efficiency?
- From intermediaries' balance sheet:

$$\eta_t \sigma_t^{\eta} = (\chi_t - \eta_t)(1 - \vartheta_t)\sigma_t^{q^K} + (\eta_t^L - \eta_t)\vartheta_t \vartheta^L \sigma_t^{P^L}$$

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- Indirect aggregate wealth effect:
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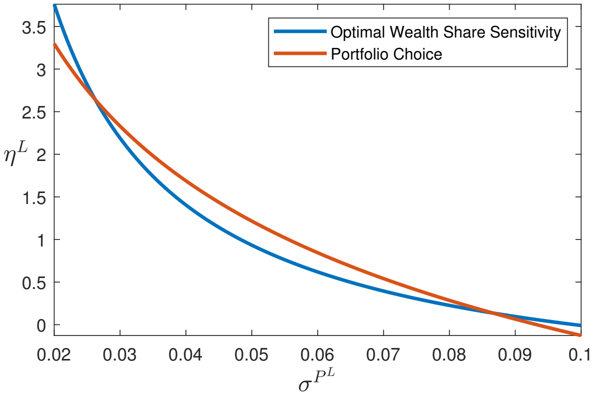
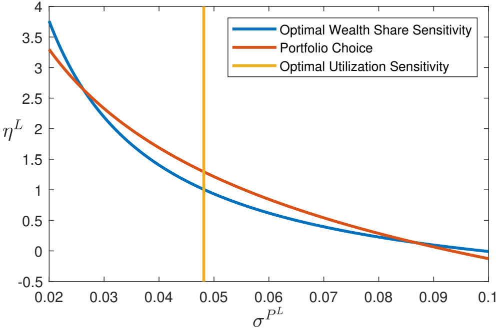
  - Capital price increase benefits levered intermediaries
- Challenge: endogenous bond distribution η_t^L

Distributional Efficiency Fixing ϑ^L



Aggregate and Distributional Efficiency Fixing ϑ^L



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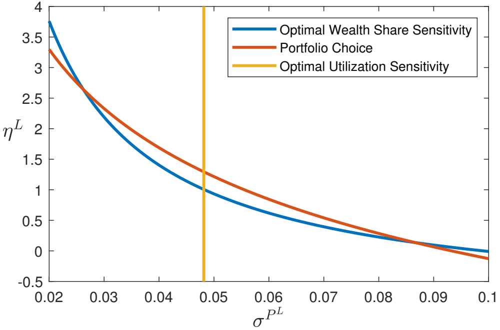

- Requires active balance sheet management!

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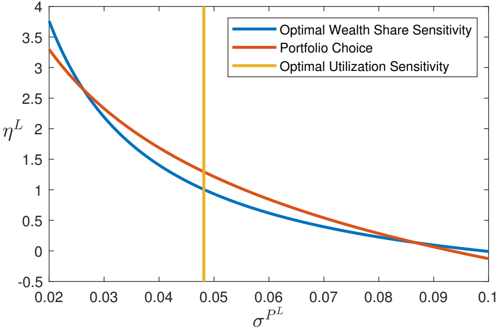

- Requires active balance sheet management!
- Existence and uniqueness of an optimal policy mix under certain conditions
  - Intermediate degree of bond market segmentation

When can CB implement Aggr. + Distr. Efficiency?

1. Balance sheet policy affects the distribution if private agents do not ‘undo’ QE/QT
   $\Rightarrow$ requires that CB can manipulate endogenous bond and wealth distributions

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   - Existence of optimal policy mix requires $\mathfrak{t}(\nu_t)$ to be sufficiently steep
   - $\mathfrak{t}'(\nu_t) = 0 \;\Rightarrow$ agents trade bonds & deposits to perfectly share aggregate risk

     $\Rightarrow$ no redistributive role of MP
   - $\mathfrak{t}'(\nu_t) > 0 \;\Rightarrow$ convenience yield on deposits prevents perfect aggregate risk sharing

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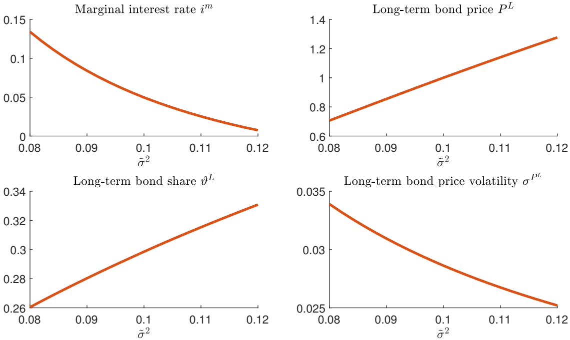
2. With full segmentation or short sell constraints on gov. bonds:

   $\Rightarrow$ CB is constrained in manipulating wealth distribution

   $\Rightarrow$ Competitive equilibrium is generically inefficient with full segmentation

Optimal Policy over the Cycle

- **Proposition 5.** Let $\tilde{\sigma}_t^2$ be small. Then, an increase in $\tilde{\sigma}_t^2$ leads to:
  - a cut in the interest rate i_t
  - a milder interest rate policy going forward (lower $\sigma_t^{P^L}$)
  - a rebalancing towards long-term bonds (higher ϑ_t^L)



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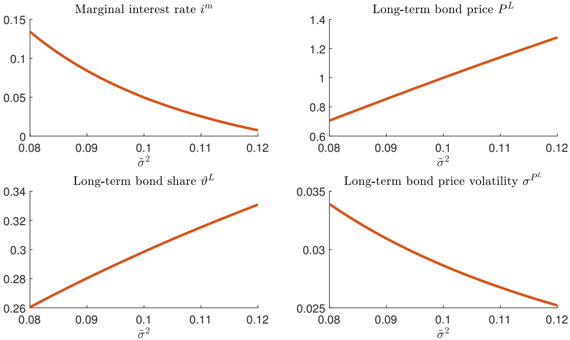

- Reduce bond price volatility, otherwise intermediaries scale down
- Compensate by larger quantity of bonds

Summary

- Macrofinance model to study optimal policy mix
  - Role for aggregate and distributional efficiencies
- Preparatory role of balance sheet policies: efficient exposure to future shocks
- Larger CB balance sheet requires more aggressive interest rate policy subsequently
- Joint aggregate and distributional efficiency requires active balance sheet management over the cycle

How much does duration risk exposure move in the data?

- In the model, $\sigma_t^{\mathcal{B}} = \vartheta_t^L \sigma_t^{P^L} \approx \vartheta_t^L \frac{\partial \log P_t^L}{\partial i_t^m} \sigma_{i^m,t}$
- In the data, $\vartheta_t^L \frac{\partial \log P_t^L}{\partial i_t^m} \approx$ total nominal wealth duration in private hands

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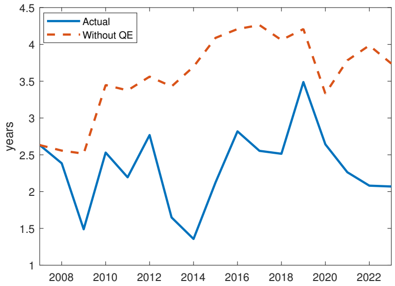

- 100bp interest rate hike in 2007 $\approx$ 177bp hike in 2009
- 100bp interest rate hike in 2023 without QE $\approx$ 181bp hike with QE

Household's Problem

$$\max_{c_t^H, \upsilon_t, \iota_t, \nu_t, \theta_t^{D,H}, \theta_t^{L,H}, \theta_t^K, \chi_t} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log(c_t^H) - b(\upsilon_t)\right) dt\right] \qquad \text{s.t.}$$

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H} dt + \theta_t^{D,H} dr_t^D + \theta_t^{L,H} dr_t^L + \theta_t^K \left(dr_t^K(\upsilon_t, \iota_t, \nu_t) - \chi_t dr_t^{x,H}\right) + \tau_t^H dt$$

$$1 = \theta_t^{D,H} + \theta_t^{L,H} + \theta_t^K(1 - \chi_t) \qquad \nu_t \theta_t^{D,H} = \theta_t^K$$

- $dr_t^K = \left[\frac{p_t a \upsilon_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}_t(\nu_t) + \mu_t^{q^K} + g(\iota_t)\right] dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^{x,H} = r_t^x dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^D = i_t^D dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[i_t^D - \pi_t\right] dt$
- $dr_t^L = \frac{i^L}{P_t^L} dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right] dt + \sigma_t^{P^L} dZ_t$

Back

Intermediary's Problem

$$\max_{c_t^I,\theta_t^{\mathcal{R}},\theta_t^{L,I},\theta_t^{D,I},\theta_t^{x,I}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(c_t^I)dt\right] \qquad \text{s.t.}$$

$$\frac{dn_t^I}{n_t^I} = -\frac{c_t^I}{n_t^I}dt + \theta_t^{\mathcal{R}} dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) + \theta_t^{D,I} dr_t^D + \theta_t^{L,I} dr_t^L + \theta_t^{x,I} dr_t^{x,I} + \tau_t^I dt$$

$$1 = \theta_t^{\mathcal{R}} + \theta_t^{D,I} + \theta_t^{L,I} + \theta_t^{x,I} \qquad \theta_t^{\mathcal{R}} \geq \underline{\theta}_t^{\mathcal{R}}$$

- $dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) = i(\theta_t^{\mathcal{R}})dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[\frac{\underline{\theta}_t^{\mathcal{R}}\underline{i}_t + (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}})i_t}{\theta_t^{\mathcal{R}}} - \pi_t\right] dt$
- $dr_t^{x,I} = (r_t^x + \tau_t^x)\, dt + \sigma_t^{q^K} dZ_t + \varphi\tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^D = i_t^D dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[i_t^D - \pi_t\right] dt$
- $dr_t^L = \frac{i^L}{P_t^L} dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right] dt + \sigma_t^{P^L} dZ_t$

Back

Monopolistic Firms

- Monopolistic producers add variety to a common good produced by HH:
  - Linear technology: $Y_t^j = y_t^j$, set prices P_t^j s.t. Rotemberg frictions:

$$\int_0^\infty \Xi_t^H \left[\left(\frac{P_t^j}{P_t} \right)^{1-\varepsilon} - p_t(1-\tau^F) \left(\frac{P_t^j}{P_t} \right)^{-\varepsilon} - \frac{\kappa}{2} \left(\pi_t^j \right)^2 - T_t^F \right] Y_t dt$$

- Perfectly competitive final good producers
  - Bundle varieties into consumption good using CES aggregator
- NKPC:

$$\frac{\mathbb{E}\left[d\pi_t\right]}{dt} = \left(r_t^{f,H} - \frac{\mathbb{E}\left[dY_t\right]}{Y_t dt} + \varsigma_t^{C,H} \sigma_t^Y \right) \pi_t - \frac{\varepsilon}{\kappa} \left(p_t(1-\tau) - \frac{\varepsilon - 1}{\varepsilon} \right)$$

$$\pi_t = \frac{\varepsilon}{\kappa Y_t} \mathbb{E}_t \int_t^\infty e^{-\int_t^s r_\tau^f d\tau} Y_s \left(p_s(1-\tau) - \frac{\varepsilon - 1}{\varepsilon} \right) ds$$

Back

Constrained Efficiency

- **Proposition 1.** Under some assumption on $\{\lambda, \varphi\}$, there exists a unique solution to the planner's problem for $\eta^*(\tilde{\sigma}_t) > \lambda$ and the constrained efficient allocation has the following properties:
  - Capital utilization $\upsilon^*(\tilde{\sigma}_t)$ is constant ($\mu_t^{\upsilon,*} = \sigma_t^{\upsilon,*} = 0$)
  - Intermediaries' wealth and risk shares $\eta^*(\tilde{\sigma}_t)$ and $\chi^*(\tilde{\sigma}_t)$ are increasing in $\tilde{\sigma}_t$
  - Nominal wealth share $\vartheta^*(\tilde{\sigma}_t)$ is increasing in $\tilde{\sigma}_t$
  - Investment rate $\iota^*(\tilde{\sigma}_t)$ is decreasing in $\tilde{\sigma}_t$

App

Net Worth and Risk Distributions

- **Net worth distribution:**

$$
\begin{tikzcd}
\boxed{N_t^H} & \boxed{N_t} \arrow[l, "1-\eta_t"'] \arrow[dl, "\eta_t"'] \arrow[r, "1-\vartheta_t"] \arrow[dr, "\vartheta_t"] & \boxed{q_t^K K_t} & & \\
\boxed{N_t^I} & & \boxed{\mathcal{B}_t/\mathcal{P}_t} \arrow[r, "1-\vartheta_t^L"] \arrow[dr, "\vartheta_t^L"] & \boxed{\mathcal{R}_t/\mathcal{P}_t} & \\
& & \boxed{\mathcal{B}_t = \mathcal{R}_t + P_t^L L_t} & \boxed{P_t^L L_t/\mathcal{P}_t} \arrow[r, "1-\alpha_t"] \arrow[dr, "\alpha_t"'] & \boxed{\text{Households}} \\
& & & & \boxed{\text{Intermediaries}}
\end{tikzcd}
$$

- **Idiosyncratic risk distribution:** χ_t held by Intermediaries

  $1-\chi_t$ held by Households

Equilibrium

- Key variables: $\tilde{\sigma}_t$, η_t, υ_t, ϑ_t, P^L_t, π_t
- Markovian equilibrium with state variables $S \equiv \{\tilde{\sigma}, \eta, \upsilon\}$:
  - Laws of motion for S:

$$d\tilde{\sigma}^2_t = -b_s(\tilde{\sigma}^2_t - \tilde{\sigma}^2_{ss})dt + \sigma\tilde{\sigma}^2_t dZ_t$$

$$\frac{d\eta_t}{\eta_t} = \mu^\eta_t dt + \sigma^\eta_t dZ_t$$

$$\frac{d\upsilon_t}{\upsilon_t} = \mu^\upsilon_t dt + \sigma^\upsilon_t dZ_t$$

  - Policy variables $\underline{i}(S)$, $i(S)$, $\vartheta^L(S)$, $\underline{\theta}^{\mathcal{R}}(S)$, $\tau^I(S)$, $\tau^x(S)$, $\tau^K(S)$
  - Mappings $\vartheta(S), P^L(S), \pi(S)$

satisfying agents' optimality and market clearing

Efficient Consumption & Risk Allocation Only: Implementation

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*)\sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^*(\alpha_t - \chi_t^*))\vartheta_t^L \sigma_t^{P^L}$$

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*}\sigma_t^{P^L} = \nu_t^2 \mathfrak{t}'(\nu_t)$$

$$\nu_t \left[\chi_t^* - \eta_t^* + \vartheta_t^*(1 - \chi_t^*) - (1 - \alpha_t)\vartheta_t^L \vartheta_t^*\right] = 1 - \vartheta_t^*$$

Back

Consolidated Government

$$\mu_t^{\mathcal{R}}\mathcal{R}_t + P_t^L\mu_t^L L_t + \mathcal{P}_t\tau_t^K K_t = \underline{i}_t\underline{\mathcal{R}}_t + i_t(\mathcal{R}_t - \underline{\mathcal{R}}_t) + i^L L_t - \sigma_t^{P^L}\sigma_t^L P_t^L L_t$$

$$\sigma_t^{\mathcal{R}}\mathcal{R}_t + P_t^L\sigma_t^L L_t = 0$$

Back

Production Efficiency Only: Implementation

- ϑ_t is an equilibrium 'mapping'

  $\Rightarrow$ implement ϑ_t^* by appropriate capital taxes τ_t^K along the equilibrium path
- υ_t is a state variable $\Rightarrow$ need to ensure $\mu_t^\upsilon = \sigma_t^\upsilon = 0 \ \forall t$
  - Drift is targeted by $\underline{i}_t$
  - Volatility loading is targeted by i_t and ϑ_t^L
- From goods market clearing:

$$a\upsilon_t = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \iota_t = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \frac{q_t^K - 1}{\phi} = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \frac{q_t^{\mathcal{B}}(1-\vartheta_t)}{\phi \vartheta_t} - \frac{1}{\phi}$$

$$q_t^{\mathcal{B}} = \frac{\mathcal{B}_t}{\mathcal{P}_t K_t} \qquad \mathcal{B}_t = \mathcal{R}_t + P_t^L L_t$$

back