Economies with Financial Frictions

Key Techniques

Princeton Initiative

1

What this class is

A mental map for continuous-time macro-finance.

You have an idea and are trying to write a model – how can you predict which way it will go?

- In what way will it be innovative?
- Will it be easy to/possible to solve?
- What is the map between assumptions and results?
- How is it related to the field.

The landscape, rather than one point in it.

How to take complex ideas, and find a simple way to think about them?

2

Expertise is a matter of representation

1

Chess

De Groot: masters did not search deeper or consider more moves. Their first candidate move was usually the right one.

2

Physics problems

Chi, Feltovich & Glaser: novices sort by inclined planes and pulleys. Experts sort by conservation of energy.

3

Macro-finance

We read papers one at a time, and each is framed as unique — so we miss the forest. Underneath sit deeper questions: what are we studying, and what does our methodology let us see, or hide?

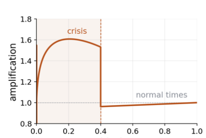
What improves with practice is the representation itself — not speed, not memory.

3

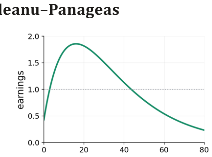
Seven models

Basak–Cuoco the ancestor — the simplest of the seven

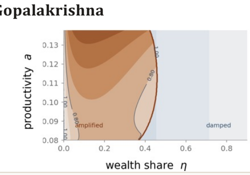
Brunnermeier–Sannikov



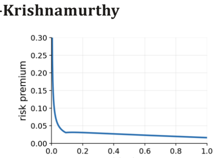
Gârleanu–Panageas



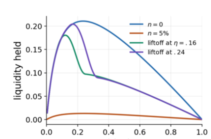
Gopalakrishna



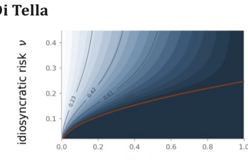
He–Krishnamurthy



Drechsler–Savov–Schnabl



Di Tella



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Brunnermeier–Sannikov

the main example

experts

output per unit of capital $a - \iota$

CRRA γ, discount ρ

hold capital, issue debt

households

$\underline{a} - \iota$, $\underline{a} \le a$

$\bar{\gamma}$, discount $\underline{\rho} < \rho$

hold debt, sometimes capital

$$\frac{dk_t}{k_t} = \big(\Phi(\iota_t) - \delta\big)dt + \sigma\, dZ_t, \quad \Phi(\iota) = \tfrac{1}{\kappa}\log(1+\kappa\iota), \qquad \frac{dq_t}{q_t} = \mu_t^q\, dt + \sigma_t^q\, dZ_t$$

5

What an equilibrium is

A description of everything happening in an economy, subject to the equilibrium conditions:

market clearing	optimisation by individual agents

Everything is a function of η, wealth share — an equilibrium is a map from Brownian shock histories to all quantities

$$\frac{dK_t}{K_t} = \big(\Phi(\iota) - \delta\big)dt + \sigma\, dZ_t, \qquad \eta_t = \frac{N_t}{q_t K_t}$$

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The blocks

the equilibrium conditions

1 consumption, prices of risk $\frac{C}{N}$ from J_n, $\varsigma = -\sigma^{J_n}$

value functions in

2 goods market $\frac{C}{K} + \frac{\bar{C}}{K} = a\psi + \underline{a}(1-\psi) - \iota(q)$

3 asset allocation

$$\frac{a - \underline{a}}{q} \geq (\varsigma - \bar{\varsigma})(\sigma + \sigma^q)$$

with equality when $\psi < 1$

4 laws of motion of N and qK

$$\frac{dN_t}{N_t} = r\,dt + \sigma^N(dZ_t + \varsigma\,dt) - \frac{C}{N}dt$$

$$\frac{d(q_t K_t)}{q_t K_t} = r\,dt + (\sigma + \sigma^q)dZ_t + [\psi\varsigma + (1-\psi)\bar{\varsigma}](\sigma + \sigma^q)dt - \frac{a\psi + \underline{a}(1-\psi) - \iota}{q}dt$$

$$\frac{d\eta_t}{\eta_t} = \mu^\eta dt + \sigma^\eta dZ_t$$

5 endogenous risk $\sigma^q q = q_\eta \sigma^\eta \eta$

value functions out

6 time step $Rv = G + \mathrm{MU}\,v_\eta + \frac{1}{2}S^2 v_{\eta\eta} + v_t$

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1 Consumption and prices of risk

value functions are summaries of the future

Two choices — consumption, and portfolio weights. Consumption/MU volatility determines the price of risk

$$\frac{C}{N} = \rho, \quad \frac{\bar{C}}{\bar{N}} = \underline{\rho}, \qquad \varsigma = \sigma^N = \frac{\psi}{\eta}(\sigma + \sigma^q), \qquad \bar{\varsigma} = \sigma^{\bar{N}} = \frac{1-\psi}{1-\eta}(\sigma + \sigma^q)$$

With CRRA : C/N and the price of risk picks up the value functions.
Under Epstein–Zin the elasticity ε governs C/N and γ governs the price of risk, separately.

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2 Goods market

total demand for consumption equals total output

$$\underbrace{\rho}_{C/N}\eta q + \underbrace{\underline{\rho}}_{\bar{C}/\bar{N}}(1-\eta)q \;=\; a\psi + \underline{a}(1-\psi) - \iota(q), \qquad \iota(q) = \frac{q-1}{\kappa}$$

Market clearing determines the value of assets

- People want to consume more — their value functions imply a higher C/N
- To clear the market, today's wealth must drop: q falls
- A lower q discourages / makes more expensive today's consumption

With CRRA the left side rises like q^(1/γ) instead of linearly; under Epstein–Zin, like q^ε.
The lower the elasticity, the further q must travel to clear the market.

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3 Asset allocation

who bears which risk

Experts are more productive, so all else equal they should hold all the capital.

The more capital they hold, the higher their price of risk — especially when their wealth share is low.

$$\frac{a-\underline{a}}{q} \;\geq\; \left(\varsigma - \bar{\varsigma}\right)\left(\sigma + \sigma^{q}\right) \qquad \text{with equality when } \psi < 1$$

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4 Laws of motion of N and qK

any portfolio: the risk-free rate, plus risk, plus the price of that risk

$$\frac{dN_t}{N_t} = \underbrace{r\,dt}_{\text{risk-free}} + \underbrace{\frac{\psi}{\eta}(\sigma+\sigma^q)\,dZ_t}_{\text{risk}} + \underbrace{\frac{\psi}{\eta}(\sigma+\sigma^q)\,\varsigma\,dt}_{\text{price of risk}} - \underbrace{\rho\,dt}_{\text{withdrawn}}$$

$$\frac{d(q_tK_t)}{q_tK_t} = r\,dt + (\sigma+\sigma^q)\,dZ_t + \left[\psi\varsigma + (1-\psi)\bar{\xi}\right](\sigma+\sigma^q)\,dt - \frac{a\psi + \underline{a}(1-\psi) - \iota}{q}\,dt$$

$$\frac{d\eta_t}{\eta_t} = \mu^\eta\,dt + \sigma^\eta\,dZ_t, \qquad \sigma^\eta\eta = (\psi-\eta)(\sigma+\sigma^q),$$

$$\mu^\eta\eta = \eta\left[\frac{a\psi + \underline{a}(1-\psi) - \iota}{q} - \rho\right] + \left[\psi(1-\eta)\varsigma - \eta(1-\psi)\bar{\xi}\right](\sigma+\sigma^q) - \sigma^\eta\eta\,(\sigma+\sigma^q)$$

You never need to know what the portfolio holds — only how much risk it carries, and at what price.
C/N always goes last: it is the one term that is not a return.

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5 Endogenous risk

the distinguishing block — and the meat of these models

A shock hits the wealth of certain agents, which feeds into prices, which affects wealth again. A loop.

$$\sigma^q q = q_\eta \underbrace{(\psi-\eta)(\sigma+\sigma^q)}_{\sigma^\eta\eta} \qquad \Longrightarrow \qquad \frac{\sigma+\sigma^q}{\sigma} = \frac{1}{1 - \frac{q_\eta}{q}(\psi-\eta)}$$

σ^q enters on both sides, and the gradient of q matters.

In a fully dynamic model the strength varies over the cycle — quiet in normal times, violent in crises.

Bernanke–Gertler–Gilchrist · Kiyotaki–Moore

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Solving the model with log utility – an ODE for q, from q(0)

(1) goods	$\rho\eta q + \underline{\rho}(1-\eta)q = a\psi + \underline{a}(1-\psi) - \iota(q)$	$\longrightarrow \psi$
(2) allocation	$\dfrac{a-\underline{a}}{q} = \dfrac{\psi-\eta}{\eta(1-\eta)}(\sigma+\sigma^q)^2$	$\longrightarrow \sigma^q$
(3) endogenous risk	$\sigma^q q = q_\eta(\psi-\eta)(\sigma+\sigma^q)$	$\longrightarrow q_\eta$

Each condition is solved for the unknown that does not belong to it.

Where ψ reaches 1: allocation goes slack, the goods market gives q pointwise

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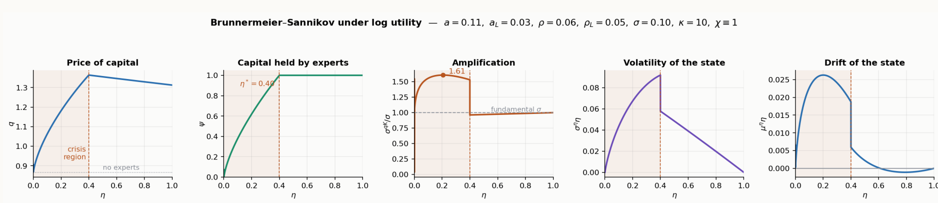
The solution



Above η* = 0.40 — experts hold everything, output is a, volatility is fundamental

Below it — ψ < 1, capital is misallocated, volatility rises to 1.6σ

Crossing in — σ^η η jumps up

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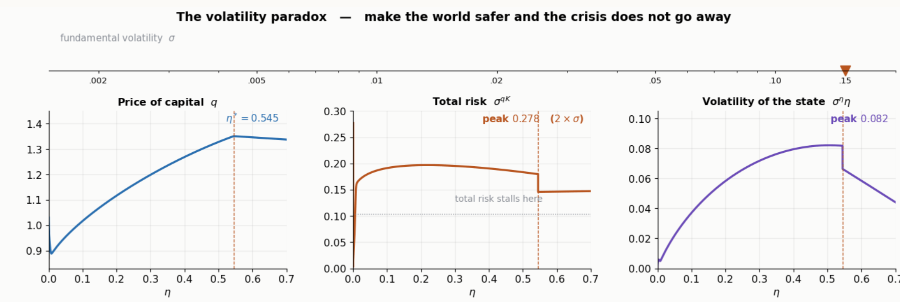
The volatility paradox



The allocation needs a certain amount of total risk to hold experts back. If the fundamental will not supply it, leverage rises until the price does.

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Two principles

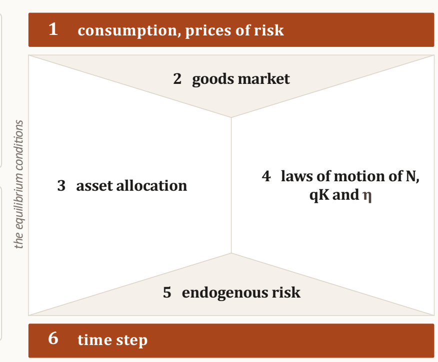
1 Have clarity in these blocks

They are interconnected, and you could cut them differently. From going through many models, this breakdown is the one that keeps working.

2 Resist the temptation to plug things in

Unless the model has a closed form, use these formulas in the right sequence rather than combining them. Errors localise; the diagnostic stays visible; the blocks are reusable.

The exception is closed form. If you can solve it on paper, substitute freely.



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Asset pricing

the one piece of theory everything rests on

For any self-financing trading strategy with value A_t :

$$A_t\,\xi_t \quad \text{is a martingale,} \qquad \xi_t = e^{-\rho t}\,u'(c_t)$$

ξ is marginal utility, discounted — the stochastic discount factor.

Valuation the price of capital q

Allocation who holds what — with equality when an agent holds a positive amount

Evolution of wealth, and of the wealth distribution

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Why this is the right thing to do

It is the first-order condition for how much wealth to put into a risky asset.

> If it does not hold, the agent can raise his utility by investing a little more — or a little less — in that asset.

So a price of risk is the compensation per unit of exposure at which he is content with what he already holds.

$$\varsigma \;=\; -\sigma^{\xi} \;=\; -\sigma^{J_n} \qquad \text{(and under CRRA, } \varsigma = \gamma\sigma^{c}\text{)}$$

Everything in Block 1 is that FOC, written for the models in front of us.

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CRRA utility: blocks 1 and 6

1 consumption, prices of risk $\frac{C}{N}$ from J_n, $\varsigma = -\sigma^{J_n}$

value functions in

2 goods market $\frac{C}{K} + \frac{\bar{C}}{K} = a\psi + \underline{a}(1-\psi) - \iota(q)$

the equilibrium conditions

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$$\frac{d\eta_t}{\eta_t} = \mu^\eta dt + \sigma^\eta dZ_t$$

5 endogenous risk $\sigma^q q = q_\eta \sigma^\eta \eta$

value functions out

6 time step $Rv = G + \mathrm{MU}\,v_\eta + \frac{1}{2}S^2 v_{\eta\eta} + v_t$

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The value function

what is forced, what is chosen, and what the choice buys

General form. Homothetic markets and CRRA: double your wealth and the whole plan doubles

$$J(\lambda n, \eta) = \lambda^{1-\gamma} J(n, \eta) \qquad \Longrightarrow \qquad J(n, \eta) = \frac{n^{1-\gamma}}{1-\gamma} h(\eta)$$

Normalization.

$$J = \frac{1}{1-\gamma}\left(\frac{n}{\eta q}\right)^{1-\gamma} v(\eta), \qquad \bar{J} = \frac{1}{1-\bar{\gamma}}\left(\frac{n}{(1-\eta)q}\right)^{1-\bar{\gamma}} \underline{v}(\eta)$$

What it buys. Representative agent utility

$$n = N = \eta q K \qquad \Longrightarrow \qquad J = \frac{K^{1-\gamma}}{1-\gamma} v(\eta)$$

One state variable in v · a constant discount rate in the time step · a clean power for C/K

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What comes off it

two choices, two outputs — Block 1 under CRRA

C / K The first-order condition for consumption.

$$c^{-\gamma} = J_n = n^{-\gamma}(\eta q)^{\gamma-1} v \qquad \Longrightarrow \qquad \frac{C}{N} = \frac{1}{\eta q}\left(\frac{\eta q}{v}\right)^{1/\gamma}, \qquad \frac{C}{K} = \left(\frac{\eta q}{v}\right)^{1/\gamma}$$

Under log, with v constant.

$$\gamma = 1,\ v = \frac{1}{\rho}: \qquad \frac{C}{N} = \rho, \qquad \frac{C}{K} = \rho\,\eta q$$

ς The price of risk — the volatility of that same line.

$$\varsigma = -\sigma^{J_n} = \gamma\sigma^N - (\gamma - 1)\,\sigma^{\eta q} - \sigma^v, \qquad \sigma^{\eta q} = \sigma^N - \sigma$$

$$\Longrightarrow \qquad \varsigma = \sigma^N + (\gamma - 1)\,\sigma - \sigma^v$$

$$\sigma^v = \frac{v_\eta}{v}\sigma^\eta \eta$$ — and $\sigma^\eta \eta$ is not known until Step 5

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Solving the model with CRRA – the same ODE for q

each condition still solved for the unknown that does not belong to it

(1) goods $\left(\frac{\eta q}{v}\right)^{1/\gamma} + \left(\frac{(1-\eta)q}{\underline{v}}\right)^{1/\bar{\gamma}} = a\psi + \underline{a}(1-\psi) - \iota(q) \longrightarrow \psi$

(2) allocation $\frac{a - \underline{a}}{q} = (\varsigma - \bar{\varsigma})(\sigma + \sigma^q) \longrightarrow \sigma^q$

(3) endogenous risk $\sigma^q q = q_\eta(\psi - \eta)(\sigma + \sigma^q) \longrightarrow q_\eta$

and the wedge — straight from Block 1

$$\varsigma - \bar{\varsigma} = (\sigma^N - \sigma^{\bar{N}}) + (\gamma - \bar{\gamma})\,\sigma - (\sigma^v - \sigma^{\underline{v}})$$

$$= \left[\frac{\psi}{\eta} - \frac{1-\psi}{1-\eta}\right](\sigma + \sigma^q) + (\gamma - \bar{\gamma})\,\sigma - \left(\frac{v_\eta}{v} - \frac{\underline{v}_\eta}{\underline{v}}\right)(\psi - \eta)(\sigma + \sigma^q)$$

Under log both marked terms vanish

Where ψ reaches 1: allocation goes slack, the goods market gives q pointwise

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6 Time step

the HJB for the sector — and the four coefficients the solver wants

Value of the sector

$$J = v(\eta)\,\frac{K_t^{1-\gamma}}{1-\gamma}$$

satisfies the Bellman equation

$$\rho\, v\,\frac{K^{1-\gamma}}{1-\gamma} = \frac{1}{1-\gamma}\left(\frac{C}{K}\right)^{1-\gamma} K^{1-\gamma} + \left[\mu^{\eta}\eta\, v_{\eta} + \tfrac{1}{2}\left(\sigma^{\eta}\eta\right)^{2} v_{\eta\eta}\right]\frac{K^{1-\gamma}}{1-\gamma}$$

$$+\, v\left[K^{-\gamma}\left(\Phi(\iota)-\delta\right)K - \tfrac{1}{2}\gamma K^{-\gamma-1}\sigma^{2}K^{2}\right] + \sigma^{\eta}\eta\, v_{\eta}\cdot \sigma K\cdot K^{-\gamma}$$

Divide through by $K^{1-\gamma}/(1-\gamma)$ — and every K disappears

$$\rho\, v = \left(\frac{C}{K}\right)^{1-\gamma} + \left[\mu^{\eta}\eta + (1-\gamma)\,\sigma\,\sigma^{\eta}\eta\right] v_{\eta} + \tfrac{1}{2}\left(\sigma^{\eta}\eta\right)^{2} v_{\eta\eta} + (1-\gamma)\left(\Phi(\iota) - \delta - \frac{\gamma}{2}\sigma^{2}\right) v$$

Match the coefficients of the solver

```
R(X)*F(t,X) = G(X) + MU(X)*F_x(t,X) + S(X)^2*F_xx(t,X)/2 + F_t(t,X)
```

$$R = \rho - (1-\gamma)\left(\Phi(\iota) - \delta - \frac{\gamma}{2}\sigma^{2}\right), \quad G = \left(\frac{C}{K}\right)^{1-\gamma}, \quad \mathrm{MU} = \mu^{\eta}\eta + (1-\gamma)\,\sigma\,\sigma^{\eta}\eta, \quad S = \sigma^{\eta}\eta$$

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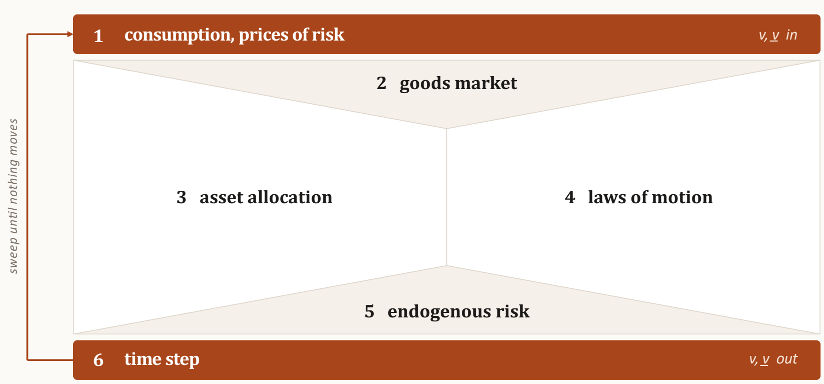
The outer loop

value functions in, value functions out



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The blocks are the code

one sweep of brusan.py, in the order this model extracts them

```
# 1  consumption      C/K = (eta q / v)**(1/gamma) = A q**(1/gamma)
A, Ab = (eta/v)**(1/g), ((1-eta)/vb)**(1/gb)
DV    = grad(v)/v - grad(vb)/vb

# 2, 3, 5  marched together in eta
q, psi, sqK, qe = march(A, Ab, DV)

# 1  prices of risk       they need sigma^eta eta, so block 1 returns
se  = (psi - eta)*sqK
vs  = psi/eta*sqK         + (g -1)*sig - grad(v)/v  *se
vsb = (1-psi)/(1-eta)*sqK + (gb-1)*sig - grad(vb)/vb*se

# 4  law of motion of eta
CK, CKb = A*q**(1/g), Ab*q**(1/gb)
mu = drift(q, psi, sqK, vs, vsb, CK)

# 6  time step
v  = step(rate(q, g,  rho ), mu + (1-g )*sig*se, se, CK **(1-g ), v )
vb = step(rate(q, gb, rhob), mu + (1-gb)*sig*se, se, CKb**(1-gb), vb)
```

1 → (2, 3, 5) → 1 → 4 → 6

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The march is three residuals

three conditions, three unknowns, one Newton step per node

```
def resid(u):
    qq, pp, ss = u                   # q, psi, sigma^qK
    x = pp - e                       # sigma^eta eta / sigma^qK
    wedge = (pp/e - (1-pp)/(1-e) - x*dv)*ss + (g-gb)*sig
    return np.array([
        A[n]*qq**(1/g) + Ab[n]*qq**(1/gb) - (a*pp + ab*(1-pp) - iota(qq)),
        ss*(qq - (qq - qp)*x/dn) - sig*qq,
        (a - ab)/qq - wedge*ss])
```

2 goods market
5 endogenous risk
3 asset allocation

$q\eta$ is the backward difference (q − q_prev)/d. That is what makes this a march rather than a pointwise solve — and it is the only place the grid enters the economics.

Where ψ reaches 1 the allocation goes slack: the goods market delivers q pointwise, and endogenous risk runs in its natural direction, delivering $\sigma + \sigma q$.

Six Newton iterations per node, a thousand nodes, seven hundred sweeps.

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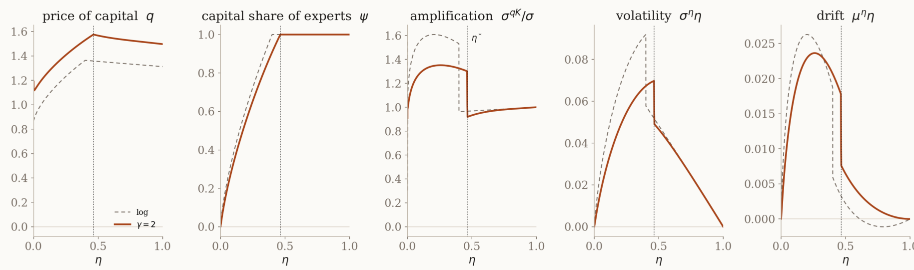
Brunnermeier–Sannikov with γ = 2

the prepared code, run — the log solution behind it



η^* moves out to 0.46, and the peak of the amplification falls to 1.35.

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Basak–Cuoco 1998

the ancestor — the simplest of the seven

significantly new · adapted · unchanged

An endowment economy in which one group simply may not hold the risky asset.

1 consumption, prices of risk *one value function — the excluded type is logarithmic*

2 goods market $\left(\frac{\eta q}{v}\right)^{1/\gamma} + \rho\,(1-\eta)\,q = a$

3 asset allocation

$\chi \equiv 1$

$$\sigma^N = \frac{\sigma^{qK}}{\eta}, \qquad \sigma^\eta \eta = (1-\eta)\,\sigma^{qK}$$

4 laws of motion

$$\mu^\eta \eta = \eta\left[\frac{a}{q} - \frac{C}{N}\right] + \sigma^\eta \eta\,(\varsigma - \sigma^{qK})$$

5 endogenous risk $\sigma^q q = q_\eta \underbrace{(1-\eta)\,\sigma^{qK}}_{\sigma^\eta \eta}$

6 time step → v

28

He–Krishnamurthy 2013

a constraint that binds only in the crisis region

significantly new · adapted · unchanged

Households reach capital only through intermediaries, can hold intermediary equity only without leverage, bound on how much risk intermediaries can offload.

1 consumption, prices of risk — *one value function — households are logarithmic*

2 goods market $\left(\frac{\eta q}{v}\right)^{1/\gamma} + \underline{\rho}\,(1-\eta)\,q = a + \ell$

3 asset allocation

$$\frac{1-\chi}{(1-\lambda)(1-\eta)} \le \frac{\chi}{\eta}$$

$$\chi = \max\left\{\underline{\chi},\ \frac{\eta}{1-\lambda+\lambda\eta}\right\}$$

4 laws of motion

$$\sigma^\eta \eta = (\chi-\eta)\,\sigma^{qK}$$

$$\mu^\eta \eta = \eta\left[\frac{a}{q} - \frac{C}{N}\right] + \sigma^\eta \eta\,(\varsigma - \sigma^{qK})$$

5 endogenous risk $\sigma^q q = q_\eta \underbrace{(\chi-\eta)\,\sigma^{qK}}_{\sigma^\eta \eta}$

6 time step — → v

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Gârleanu–Panageas 2015

no friction anywhere — and still a wealth distribution that matters

significantly new · adapted · unchanged

Overlapping generations of two types who differ in risk aversion and in elasticity of substitution. Aggregate risk is shared perfectly.

1 consumption, prices of risk — *Epstein–Zin, two value functions*

2 goods market $\frac{C}{K} + \frac{\bar{C}}{K} = a$

3 asset allocation

$$\varsigma = \bar{\varsigma}$$

$$\left[1 - \eta(1-\eta)\left(\frac{v_\eta}{v} - \frac{\underline{v}_\eta}{\underline{v}}\right)\right]\sigma^\eta \eta = \eta(1-\eta)\,(\bar{\gamma}-\gamma)\,\sigma$$

4 laws of motion

$$\mu^\eta \eta = \eta\left[\frac{a}{q} - \frac{C}{N}\right] + \sigma^\eta \eta\,(\varsigma - \sigma^{qK}) + \delta\,\frac{h_0}{q}(v-\eta)$$

5 endogenous risk $\sigma^q q = q_\eta\, \sigma^\eta \eta$

6 time step — → v, $\underline{v}$, and h_0 — the price of a newborn's income stream

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Drechsler–Savov–Schnabl 2018

the wedge is a policy instrument

significantly new · *adapted* · *unchanged*

Banks, with much lower risk aversion than households, accept deposits and hold a liquid asset — provided by the monetary authority — to offset withdrawal risk.

1 consumption, prices of risk — *Epstein–Zin, two value functions*

2 goods market $\frac{C}{K} + \frac{\bar{C}}{K} = a$ $\Lambda = \lambda(\psi - \eta)$

3 asset allocation

$$\bar{\varsigma} - \varsigma = \frac{\lambda n/m}{\sigma^{qK}}$$

$$\sigma^\eta \eta = \eta(1-\eta)\left[(\bar{\gamma} - \gamma)\sigma + \sigma^{v} - \sigma^{\underline{v}} - \frac{\lambda n/m}{\sigma^{qK}}\right]$$

4 laws of motion

$$\frac{dN_t}{N_t} = r\,dt + \underbrace{\sigma^{qK}(dZ_t + \bar{\varsigma}\,dt)}_{\text{unlevered: untaxed}} + \underbrace{\frac{\psi - \eta}{\eta}\sigma^{qK}(dZ_t + \varsigma\,dt)}_{\text{funded with deposits: taxed}}$$

$$+ \frac{\Lambda n}{m}dt + \kappa\left(\frac{\bar{\eta}}{\eta} - 1\right)dt - \frac{C}{N}dt$$

$$\mu^\eta \eta = \eta\left[\frac{a}{q} - \frac{C}{N} + \frac{\Lambda n}{m}\right] + \sigma^\eta \eta(\varsigma - \sigma^{qK}) + \kappa(\bar{\eta} - \eta)$$

5 endogenous risk $\sigma^q q = q_\eta \underbrace{(\psi - \eta)\sigma^{qK}}_{\sigma^\eta \eta}$, $\sigma^N = \frac{\psi}{\eta}\sigma^{qK}$

6 time step → v, $\underline{v}$

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Di Tella 2017

the only model here with idiosyncratic risk

significantly new · *adapted* · *unchanged*

Idiosyncratic risk that cannot be shared perfectly; the aggregate risk market is complete. The level of idiosyncratic risk is a second, exogenous state variable.

1 consumption, prices of risk — *two value functions, and two more prices of risk — for the idiosyncratic shock*

2 goods market $\frac{C}{K} + \frac{\bar{C}}{K} = a\psi + \underline{a}(1-\psi) - \iota(q)$

3 asset allocation

$$\frac{a - \underline{a}}{q} \geq (\tilde{\varsigma} - \bar{\tilde{\varsigma}})\phi\nu = \phi^2\nu^2\left[\frac{\gamma\psi}{\eta} - \frac{\bar{\gamma}(1-\psi)}{1-\eta}\right], \qquad \text{equality if } \psi < 1$$

$$\psi = \min\left\{\frac{(a - \underline{a})/(q\phi^2\nu^2) + \bar{\gamma}/(1-\eta)}{\gamma/\eta + \bar{\gamma}/(1-\eta)},\ 1\right\}$$

Di Tella is the corner $\underline{a} \to -\infty$: $\psi \equiv 1$ — households cannot operate capital at all

4 laws of motion

$$\frac{dN_t}{N_t} = r\,dt + \sigma^N(dZ_t + \varsigma\,dt) + \underbrace{\frac{\psi}{\eta}\phi\nu\tilde{\varsigma}\,dt}_{\text{idiosyncratic premium}} - \frac{C}{N}dt$$

$$\left[1 - \eta(1-\eta)\Delta_\eta\right]\sigma^\eta \eta = \eta(1-\eta)\left[(\bar{\gamma} - \gamma)\sigma + \Delta_\nu \sigma^\nu \nu\right]$$

$$\mu^\eta \eta = \sigma^\eta \eta(\varsigma - \sigma^{qK}) + \eta(1-\eta)\left[\frac{\tilde{\varsigma}^2}{\gamma} - \frac{\bar{\tilde{\varsigma}}^2}{\bar{\gamma}} + \frac{\bar{C}}{\bar{N}} - \frac{C}{N}\right]$$

5 endogenous risk $\sigma^q q = q_\eta \sigma^\eta \eta + q_\nu \sigma^\nu \nu$

6 time step → $v(\eta, \nu)$, $\underline{v}(\eta, \nu)$ — two-dimensional

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Gopalakrishna 2021

a second state — and crisis dynamics that are not in the one-state model

significantly new · *adapted* · *unchanged*

Productivity is a second, exogenous state. Every volatility becomes a vector, the allocation condition a dot product, and the march runs a whole row of the grid at a time.

1 consumption, prices of risk — *two value functions of (η, a)*

2 goods market $\frac{C}{K}+\frac{\bar{C}}{K} = a_t\,\psi + \underline{a}(1-\psi) - \iota(q)$

3 asset allocation

$$\chi = \max\{\underline{\chi},\ \eta\}$$

$$\frac{a_t-\underline{a}}{q} \ge \chi(\varsigma-\bar{\varsigma})\cdot\sigma^{qK}, \qquad \text{equality if } \psi<1$$

— a quadratic in $\|\sigma^{qK}\|$

4 laws of motion

$$\sigma^{\eta}\eta = (\chi\psi-\eta)\,\sigma^{qK} \qquad \text{— a vector}$$

$$\mu^{\eta}\eta = \eta\left[\frac{a_t\psi+\underline{a}(1-\psi)-\iota}{q} - \frac{C}{N}\right] - \tau(\eta,a)\,\eta$$
$$+\ \sigma^{\eta}\eta\cdot(\varsigma-\sigma^{qK}) + \eta(1-\chi\psi)(\varsigma-\bar{\varsigma})\cdot\sigma^{qK}$$

$$\tau(\eta,a)=\tau_c \text{ where } \psi<1, \quad \tau \text{ where } \psi=1$$

5 endogenous risk $\underbrace{\left(\sigma^{qK}-(\sigma,0)\right)}_{\sigma^{q}} q \;=\; q_\eta\,\underbrace{(\chi\psi-\eta)\,\sigma^{qK}}_{\sigma^{\eta}\eta} + q_a\,\sigma^{a}$

6 time step — *→ v(η, a), $\underline{v}$(η, a) — two-dimensional*

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What you will do

one hour, in groups — two questions, and one of them you answer before you touch the solver

1 The map

written, half a page

You have now seen seven models. In your own words, how are they related to one another?

If you had to organise them — a map, a tree, a table, whatever form suits you — how would you do it, and why that way?

There is no single right answer. We are interested in the organising principle you choose — and in what it buys you.

2 Your model

with the solver in front of you

a Which is your group's model?

b Before you touch it: which one or two parameters carry the most economics, and which quantities would show it? Write down direction and rough magnitude. Commit before you look.

c Now move them. What do you find — qualitatively, and with numbers off the panels? Where the prediction was wrong, say why. That is the interesting part.

d Do they move the curves, or move where the economy spends its time, or both? Which of the two is doing the work for the point your paper makes?

Commit before you look. The prediction is the exercise — the solver only settles it.

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Best of luck with your research

Build useful mental models (like the six-step model for economies with financial frictions) – this gives you speed, perspective, the ability to anticipate solution

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