Countercyclical Bank Regulation: A Quantitative Approach

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Markus Brunnermeier (Princeton)
Goutham Gopalakrishna (Toronto)
Sebastian Merkel (Bristol)

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Outline

1. Introduction
2. Model
3. Mechanisms
4. Planner problem
5. Quantitative Analysis
6. Conclusion

Introduction

Objectives

1. Study the **optimal** capital regulation in a macrofinance model with
   - ⋆ Heterogeneous agents with idiosyncratic + aggregate shocks and macro growth
   - ⋆ Regulatory requirements on capital
   - ⋆ Risk dynamics and growth-resilience trade off
2. Welfare analysis
   - ⋆ Decomposing welfare into default, growth, distribution, capital allocation effects
   - ⋆ Asymmetry of over- vs under-regulation
   - ⋆ Countercyclical regulation
3. Estimate the model using a machine learning based technique
   - ⋆ Build a surrogate model that maps the structural parameters to model moments
   - ⋆ Balances numerical accuracy on equilibrium solution with computational efficiency on estimation. Easily generalizes to models with non-trivial equilibrium selection

Results

- Growth-resilience trade off: tighter regulation implies

  $\star$ Amplification: less pronounced

  Recovery: faster

  $\left.\begin{array}{l}\text{Amplification: less pronounced}\\ \text{Recovery: faster}\end{array}\right\}$ Resilience $\uparrow$

  $\star$ Growth $\downarrow$

  Safe asset demand crowds-out capital investment

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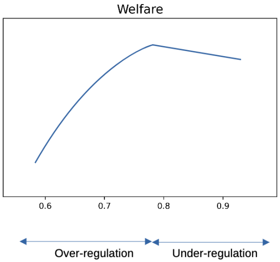

- Welfare analysis:
  - ⋆ Asymmetric welfare costs of over- vs under-regulation $\rightarrow$ **Gradualism.**
  - ⋆ Decomposition results: Idio. risk > Growth > Distribution > Capital allocation.
  - ⋆ Welfare gain from no regulation to optimal 6% requirement is 0.2%
  - ⋆ Countercyclical capital buffer increases welfare

Literature review (selected)

- Quantitative macro models with bank capital regulation

  [Abad-Martinez-Miera-Suarez, 2025], [Begenau, 2020], [Davydiuk, 2017],[Begenau-Landvoight, 2022],[Corbae-D'Erasmo, 2021], [Elenev-Landvoight-Nieuwerburgh, 2021], [Mendicino-Nikolov-Suarez-Supera, 2018] ...

  - ⋆ *This paper:* Asymmetric welfare costs, distributional implications.

- Dynamic banking and capital structure models
  [Gersbach-Rochet, 2017], [hugonnier-Morellec, 2017], [Sundaresan-Wang, 2023], ...

  - ⋆ *This paper:* Macro growth and asset price implications.

- Financial accelerator and intermediary asset pricing
  [Brunnermeier-Sannikov, 2014],[He-Krishnamurthy, 2013], [Krishnamurthy-Li, 2025] ....

  - ⋆ *This paper:* Studies optimal bank capital regulation.

- Idiosyncratic risk, I-theory and money.
  [Brunnermeier-Sannikov, 2016], [Merkel, 2020], [Alexandrov-Brunnermeier, 2025] ....

  - ⋆ *This paper:* Capital regulation.

→ $\boxed{\textit{Growth-Resilience trade-off, dispersion of recovery}}$ [Novel in this paper].

Model

Environment

- Continuous time $t \in [0, \infty)$. One perishable consumption good.
- Two types of agents:
  - ⋆ **Households**: operate firms, produce consumption goods. Firms own physical capital in
    - bank dependent sector, a fraction of risk shared with intermediaries
    - bank independent sector, a fraction of risk shared with other HH.

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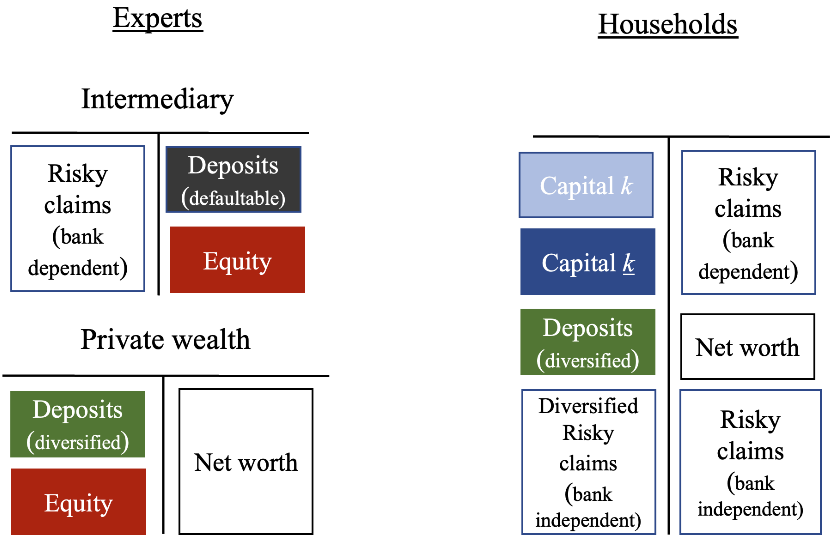
    - bank independent sector, a fraction of risk shared with other HH.
  - ⋆ **Experts**: operate intermediary firms with
    - *diversification* technology $\rightarrow$ absorbs idio. risk
    - *operational risk* $\rightarrow$ default risk

    Experts own intermediary equity, hold money in private wealth. Can open new intermediary upon default with pvt. wealth.



Experts
Intermediary
Risky claims (bank dependent)
Deposits (defaultable)
Equity
Private wealth
Deposits (diversified)
Equity
Net worth
Households
Capital $\bar{k}$
Capital $\underline{k}$
Deposits (diversified)
Diversified Risky claims (bank independent)
Risky claims (bank dependent)
Net worth
Risky claims (bank independent)

Environment

- AK production technology with bank dependent k_t and bank independent ($\underline{k}_t$) capital

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)\, dt + \sigma d Z_t + + \tilde{\sigma} d \tilde{Z}_t; \quad \frac{d\underline{k}_t}{\underline{k}_t} = (\Phi(\iota_t) - \delta)\, dt + \underline{\sigma} d \underline{Z}_t + \tilde{\sigma} d \tilde{Z}_t$$

where $Z_t, \underline{Z}_t$, and $\tilde{Z}_t$ are the aggregate shocks and idiosyncratic Brownian shocks

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  where $Z_t, \underline{Z}_t$, and $\tilde{Z}_t$ are the aggregate shocks and idiosyncratic Brownian shocks
- Total output is given by (Leontief)

$$Y_t = a \min \left\{ \frac{K_t}{\kappa}, \frac{\underline{K}_t}{1-\kappa} \right\} K_t = aK_t$$

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta_t)\, dt + \boldsymbol{\sigma}^{\boldsymbol{K}\top} \boldsymbol{dZ_t}; \quad \boldsymbol{\sigma}^{\boldsymbol{K}\top} = [\kappa\sigma \quad (1-\kappa)\sigma]^\top; \quad \boldsymbol{dZ_t} = [Z_t \quad \underline{Z}_t]^\top$$

Environment

- Intermediaries hold risky claims of bank dependent sector issued by firms
  - Diversify $\tilde{Z}_t$ shocks
  - Face $\tilde{J}_t$ shocks w.p. $\tilde{\lambda}_j$ of size $\tilde{j} \sim U[\underline{j}, 0]$
- Return on risky claims

$$\text{[Firms]}: \quad dR_t^{h,L} = r_t^L dt + \left(\boldsymbol{\sigma} + \boldsymbol{\sigma}_t^{q^K}\right)^\top \boldsymbol{dZ_t} + \tilde{\sigma} d\tilde{Z}_t$$

$$\text{[Intermediaries]}: \quad dR_t^{I,L} = r_t^L dt + \left(\underline{\boldsymbol{\sigma}} + \boldsymbol{\sigma}_t^{q^K}\right)^\top \boldsymbol{dZ_t} + \left(\tilde{j} - \mathcal{D}1_{\tilde{j}<\hat{j}}\right) d\tilde{J}_t - E[\tilde{j}]\tilde{\lambda}_j dt$$

where $\mathcal{D}$ is the default cost paid by depositors, and $\hat{j}$ is the endogenous default boundary

- When an intermediary defaults, it can use private wealth to open up a new bank
- Default destroys capital growth: $\delta_t = \delta + \tilde{\lambda}_j \mathcal{D}\mathbb{P}(\tilde{j} < \hat{j})\chi_t$, where χ_t is the fraction of outside equity firm issues on bank dependent sector

Environment

- Govt. issues outside money whose nominal value is $\mathcal{MB}$
- The real price of bonds (per unit of K_t) is denoted by $q_t^B = \frac{\mathcal{MB}}{\mathcal{P}_t K_t}$

  The real price of capital is $q_t^K K_t$. Total wealth is $N_t = q_t^B K_t + q_t^K K_t$

Households

- Operates firms, holds capital, deposits
  - ⋆ [Bank dependent k_t]: Issues outside equity χ_t to I subject to constraint $\chi_t \leq \overline{\chi}$
  - ⋆ [Bank independent $\underline{k}_t$]: Keeps χ_x, holds rest in diversified index fund
- Maximize lifetime utility subject to constraints

  Choose portfolio allocations (capital, deposit, outside equity), investment ι_t

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Formally, the problem is

$$\max_{c_t^h, \iota_t^h, \theta_t^{h,D}, \theta_t^k, \theta_t^{h,L}, \underline{\theta}_t^k, \underline{\theta}_t^{h,L}, \underline{\check{\theta}}_t^{h,L}} E_0 \left[\int_0^\infty e^{-\rho t} \log c_t^h dt \right]$$

subject to wealth dynamics

FOCs Returns

$$\frac{dn_t^h}{n_t^h} = -\frac{c_t^h}{n_t^h} dt + \theta_t^{h,D} dR_t^D + \theta_t^k dR_t^k(\iota_t) + \theta_t^{h,L} dR_t^{h,L} + \underline{\theta}_t^k \underline{dR}_t^k(\iota_t) + \underline{\theta}_t^{h,L} \underline{dR}_t^{h,L} + \underline{\check{\theta}}_t^{h,L} \underline{d\check{R}}_t^{h,L}$$

$$\frac{\chi_t}{\kappa} = -\frac{\theta_t^{h,L}}{\theta_t^y} \leq \frac{\overline{\chi}}{\kappa}; \qquad -\frac{\underline{\theta}_t^{h,L}}{\underline{\theta}_t^k} \leq \frac{\overline{\chi}_x}{1-\kappa}$$

Experts

- Do not produce. Holds intermediary equity invested in risky firms financed by deposits
- Keep private wealth outside of intermediary equity invested in money
- Maximize lifetime utility subject to constraints

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- Keep private wealth outside of intermediary equity invested in money
- Maximize lifetime utility subject to constraints

Formally, they solve

FOCs

$$\max_{c_t^I, \iota_t, \theta_t^E, \theta_t^I} E_0 \left[\int_0^\infty e^{-\rho t} \log c_t^I dt \right]$$

subject to

$$\frac{dn_t^I}{n_t^I} = -\frac{c_t^I}{n_t^I} dt + (1-\theta_t^E) dR_t^{\mathcal{MB}} + \theta_t^E dR_t^E; \quad dR_t^E = \theta_t^I \underbrace{dR_t^{I,L}}_{= \int_i dR_t^{h,L}(i) + \text{jump risk}} + (1-\theta_t^I) dR_t^{D,i}$$

$$\theta_t^I \leq \frac{1}{1-\ell}; \text{ where } \ell \text{ is max leverage.}$$

Model Dynamics

Evolution of state variable

$$\frac{d\eta_t}{\eta_t} = \left[-\rho + \hat{r}_t^B + (1-\theta_t^E)\hat{r}_t^B + \theta_t^E \hat{r}_t^{OE}\right] dt + \boldsymbol{\sigma}_t^{\boldsymbol{\eta}\top} \boldsymbol{dZ_t}$$

The equilibrium functions depend on the share of nominal wealth in bonds ϑ_t given by

$$\vartheta_t = \underbrace{\mathbb{E}_t \left[\int_t^\infty e^{-\rho(t'-t)} \left(\eta_{t'} (\boldsymbol{\sigma}_{t'}^{\boldsymbol{\eta}\top} \boldsymbol{\sigma}_{t'}^{\boldsymbol{\eta}}) + (1-\eta_{t'})(\boldsymbol{\sigma}_{t'}^{\boldsymbol{1-\eta}\top} \boldsymbol{\sigma}_{t'}^{\boldsymbol{1-\eta}}) \right) dt' \right]}_{\text{Aggregate terms}}$$

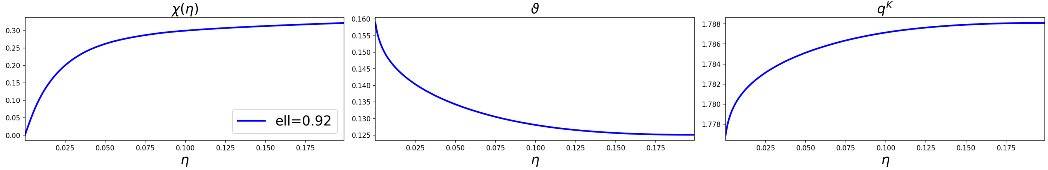
$$+ \underbrace{\mathbb{E}_t \left[\int_t^\infty e^{-\rho(t'-t)} \left(\eta_{t'} \tilde{j}_{t'}^{\eta} + (1-\eta_{t'})(\tilde{\sigma}_{t'}^{1-\eta})^2 \right) dt' \right]}_{\text{Idiosyncratic terms}}$$

↳ Larger aggregate and/or idiosyncratic risk push up safe asset demand

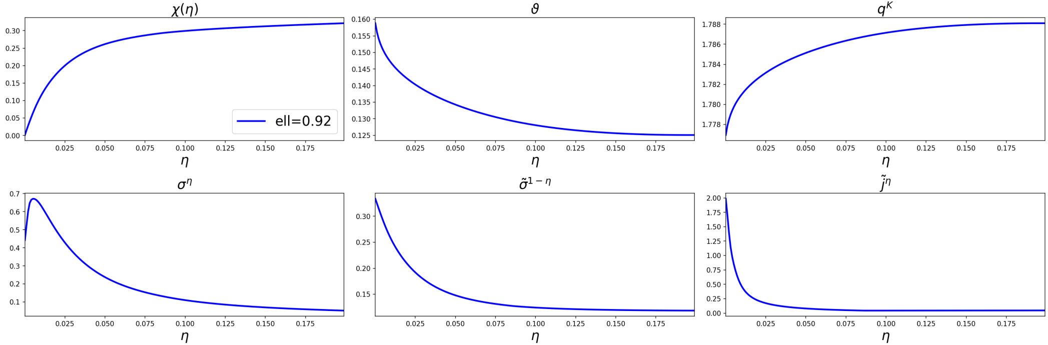
Equilibrium

Mechanisms

Model solution



Model solution



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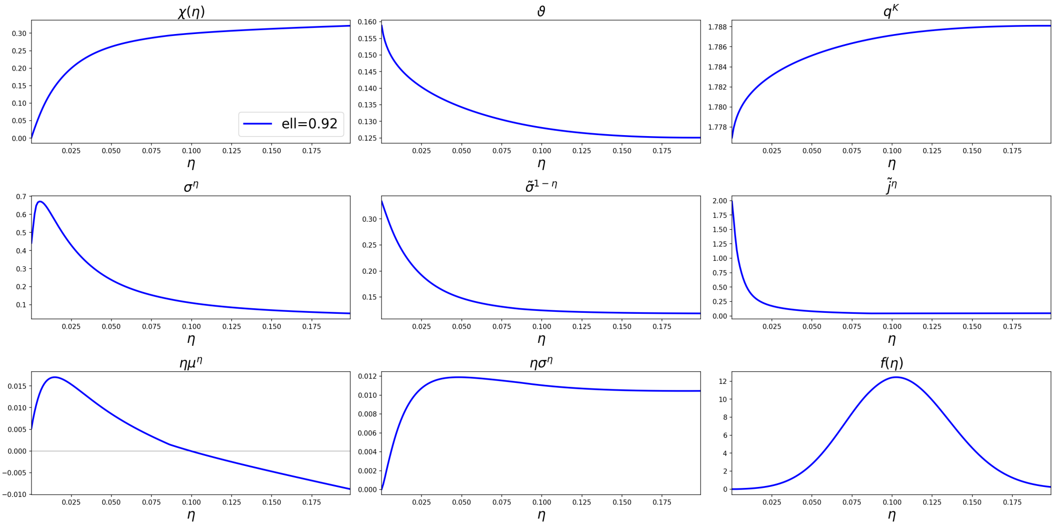
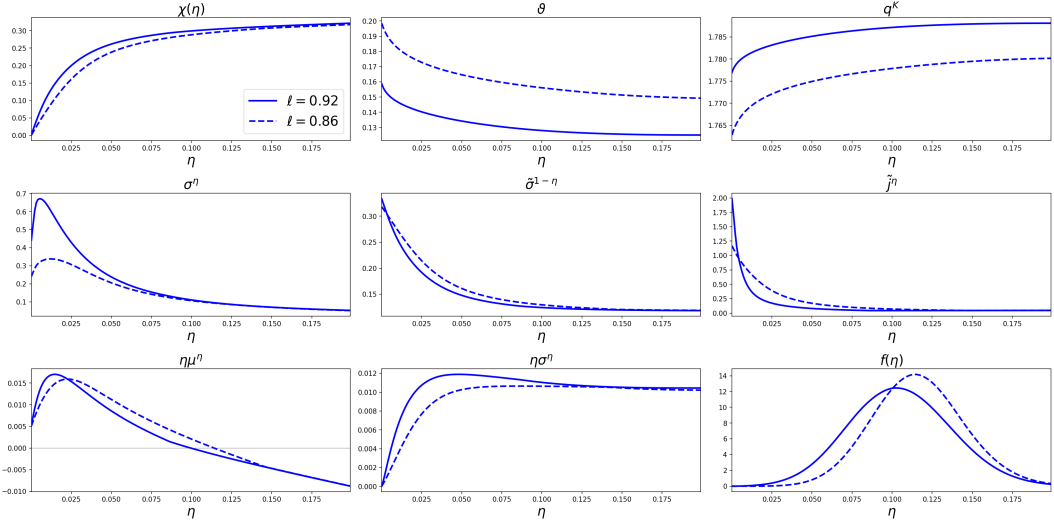
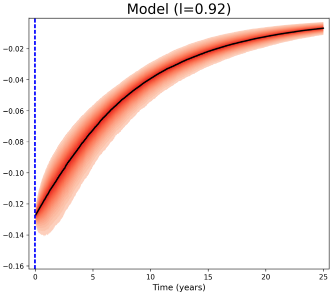


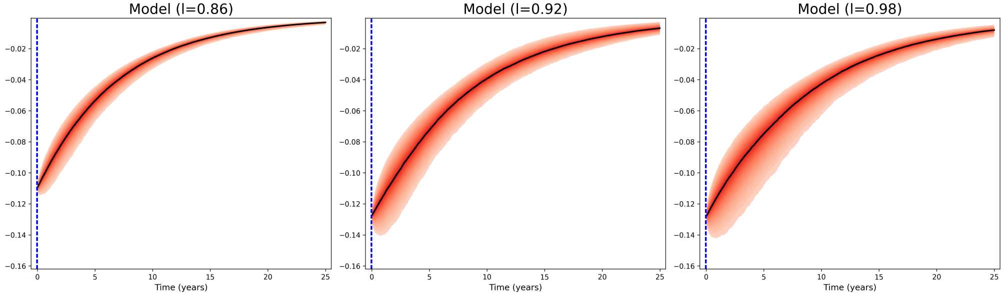
Model solution



Resilience


Model (l=0.92)
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
0
5
10
15
20
25
Time (years)

Resilience

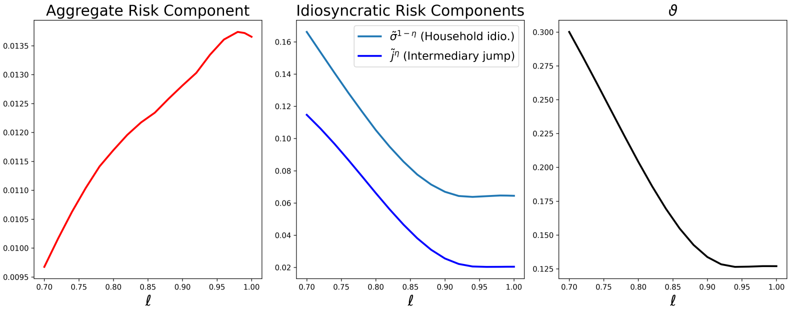


Tighter capital regulation leads to

- Amplification ↓ vs Recovery ↑ and less dispersed

Safe asset demand & growth

Aggregate risk + HH idiosyncratic risk + intermediary idiosyncratic jump risk = Safe asset



- Tight regulation $\implies \downarrow$ aggregate risk but $\uparrow$ idiosyncratic risk $\implies \uparrow$ safe asset demand
- More safe asset demand $\rightarrow \downarrow$ investment

$$\iota_t = \frac{q_t^K - 1}{\phi} = (1 + a\phi)\frac{1 - \vartheta_t}{1 - \vartheta_t + \phi\rho}$$

Welfare

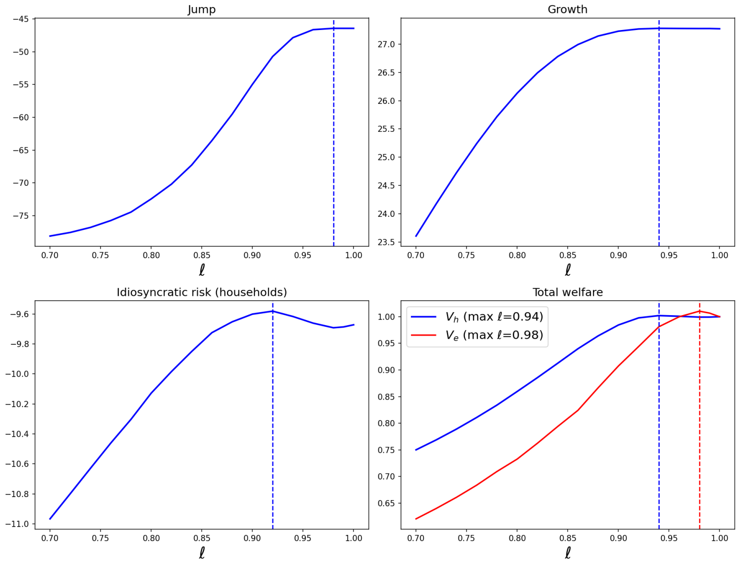
Total welfare: $W = \int_0^{1/2} W^e(i)di + \int_{1/2}^1 W^h(i)di$.
Welfare of agent i is

$$W(i) = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right]$$
$$= \mathbb{E}\left[\int_0^\infty e^{-\rho t} \left(\underbrace{\log \check{\eta}_t{}^i}_{\text{Distribution}} + \underbrace{\log A(\kappa_t)}_{\text{Capital allocation}} + \underbrace{\frac{\Phi(\iota_t) - \delta}{\rho}}_{\text{Growth}} - \frac{\sigma^2}{2\rho} \right) dt\right]$$

where

- $\check{\eta}_t{}^i$ is the net worth share of agent i.
- $A(\kappa_t)$ is the aggregate productivity driven by capital allocation.
- $\Phi(\iota_t) - \delta$ is the growth rate of economy.
- σ, ρ are exogenous volatility and discount rate parameters.

Welfare



Jump
Growth
Idiosyncratic risk (households)
Total welfare
V_h (max ℓ=0.94)
V_e (max ℓ=0.98)
ℓ

Planner problem

Planner problem: environment [WIP]

Assume no default ($\tilde{j} = 0$) and $\theta_t^E = 1$.

$\rightarrow$ Planner chooses issuance $\chi_t \in [0, \overline{\chi}]$ each state

Two margins, as in the quantitative model:

- ⋆ Risk sharing: issuing more moves idiosyncratic risk to the agent who can diversify it
- ⋆ Distribution and safe asset demand: χ_t moves η_t and ϑ_t, which households and intermediaries take as given

Planner problem: no commitment

Pareto weight ω on intermediaries. With fixed capital, the growth and capital allocation terms drop out of the welfare decomposition and

Dynamics

$$W = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \Big(\underbrace{\omega \log \eta_t + (1-\omega) \log(1-\eta_t)}_{\text{Distribution}} - \underbrace{\frac{1}{2\rho} \left[\omega (\tilde{\varsigma}_t^I)^2 + (1-\omega)(\tilde{\varsigma}_t^h)^2 \right]}_{\text{Idiosyncratic risk}} \Big) dt \right]$$

where $\tilde{\varsigma}_t^I = \theta_t^I \varphi \tilde{\sigma}$ and $\tilde{\varsigma}_t^h = \theta_t^h \tilde{\sigma}$ are the prices of idiosyncratic risk

- Maximized subject to the evolution of η_t, given η_0, and the equation for ϑ_t
- ϑ_t is *forward looking*: past promises as a state variable $\rightarrow$ time inconsistency

$\rightarrow$ We solve the **no-commitment** problem: the planner takes the equilibrium function $\vartheta(\eta)$ as given and re-optimizes at every instant

Planner problem: Stochastic maximum principle

The Hamiltonian is

Dynamics

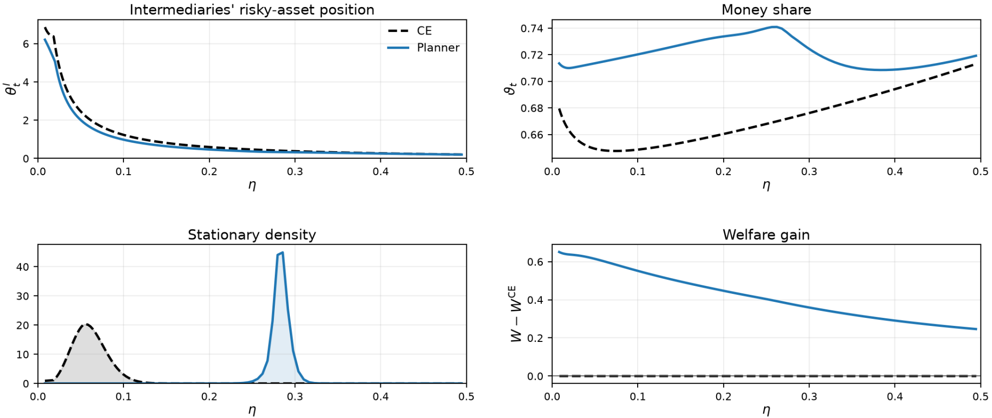
$$H_t = e^{-\rho t} w\left(\chi_t, \eta_t, \vartheta_t, \sigma_{\vartheta,t}\right) + \xi_t \, \mu^{\eta}\left(\chi_t, \eta_t, \vartheta_t, \sigma_{\vartheta,t}\right) + \sigma_{\xi,t} \, \sigma_{\eta}\left(\chi_t, \eta_t, \vartheta_t, \sigma_{\vartheta,t}\right)$$

$$[\chi_t]: \quad \frac{\partial H_t}{\partial \chi} = 0$$

$$d\vartheta_t = \mu^{\vartheta}\left(\chi_t, \eta_t, \vartheta_t, \sigma_{\vartheta,t}\right) dt + \sigma_{\vartheta,t} \, dZ_t,$$

$$d\xi_t = -\left[\frac{\partial H_t}{\partial \eta_t} + \partial_\eta \vartheta_t \frac{\partial H_t}{\partial \vartheta_t} + \frac{\partial H_t}{\partial \sigma_{\vartheta,t}} \frac{d\langle \partial_\eta \vartheta_t, Z_t \rangle}{dt}\right] dt + \sigma_{\xi,t} \, dZ_t, \qquad \vartheta_T = \overline{\vartheta}, \quad \xi_T = 0$$

Planner solution



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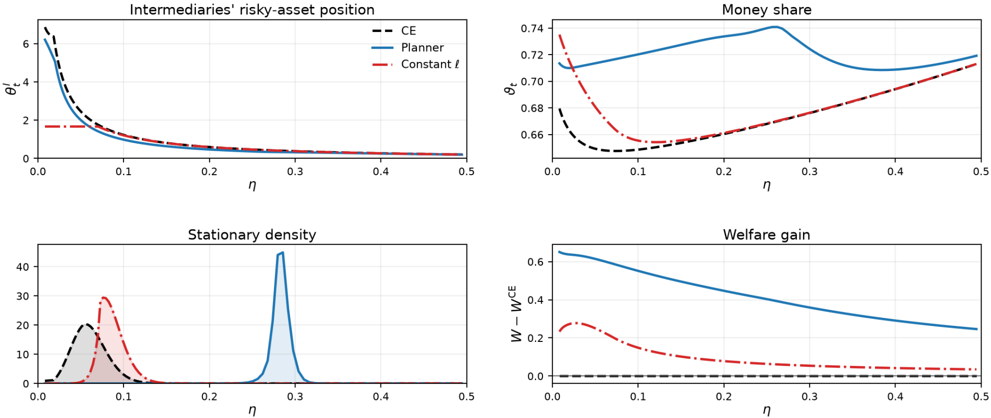
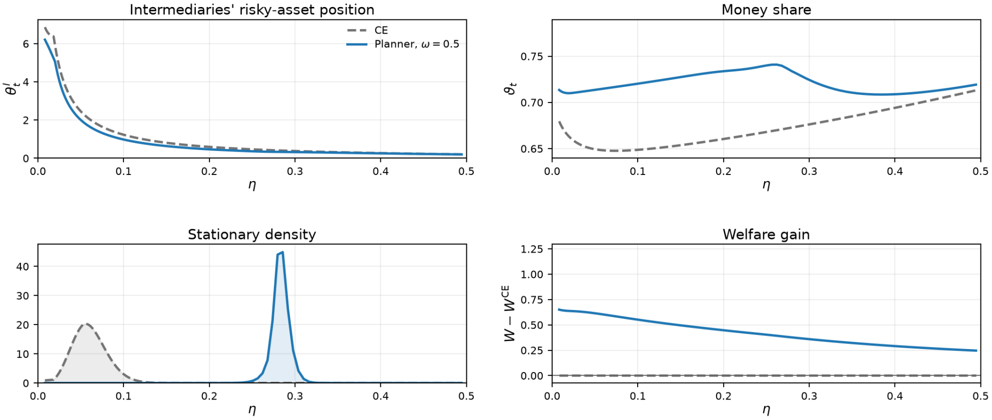
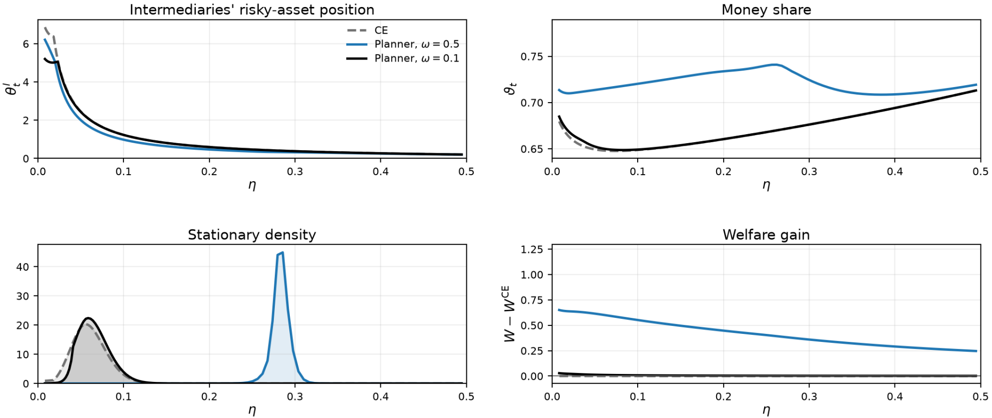


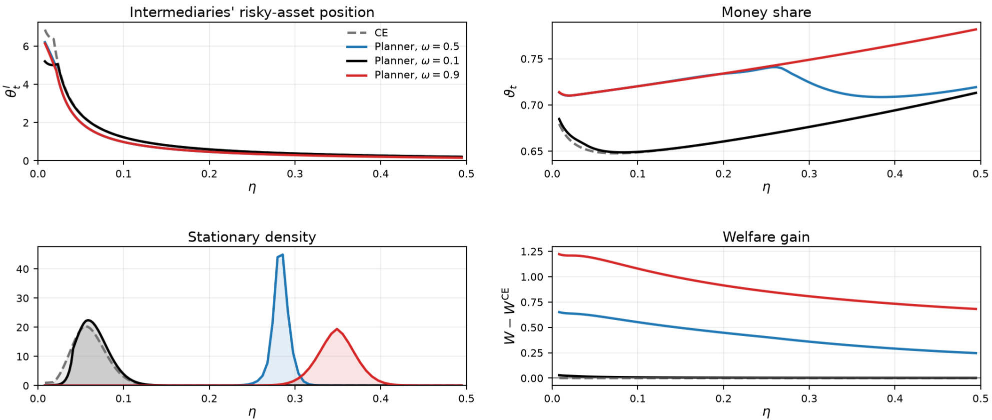
Planner solution: Pareto weights



Planner solution: Pareto weights



Planner solution: Pareto weights



Estimation

ML Calibration Strategy

Parameter	Variable definition
$\sigma(\underline{\sigma})$	Fundamental vol. bank dep. (bank indep.)
$\tilde{\sigma}$	Idiosyncratic vol.
κ	Capital share in bank dep. sector
$\mathcal{D}$	Bankruptcy cost
ϕ	Investment friction
ρ	Discount rate

Target moment	Value	Model	Target moment	Value	Model
GDP growth volatility	3.8%	4.3%	Equity risk premium	6.0%	4.3%
Bank Leverage mean	10	10.8	Risk-free rate	2%	2.2%
GDP growth rate	2%	2.5%	Bank Leverage vol.	2.1	1.9

Table 1: Estimated parameters (top panel) and data target moments (bottom panel).

ML Calibration Strategy [WIP]

- Let $\Psi \in \Omega^{\Psi}$ denote the structural parameters to be estimated
- Let $\varphi(\Psi) = (\varphi_1(\Psi), ..., \varphi_N(\Psi))$ denote the corresponding model moments
- Let $\tilde{\varphi} = (\tilde{\varphi_1}, ..., \tilde{\varphi_N})$ denote the data moments

Overview

Step-1: Use an ML model to learn mapping $\widehat{\varphi}(\Psi; \Theta)$. Model is trained on a set of simulated model moments $\varphi(\Psi)$ for different Ψ

Step-2: Use the learned ML model to predict the model moments $\widehat{\varphi}(\Psi; \Theta^*)$ for any given parameter Ψ and optimal ML model parameters Θ^*

Step-3: Use the predicted model moments to estimate the parameters Ψ by minimizing loss fn

$$\hat{\Psi} = \arg\min \sum_{i}^{N} \left(\frac{\tilde{\varphi}_i - \widehat{\varphi}_i(\Psi; \Theta^*)}{\tilde{\varphi}_i} \right)^2$$

ML Calibration Strategy [WIP]

We follow an iterative procedure to alleviate the curse of dimensionality in Step 1.

Draw parameters $\Psi^0 \sim \mathcal{U}(\Omega^\Psi)$. Set $\Psi = \Psi^0$.

while Loss $>$ tolerance **do**

  Simulate model, learn $\widehat{\varphi}(\Psi;\Theta^*)$ via ML model.

  Compute train/val losses, pick best Θ^*.

  Gradient Descent on (1) to find Ψ^*.

  Obtain perturbed parameter set Ψ^j by perturbing Ψ^*, i.e., $\Psi^j = \Psi^* + \epsilon$, where ϵ is $N(0, \sigma^2)$, where σ is equal to perturbation range.

  Draw new parameters Ψ^j from a normal distribution $\mathcal{N}(\overline{\Psi}_i, 1)$, where $\overline{\Psi}_i$ is the mean parameter value that generates model moments with the smallest $||\widehat{\varphi}(\Psi;\Theta^*)$ - $\tilde{\varphi}^j||$.

  Append the new parameter set $\Psi \leftarrow \Psi \cup \Psi^j$

  $i \leftarrow i + 1$

end while

Conclusion

- Studied optimal capital regulation with macro growth and financial stability implications
- Growth-resilience trade off: tighter regulation implies
  - ⋆ Amplification: less pronounced.
    Recovery: faster
    } Resilience $\uparrow$
  - ⋆ Growth: $\downarrow$
- Welfare analysis reveals
  - ⋆ Non-linear effect: tighter regulation increases welfare but drops off sharply beyond a point $\rightarrow$ argues for *gradual experimentation*
  - ⋆ Welfare decomposition reveals that default and growth effects are important
  - ⋆ Welfare gains from current to optimal regulation is 0.2%
  - ⋆ Planner choosing countercyclical capital regulation increases welfare

Thank you!

Part I
Appendix

Asset returns: households

The return on capital is given by

$$
\begin{aligned}
d\hat{R}_t &= \frac{a-\iota_t}{q_t^K}dt + \frac{d((1-\vartheta_t)k_t^x/K_t)}{(1-\vartheta_t)k_t^x/K_t} \\
&= \underbrace{\left(\frac{a-\iota_t}{q_t^K} + \mu_t^{1-\vartheta} + \sigma\kappa\left(\sigma\kappa - \sigma(1-\kappa)\right) + \kappa\sigma\sigma_t^{1-\vartheta,x} - \kappa\sigma\sigma_t^{1-\vartheta,y}\right)}_{\hat{r}_t^x} dt + \delta_t dt \\
&\quad + \underbrace{\left(\sigma_t^{1-\vartheta,x} + \kappa\sigma\right)}_{\hat{\sigma}_t^{x,x}} d{\color{red}Z_t^x} + \underbrace{\left(\sigma_t^{1-\vartheta,y} - \kappa\sigma\right)}_{\hat{\sigma}_t^{x,y}} d{\color{red}Z_t^y} + \bar{\chi}_x\tilde{\sigma}d{\color{gray}\tilde{Z}}_t \\
d\hat{R}_t^y &= \underbrace{\left(\frac{a-\iota_t}{q_t^K} + \mu_t^{1-\vartheta} + \sigma(1-\kappa)\left(\sigma(1-\kappa) - \sigma\kappa\right) - \sigma(1-\kappa)\sigma_t^{1-\vartheta,x} + \sigma(1-\kappa)\sigma_t^{1-\vartheta,y}\right)}_{\hat{r}_t^y} dt + \delta_t dt \\
&\quad + \underbrace{\left(\sigma_t^{1-\vartheta,x} - (1-\kappa)\sigma\right)}_{\hat{\sigma}_t^{y,x}} d{\color{red}Z_t^x} + \underbrace{\left(\sigma_t^{1-\vartheta,y} + (1-\kappa)\sigma\right)}_{\hat{\sigma}_t^{y,y}} d{\color{red}Z_t^y} + \tilde{\sigma}d{\color{gray}\tilde{Z}_t}
\end{aligned}
$$

Note: We denote y (x) to be bank dependent (independent) sector in the appendix.

Back

Asset returns: households

The return on risky claims are given by

$$d\hat{R}_t^{H,L} = \hat{r}_t^L dt + (\hat{\boldsymbol{\sigma}}_t^y)^T \boldsymbol{dZ_t} + \tilde{\sigma} d\tilde{Z}_t$$
$$d\hat{R}_t^{I,L} = \hat{r}_t^L dt + (\hat{\boldsymbol{\sigma}}_t^y)^T \boldsymbol{dZ_t} + (\tilde{j} - \mathcal{D}1_{\tilde{j}<\hat{j}_t}) d\tilde{J}_t - E[\tilde{j}]\tilde{\lambda}_j dt$$

where $\hat{\boldsymbol{\sigma}}_t^y = [\hat{\sigma}_t^{y,x}, \hat{\sigma}_t^{y,y}]^T$. The return on bond is

$$d\hat{R}_t^B = i_t dt + \frac{d(\vartheta_t/B_t)}{(\vartheta_t/B_t)} = i_t dt + \left(\mu_t^\vartheta - \mu_t^B\right) dt + \sigma_t^\vartheta \boldsymbol{dZ_t}$$
$$= \underbrace{\left(\mu_t^\vartheta - \check{\mu}_t{}^B\right)}_{\hat{r}_t^B} dt + \underbrace{\sigma_t^{\vartheta,x}}_{\hat{\sigma}_t^{B,x}} dZ_t^x + \underbrace{\sigma_t^{\vartheta,y}}_{\hat{\sigma}_t^{B,y}} dZ_t^y$$

Return on defaultable deposits is

$$d\hat{R}_t^{D,i} = d\hat{R}_t^D + \alpha_t dt + \tilde{j}_t^D d\tilde{J}_t$$

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Household FOCs

$$
\begin{aligned}
[\theta_t^x]: \quad & \hat{r}_t^x - \hat{r}_t^D = \varsigma_t^{h,x}\left(\hat{\sigma}_t^{x,x} - \hat{\sigma}_t^{B,x}\right) + \varsigma_t^{h,y}\left(\hat{\sigma}_t^{x,y} - \hat{\sigma}_t^{B,y}\right) + \tilde{\varsigma}_t^h \tilde{\sigma}\bar{\chi}_x - \delta_t \\
[\theta_t^y]: \quad & \hat{r}_t^y - \hat{r}_t^D = \frac{\chi_t}{\kappa}\left(\hat{r}_t^{OE} - \hat{r}_t^D\right) + \varsigma_t^{h,x}\left(\hat{\sigma}_t^{y,x} - \hat{\sigma}_t^{B,x}\right)\left(1 - \frac{\chi_t}{\kappa}\right) \\
& \quad + \varsigma_t^{h,y}\left(\hat{\sigma}_t^{y,y} - \hat{\sigma}_t^{B,y}\right)\left(1 - \frac{\chi_t}{\kappa}\right) + \tilde{\varsigma}_t^h\left(1 - \frac{\chi_t}{\kappa}\right)\tilde{\sigma} - \delta_t \\
[\chi_t^h]: \quad & \hat{r}_t^{OE} - \hat{r}_t^D = \varsigma_t^{h,x}\left(\hat{\sigma}_t^{y,x} - \hat{\sigma}_t^{B,x}\right) + \varsigma_t^{h,y}\left(\hat{\sigma}_t^{y,y} - \hat{\sigma}_t^{B,y}\right) + \tilde{\varsigma}_t^h \tilde{\sigma} - \lambda_t^h \\
[\lambda_t]: \quad & 0 = \lambda_t^h(\bar{\chi} - \chi_t)
\end{aligned}
$$

With log utility, we have $\hat{c}_t^h = \rho$. We can combine some FOCs to get

$$
\hat{r}_t^y - \hat{r}_t^D = \varsigma_t^{h,x}\left(\hat{\sigma}_t^{y,x} - \hat{\sigma}_t^{B,x}\right) + \varsigma_t^{h,y}\left(\hat{\sigma}_t^{y,y} - \hat{\sigma}_t^{B,y}\right) + \tilde{\varsigma}_t^h \tilde{\sigma} - \lambda_t \chi_t^h / \kappa - \delta_t
$$

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Asset returns: intermediaries

$$d\hat{R}_t^E = \left(\theta_t^I\left[\hat{r}_t^L - \hat{r}_t^D + \lambda_j E[\tilde{j}_t^D] - \tilde{\lambda}_j E[\tilde{j}]\right] + \hat{r}_t^D - \lambda_j E[\tilde{j}_t^D]\right) dt + \left[\theta_t^I(\hat{\sigma}_t^y - \hat{\sigma}_t^D) + \hat{\sigma}_t^D\right] \boldsymbol{dZ_t}$$
$$+ \underbrace{\left[\tilde{j}_t^D + \theta_t^I(\tilde{j} - \mathcal{D}1_{\tilde{j}<\hat{j}_t} - \tilde{j}_t^D)\right]}_{j_t^E} d\tilde{J}_t$$

$$\tilde{j}_t^D = \begin{cases} \dfrac{-1 - \theta_t^I(\tilde{j} - \mathcal{D}1_{\tilde{j}<\hat{j}_t})}{1 - \theta_t^I} & \text{if } \underline{j} < \tilde{j} < \hat{j}(\theta^I), \\ 0 & \text{otherwise.} \end{cases}$$

where $\hat{j}(\theta_t^I) = -\frac{1}{\theta_t^I}$.

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Intermediary FOCs

$$[\theta_t^E] \quad 0 = \theta_t^I \left(\hat{r}_t^L - \hat{r}_t^D - \tilde{\lambda}_j E[\tilde{j}] + \lambda_j E[\tilde{j}_t^D]\right) - \lambda_j E[\tilde{j}_t^D] - \theta_t^I(\hat{\sigma}_t^y - \hat{\sigma}_t^D)\left(\hat{\sigma}_t^D + \theta_t^E \theta_t^I(\hat{\sigma}_t^y - \hat{\sigma}_t^D)\right)$$
$$+ \tilde{\lambda}_j E_t \left[\frac{j_t^E(\tilde{j};\theta_t^I)}{1 + \theta_t^E j_t^E(\tilde{j};\theta_t^I)}\right]$$
$$[\theta_t^I] \quad 0 = \theta_t^E \left[\hat{r}_t^L - \hat{r}_t^D - \tilde{\lambda}_j E[\tilde{j}] + \lambda_j E[\tilde{j}_t^D(\theta_t^I)] + \lambda_j(\theta_t^I - 1)\frac{dE[\tilde{j}_t^D(\theta_t^I)]}{d\theta_t^I}\right] - \theta_t^E(\hat{\sigma}_t^y - \hat{\sigma}_t^D)\left(\hat{\sigma}_t^D + \theta_t^E \theta_t^I(\hat{\sigma}_t^y - \hat{\sigma}_t^D)\right)$$
$$+ \tilde{\lambda}_j E \left[\frac{\theta_t^E \frac{\partial j_t^E}{\partial \theta_t^I}}{1 + \theta_t^E j_t^E}\right]$$

where

$$E[\tilde{j}_t^D] = \begin{cases} \frac{(1+\theta_t^I \underline{j})(1+\theta_t^I \underline{j} - 2\mathcal{D}\theta_t^I)}{-2\underline{j}\theta_t^I(1-\theta_t^I)} & \text{if } \underline{j} < \hat{j} < 0 \\ 0 & \textit{otherwise} \end{cases}$$

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Markov Equilibrium

We solve for a Markov equilibrium in the wealth share of intermediaries $\eta_t = N_t^I / N_t$.

1. Goods market: $AK_t - \iota_t K_t = C_t^h + C_t^I$, where capital letters denote aggregate quantities (e.g., $C_t^h = \int c_t^h dh$) Rewrite this as

$$(A - \iota_t)(1 - \vartheta_t) = q_t^K \left[(1 - \eta_t) \frac{C_t^h}{N_t^h} + \eta_t \frac{C_t^I}{N_t^I} \right]$$

2. Capital market:
   - Tech. x: $(1 - \kappa) q_t^K K_t = \theta_t^x N_t^h$
   - Tech. y: $\kappa q_t^k K_t \quad = \theta_t^y N_t^h$
3. Risky claims market: $\eta_t \theta_t^I + (1 - \eta_t) \theta_t^{h,OE} = 0$
4. Deposit market: $\eta_t \theta_t^{I,D} + (1 - \eta_t) \theta_t^{h,D} = 0$

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Planner problem: state dynamics

Wealth share of intermediaries, with $\vartheta_t = \vartheta(\eta_t)$ and $\sigma_t^\vartheta = \frac{\vartheta'(\eta_t)}{\vartheta(\eta_t)}\sigma_t^\eta$

$$\sigma_t^\eta = \chi_t(1-\vartheta_t)\sigma - (\chi_t - \eta_t)\sigma_t^\vartheta; \qquad \sigma_t^\vartheta = \frac{\vartheta'(\eta_t)}{\vartheta(\eta_t)} \frac{\chi_t(1-\vartheta_t)\sigma}{1+(\chi_t-\eta_t)\frac{\vartheta'(\eta_t)}{\vartheta(\eta_t)}}$$

$$\mu_t^\eta = \eta_t(1-\eta_t)(1-\vartheta_t)^2\left\{\frac{1}{1-\eta_t}\left[\frac{\eta_t-\chi_t}{\eta_t}\left(-\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)+\frac{1-\chi_t}{1-\eta_t}\sigma\right]\left(\sigma-\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)\right.$$
$$\left.+\left[\frac{\chi_t^2\Delta_t}{\eta_t^2}-\frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right]\left(\sigma-\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)^2+\left[\frac{\varphi^2\chi_t^2\Delta_t}{\eta_t^2}-\frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right]\tilde{\sigma}^2\right\} - \frac{\eta_t(1-\vartheta_t)\sigma\sigma_t^\eta}{1-\eta_t}$$

Money share, from the households' and intermediaries' conditions for the combined portfolio

$$\mu_t^\vartheta = \rho + \left(\sigma_t^\vartheta\right)^2 - (1-\vartheta_t)^2\left[\frac{\left(-(1-\chi_t)\frac{\sigma_t^\vartheta}{1-\vartheta_t}-\chi_t\sigma\right)^2}{1-\eta_t}+\frac{\chi_t^2\Delta_t}{\eta_t}\left(\sigma-\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)^2\right]$$
$$-(1-\vartheta_t)^2\left[\frac{(1-\chi_t)^2}{1-\eta_t}+\frac{\varphi^2\chi_t^2\Delta_t}{\eta_t}\right]\tilde{\sigma}^2, \qquad \mu_t^\vartheta\vartheta_t = \mu_t^\eta\vartheta'(\eta_t)+\frac{1}{2}\left(\sigma_t^\eta\right)^2\vartheta''(\eta_t)$$

The wedge Δ_t enters both drifts through the intermediaries' position only.

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