Nominal Wages and Aggregate Production

Kosuke Aoki and Nobuhiro Kiyotaki
Tokyo and Princeton

In monetary economy, the circulation of money is essential for improving allocation $\neq$ Economy where money is not essential, only worsening allocation

Money as a Unit of Account

Under what environment, is wage contract written in terms of money?

When is the nominal wage rate sticky instead of indexed to the price level?

Under what conditions, do the sticky nominal wage contracts improve allocation?

We develop a general equilibrium model of incomplete market and liquidity constraint

Literature on Unit of Accounts and Nominal Contracts

Incomplete market + no default

Doepke-Schneider (2017), Drenik-Kirpalani-Perez (2022)

Moral hazard + delayed CPI release + renegotiation

Jovanovic-Ueda (1997, 1998), Meh-Quadrini-Terajima (2024)

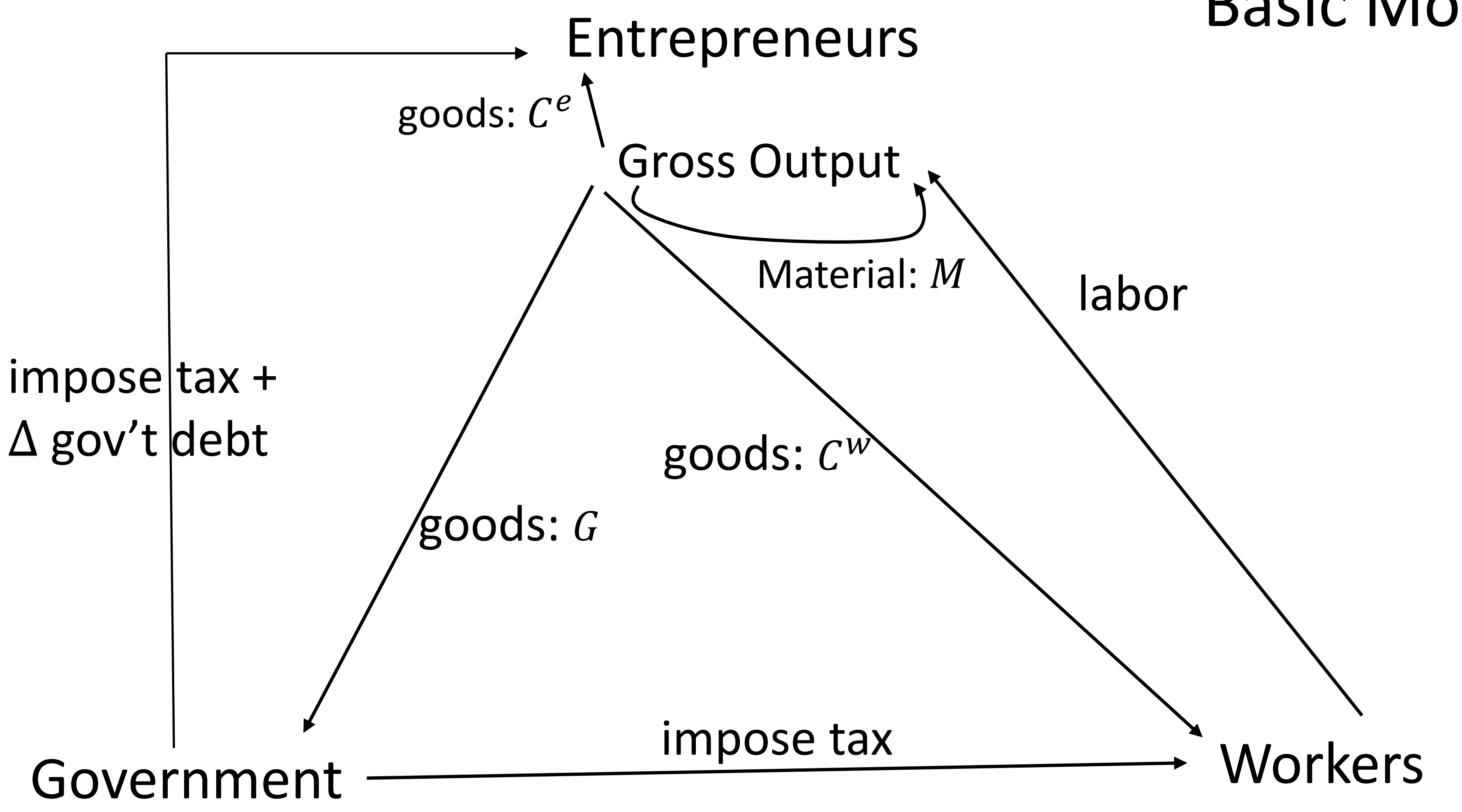
Coordination of sticky price and nominal contract

Gottfries (1992), Gopinath-Stein (2021)

Nominal contract for risk sharing against productivity shock

Gray (1976, 1978), Fischer (1977), Alexanian (2025)

Basic Model



Entrepreneurs
goods: C^e
Gross Output
Material: M
labor
impose tax +
Δ gov't debt
goods: C^w
goods: G
impose tax
Government
Workers

Basic Model: Nominal Government Bond & Almost Degenerate Idiosyncratic Shock

Government issues nominal discount bond to finance deficit

$$qB' - B = PG - \tau PY, \text{ or}$$

$$qb - \frac{b^-}{\pi} = G - \tau Y \tag{1}$$

where $b = \frac{B'}{P}$, $b^- = b_{t-1}$ and $\pi = \frac{P}{P^-}$

Monetary policy is

$$\frac{1}{1+i} \equiv q = \frac{q(\pi)}{\varepsilon^M} \text{ where } q'(\pi) < 0 \tag{2}$$

Government purchase rule is given by fiscal policy

$$G = G\left(b^-\right)\varepsilon^G, \text{ where } G'\left(b^-\right) < 0 \tag{3}$$

Entrepreneurial family consists of $[0,1]$ continuum of goods producers

Each producer produces homogeneous goods x_i from labor n_i and material m_i as

$$x_i = A\xi_i {m_i}^{\alpha} {n_i}^{\gamma}, \text{ where } \alpha + \gamma \in (0,1)$$

where A and ξ_i are aggregate and idiosyncratic productivity shocks

The idiosyncratic shock is almost degenerate

$$\xi_i = \begin{cases} \overline{\xi} = 1 + \delta, \text{ with probability } 1 - \delta \\ \underline{\xi} = \delta, \text{ with probability } \delta \rightarrow +0 \end{cases}$$

Employment is determined in the previous period n^-

After shocks realize, each producer chooses (x_i, m_i) to maximize the value added $x_i - m_i$ for a given $n_i = n^-$

Aggregate value added is

$$\begin{aligned} X - M &= Y = a \cdot (n^-)^{\frac{\gamma}{1-\alpha}}, \\ \text{where } a &= (1-\alpha)\left(\alpha^\alpha A\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{4}$$

Producers hire workers for the next period by offering a wage contract to pay in terms of today's final goods and the next period's final goods (w_m, w_y)

Worker family chooses consumption, labor supply and saving to maximize their utility

$$V_t^w = \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} [u(C_s^w) - \varphi\left(n_s^-\right)] \right\}$$

subject to the budget constraint

$$C^w + qb^w = (1-\tau)(w_y^- + \frac{w_m^-}{\pi})n^- + \frac{b^{w-}}{\pi}, \;\; b^w \geq 0$$

We can show $qu'(C^w) > \beta \mathbb{E}_t \left[\frac{1}{\pi'} u'(C^{w\prime}) \right]$ and

$$C_t^w = (1-\tau)\left(w_y^- + \frac{w_m^-}{\pi} \right) n^- \tag{5}$$

$$\mathbb{E}\left\{ u'(C^{w\prime}) \left[(1-\tau)\left(\frac{w_m}{\pi'} + w_y \right) \right] \right\} = u^w = \varphi'\left(n\right) \tag{6}$$

When offering wage contract (w_m, w_y) to n number of workers, each producers faces a liquidity constraint

$$B' + P'(x_i' - m_i') - (Pw_m + P'w_y)n \geq 0$$

for all idiosyncratic and aggregate shocks.

Because $\delta \longrightarrow +0$,

$$b - w_m n - \pi'_{\text{max}} w_y n \geq 0 \tag{7}$$

where $\pi'_{\text{max}} = \max_{S'=(A',\varepsilon^{M\prime},\varepsilon^{G\prime})} (\pi')$

After production, all entrepreneur members get together to share their funds to choose (C^e, b, n, w_m, w_y) to maximize expected discounted utility $\mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} [u(C_s^e)] \right\}$ subject to

$$C^e + qb = (1-\tau) \left[a \left(n^- \right)^{\frac{\gamma}{1-\alpha}} - \left(w_y^- + \frac{w_m^-}{\pi} \right) n^- \right] + \frac{b^-}{\pi}$$

and (6) for a given u^w and (7).

The FOCs are

$$qu'(C^e) = \lambda + \beta \mathbb{E} \left[\frac{1}{\pi'} u'(C^{e\prime}) \right] \tag{8}$$

$$\lambda (w_m + \pi'_{\text{max}} w_y) = (1\text{-}\tau) \beta \mathbb{E} \left\{ u'(C^{e\prime}) \left[\tilde{a}' - \left(w_y + \frac{w_m}{\pi'} \right) \right] \right\} \tag{9}$$

$$\text{where } \tilde{a}' = \frac{\gamma}{1-\alpha} a' n^{\frac{\alpha+\gamma-1}{1-\alpha}}$$

$$0 = \lambda (b - w_m n - \pi'_{\text{max}} w_y n)$$

Define the marginal rate of substitutions between real and nominal wages for workers and entrepreneurs

$$M^{w} = -\left.\frac{dw_m}{dw_y}\right|_{V^w} = \frac{\mathbb{E}\left[u'\left(C^{w\prime}\right)\right]}{\mathbb{E}\left[u'\left(C^{w\prime}\right)\frac{1}{\pi'}\right]}$$

$$M^{e} = -\left.\frac{dw_m}{dw_y}\right|_{V^e} = \frac{\mathbb{E}\left\{u'\left(C^{e\prime}\right)\left[\pi'_{\mathsf{max}}\tilde{a}' + w_m\left(1 - \frac{\pi'_{\mathsf{max}}}{\pi'}\right)\right]\right\}}{\mathbb{E}\left\{u'\left(C^{e\prime}\right)\left[\tilde{a}' + w_y\left(\frac{\pi'_{\mathsf{max}}}{\pi'} - 1\right)\right]\right\}}$$

The optimal wage contract is given by

$$\begin{aligned} w_m &> 0,\ w_y = 0,\ \text{if}\ \left.M^e\right|_{w_y=0} > \left.M^w\right|_{w_y=0} \\ w_m,\ w_y &> 0,\ \text{if}\ \left.M^e\right|_{w_m,w_y>0} = \left.M^w\right|_{w_m,w_y>0} \\ w_m &= 0,\ w_y > 0,\ \text{if}\ \left.M^e\right|_{w_m=0} < \left.M^w\right|_{w_m=0} \end{aligned} \tag{10}$$

Goods market equilibrium is given by

$$Y = C^w + C^e + G \tag{11}$$

The endogenous state variable is $K^- = \left(b^-, n^-, w_m^-, w_y^-\right)$. The exogenous state $S = (A, \varepsilon^M, \varepsilon^G)$ follows a Markov process. The recursive equilibrium is given by the endogenous variables $(q, \pi, \lambda, G, Y, C^e, C^w, K)$ as a function of (K^-, S) which satisfies $(1 - 11)$.

Non-inflationary steady state without aggregate shock is given by (w, q, b, n) which solves

$$b = wn \tag{12}$$

$$u'((1-\tau)b)\,(1-\tau)\,w = \varphi'\,(n)\,, \text{ or } n = n^S((1-\tau)\,w) \tag{13}$$

$$(1-q)b = \tau a n^{\frac{\gamma}{1-\alpha}} - G \tag{14}$$

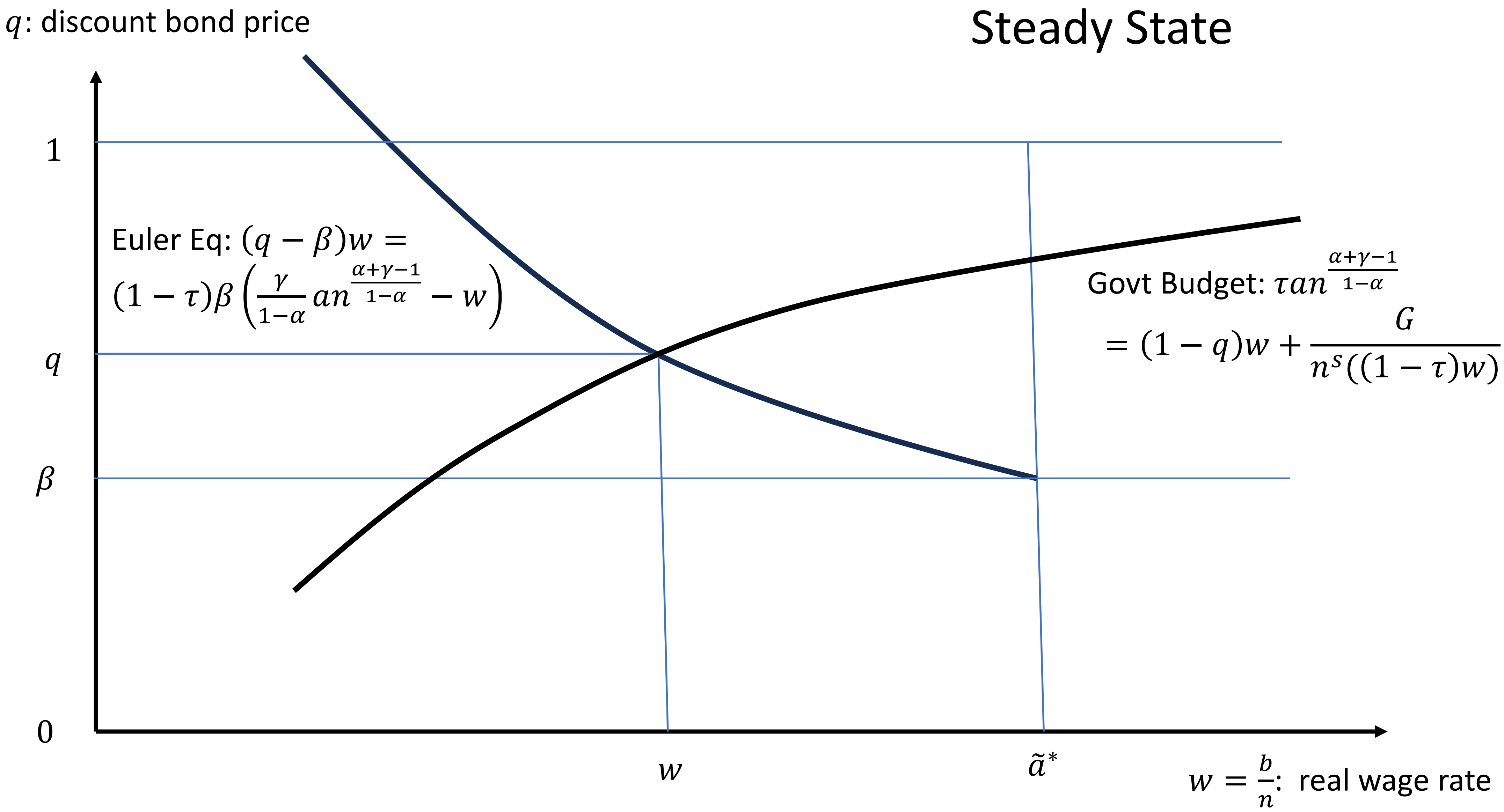
$$(q-\beta)w = (1-\tau)\beta(\tilde{a} - w) \tag{15}$$

There is a unique steady state with positive liquidity premium, $q > \beta$ and $w < \tilde{a}$, iff

$$G > a n^{\frac{\gamma}{1-\alpha}}\left[\tau - (1-\beta)\frac{\gamma}{1-\alpha}\right] \tag{C1}$$

where $n = n^S((1-\tau)\,\tilde{a}) = n^S\left((1-\tau)\,\frac{\gamma}{1-\alpha} a n^{\frac{\alpha+\gamma-1}{1-\alpha}}\right)$

Steady State

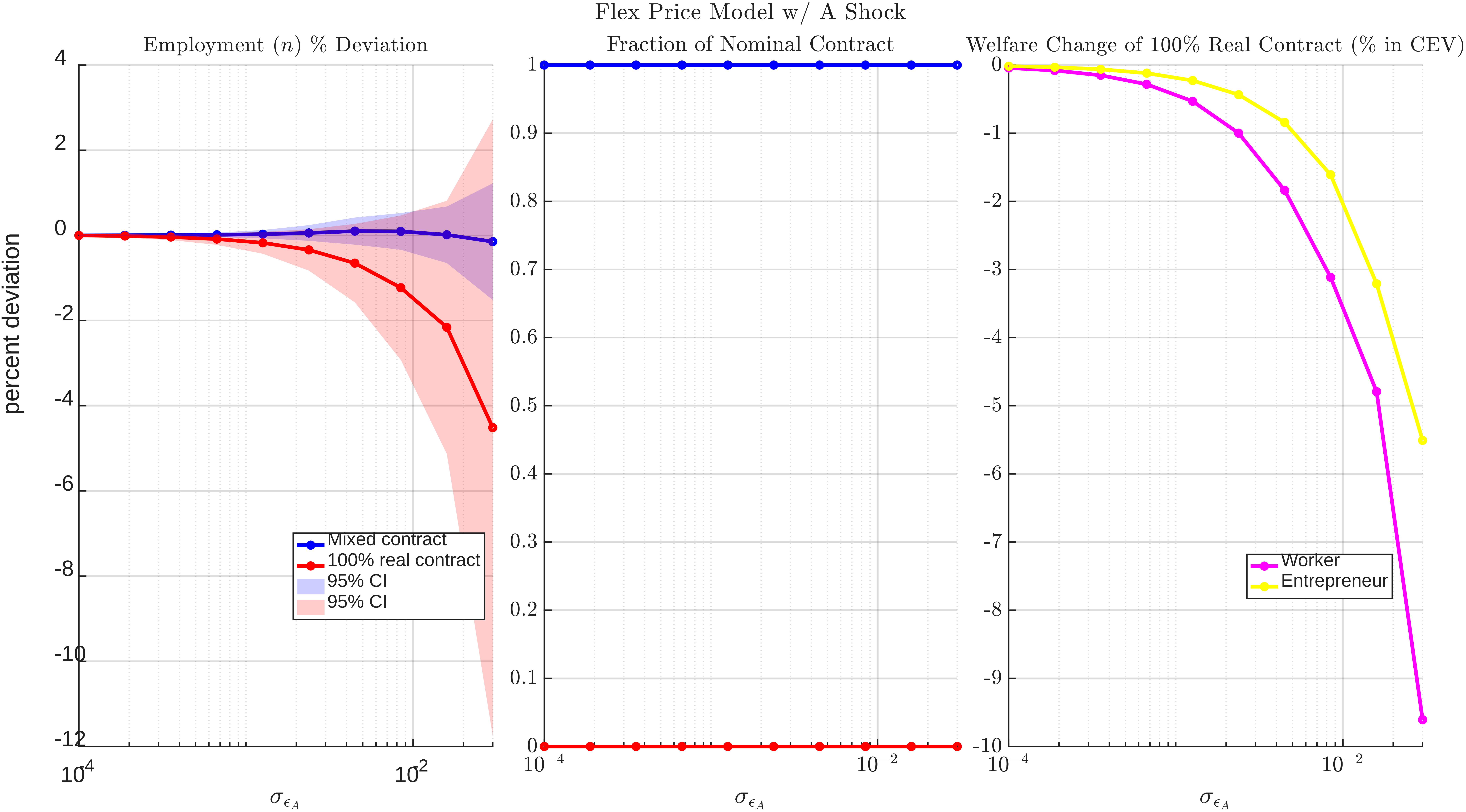


Proposition 1: Under condition (C1), there is a unique steady state in which the liquidity constraint is binding for producers with $\delta \longrightarrow 0$. In the neighborhood of the steady state, we have

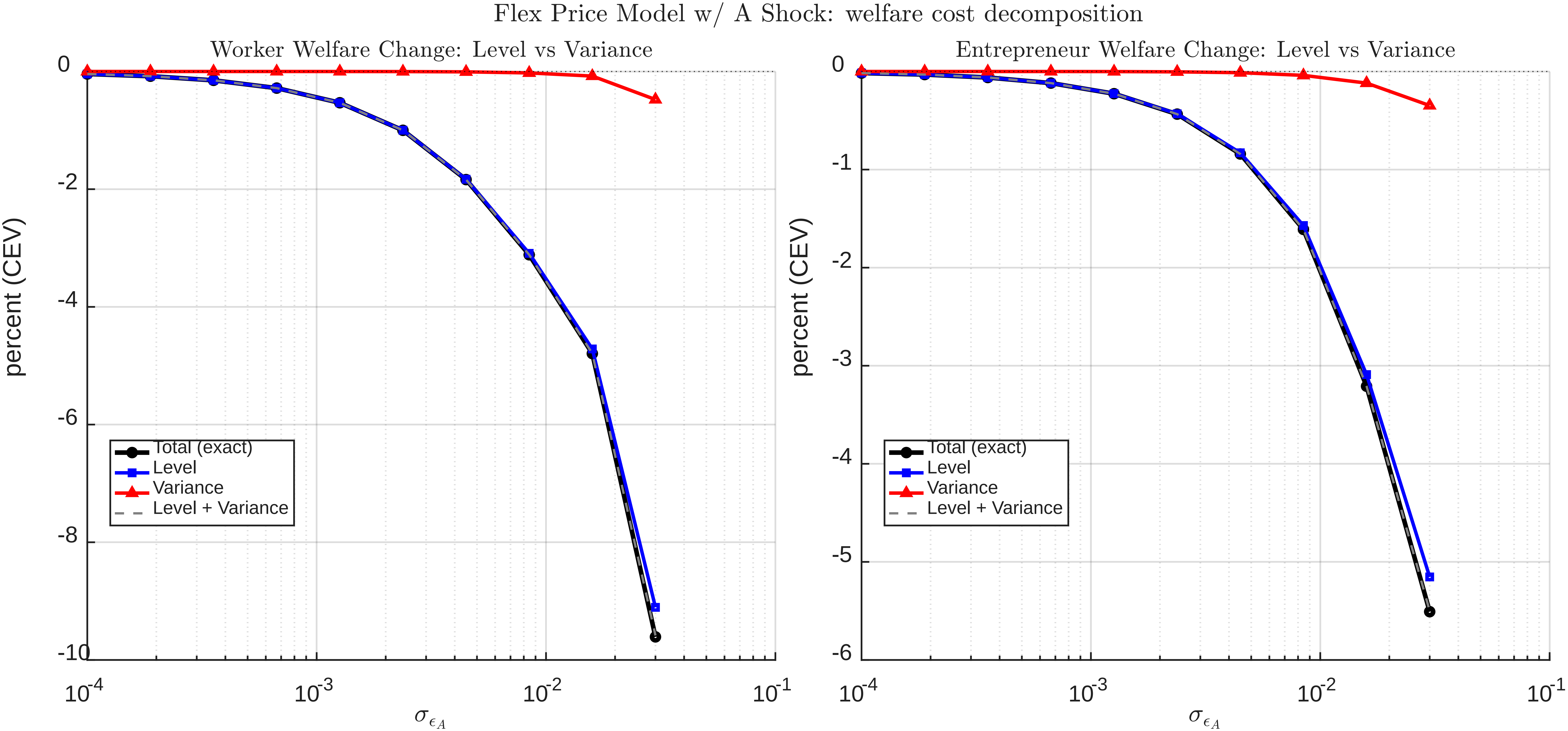
(a) The expected rate of return on government bond is lower than the time preference rate, and workers do not save

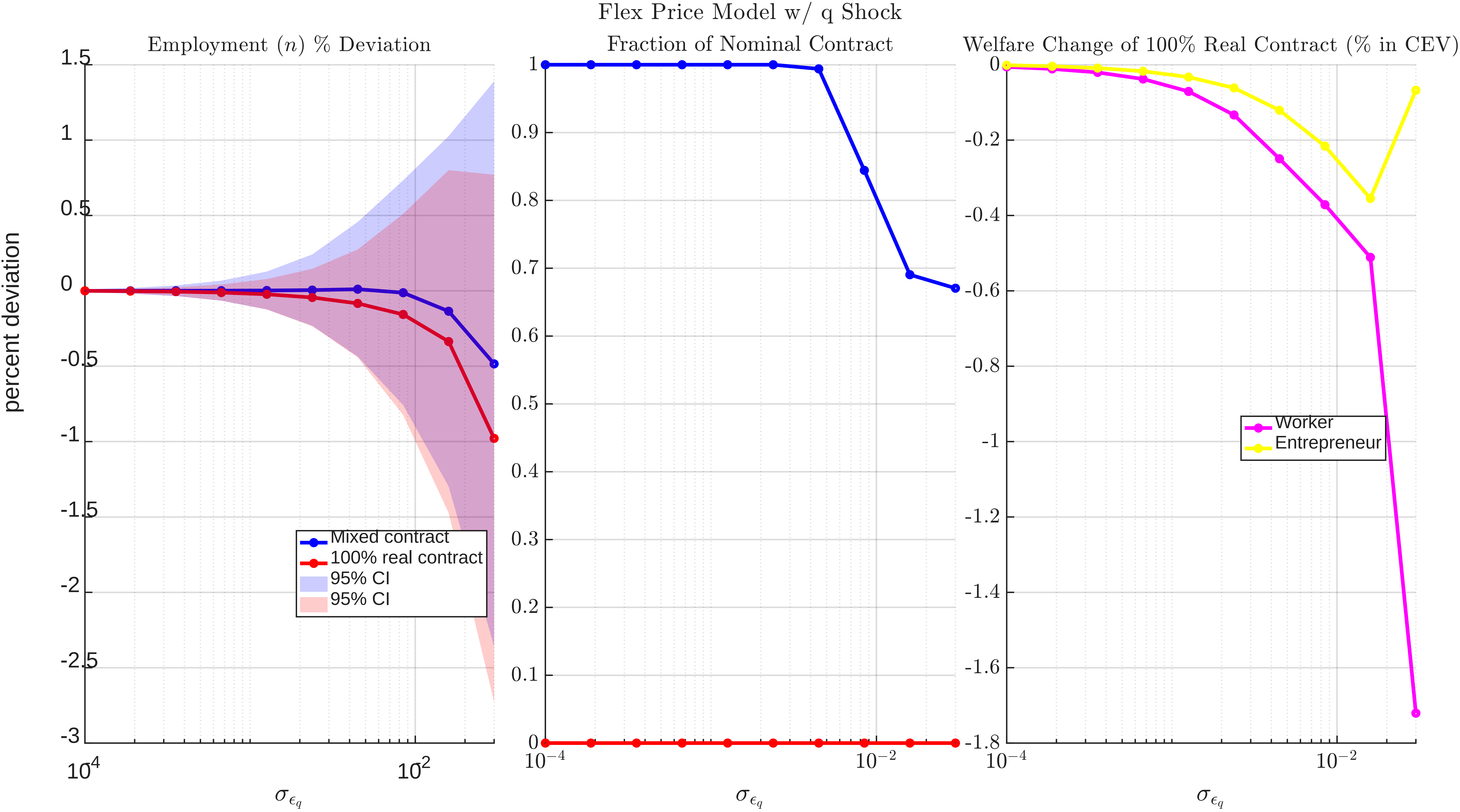
(b) Wage contract is at least partially nominal

Parameter Value (semi-annual rate)		
β	discount factor	0.97
η	elasticity of marginal utility	0.5
$\frac{\varphi'(n)}{n\varphi''(n)}$	Frisch elasticity of labor supply	1
α	share of material	0.4
γ	share of labor in steady state	0.4
τ	value added tax rate	0.3
$\frac{G}{n}$	Gov't expenditure per worker	0.13
ψ_q	Taylor coefficient	1.5
ψ_g	Fiscal rule coefficient	-1
ρ_i	persistence of shocks	0.8



Flex Price Model w/ A Shock
Employment (n) % Deviation
percent deviation
Mixed contract
100% real contract
95% CI
95% CI
σ_{ϵ_A}
Fraction of Nominal Contract
σ_{ϵ_A}
Welfare Change of 100% Real Contract (% in CEV)
Worker
Entrepreneur
σ_{ϵ_A}


Flex Price Model w/ A Shock: welfare cost decomposition
Worker Welfare Change: Level vs Variance
Entrepreneur Welfare Change: Level vs Variance
percent (CEV)
0
-2
-4
-6
-8
-10
-1
-3
-5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
σ_{ϵ_A}
Total (exact)
Level
Variance
Level + Variance



Flex Price Model w/ q Shock
Employment (n) % Deviation
Fraction of Nominal Contract
Welfare Change of 100% Real Contract (% in CEV)
percent deviation
Mixed contract
100% real contract
95% CI
95% CI
Worker
Entrepreneur
σ_{ϵ_q}
σ_{ϵ_q}
σ_{ϵ_q}
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
10^4
10^2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^{-4}
10^{-2}
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
10^{-4}
10^{-2}


Flex Price: $(\sigma_{\epsilon_q} = 0.01, \sigma_{\epsilon_g} = 0.01, \sigma_{\epsilon_A} = 0.01)$
n (employment)
Mixed contract
$w_m^-/\pi + w_y^-$ (total wage)
p_x
Fraction of Nominal Contract
99.99999999992
99.99999999991
99.9999999999
C^e (entrepreneur cons)
λ (liquidity constraint)
b^-/π (real government bond)
Y (real GDP)
$\pi - 1$ (inflation rate)
i (nominal interest rate)
G (government purchase)
ϵ_A (productivity shock)
percent deviation
percent
absolute percent deviation

Flex Price: $(\sigma_{\epsilon_q} = 0.01, \sigma_{\epsilon_g} = 0.01, \sigma_{\epsilon_A} = 0.01)$
n (employment)
$w_m^-/\pi + w_y^-$ (total wage)
p_x
Fraction of Nominal Contract
99.99999999992
99.99999999991
99.9999999999
Mixed contract
C^e (entrepreneur cons)
λ (liquidity constraint)
b^-/π (real government bond)
Y (real GDP)
$\pi - 1$ (inflation rate)
i (nominal interest rate)
G (government purchase)
ϵ_q (monetary policy shock)
percent deviation
absolute percent deviation
percent


Flex Price: $(\sigma_{\epsilon_q} = 0.01, \sigma_{\epsilon_g} = 0.01, \sigma_{\epsilon_A} = 0.01)$
n (employment)
Mixed contract
$w_m^-/\pi + w_y^-$ (total wage)
p_x
Fraction of Nominal Contract
99.99999999993
99.999999999925
99.99999999992
99.999999999915
C^e (entrepreneur cons)
λ (liquidity constraint)
b^-/π (real government bond)
Y (real GDP)
$\pi - 1$ (inflation rate)
i (nominal interest rate)
G (government purchase)
ϵ_g (fiscal policy shock)
percent deviation
absolute percent deviation
percent

Partially Indexed Government Bond

A fraction $\mu \in [0, 1]$ of government bond is nominal

$$qb - \left(1 - \mu + \frac{\mu}{\pi}\right) b^{-} = G - \tau Y \tag{16}$$

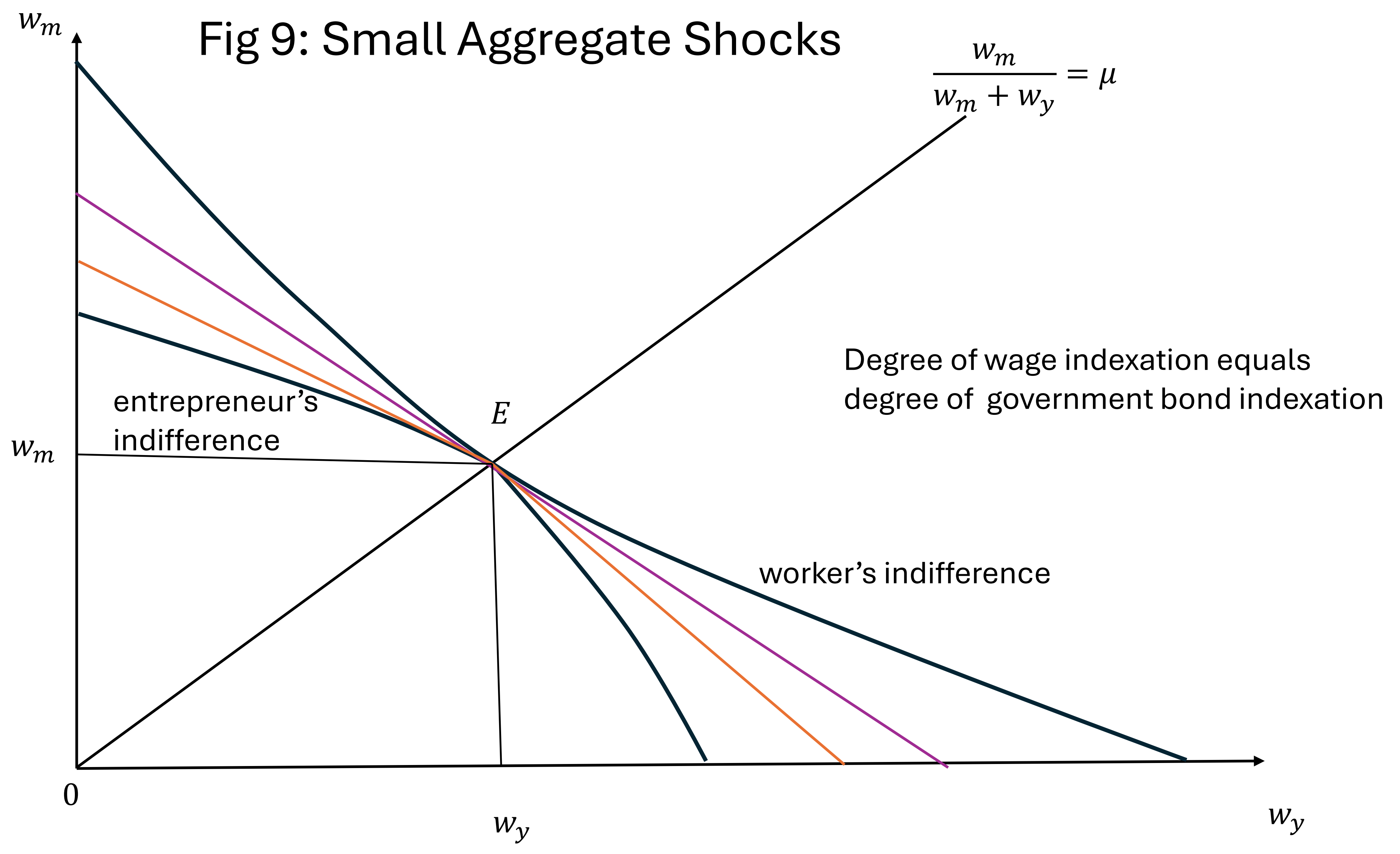
The liquidity constraint of the goods producer is

$$\left(1 - \mu + \frac{\mu}{\pi'}\right) b \geq \left(w_y + \frac{w_m}{\pi'}\right) n, \text{ for all } \pi' \tag{17}$$

Proposition 2: (i) When aggregate shock is small, the degree of wage indexation equals that of government bond

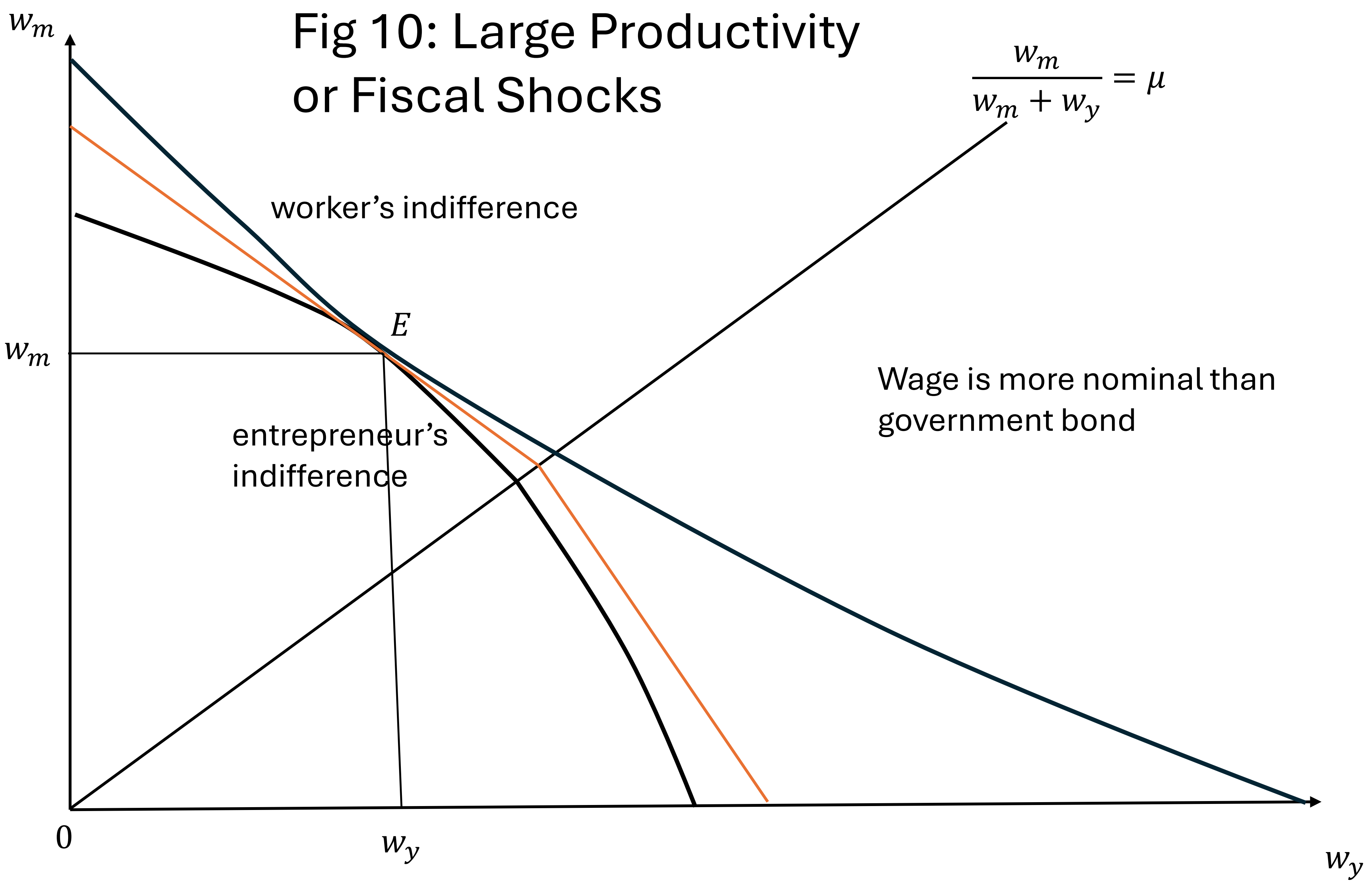
(ii) When productivity shock or fiscal shock are significant, entrepreneurs pay wage largely in nominal term

(iii) Entrepreneurs index wage with significant monetary shock

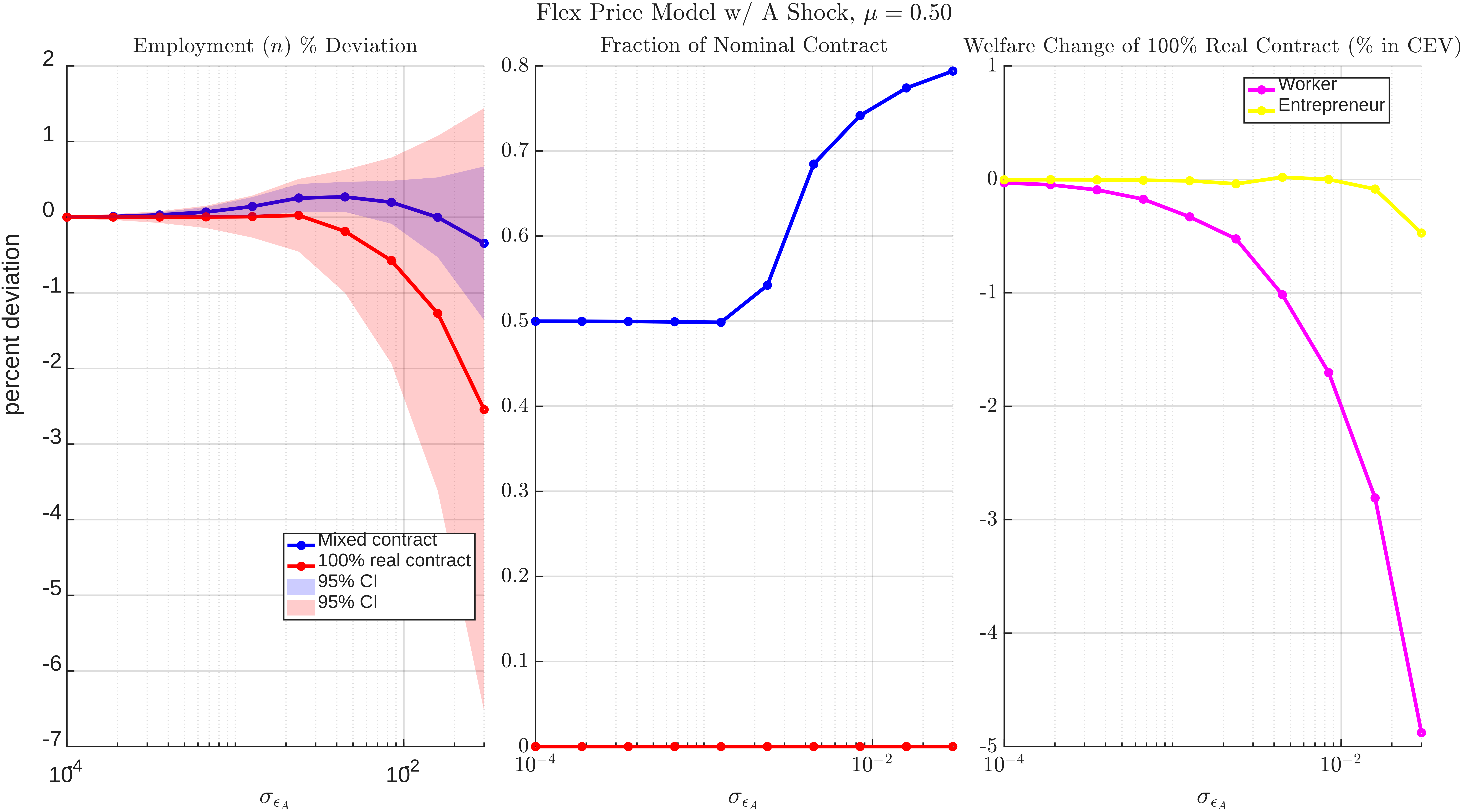


w_m
Fig 9: Small Aggregate Shocks
$\frac{w_m}{w_m + w_y} = \mu$
Degree of wage indexation equals
degree of government bond indexation
entrepreneur's
indifference
E
w_m
worker's indifference
0
w_y
w_y

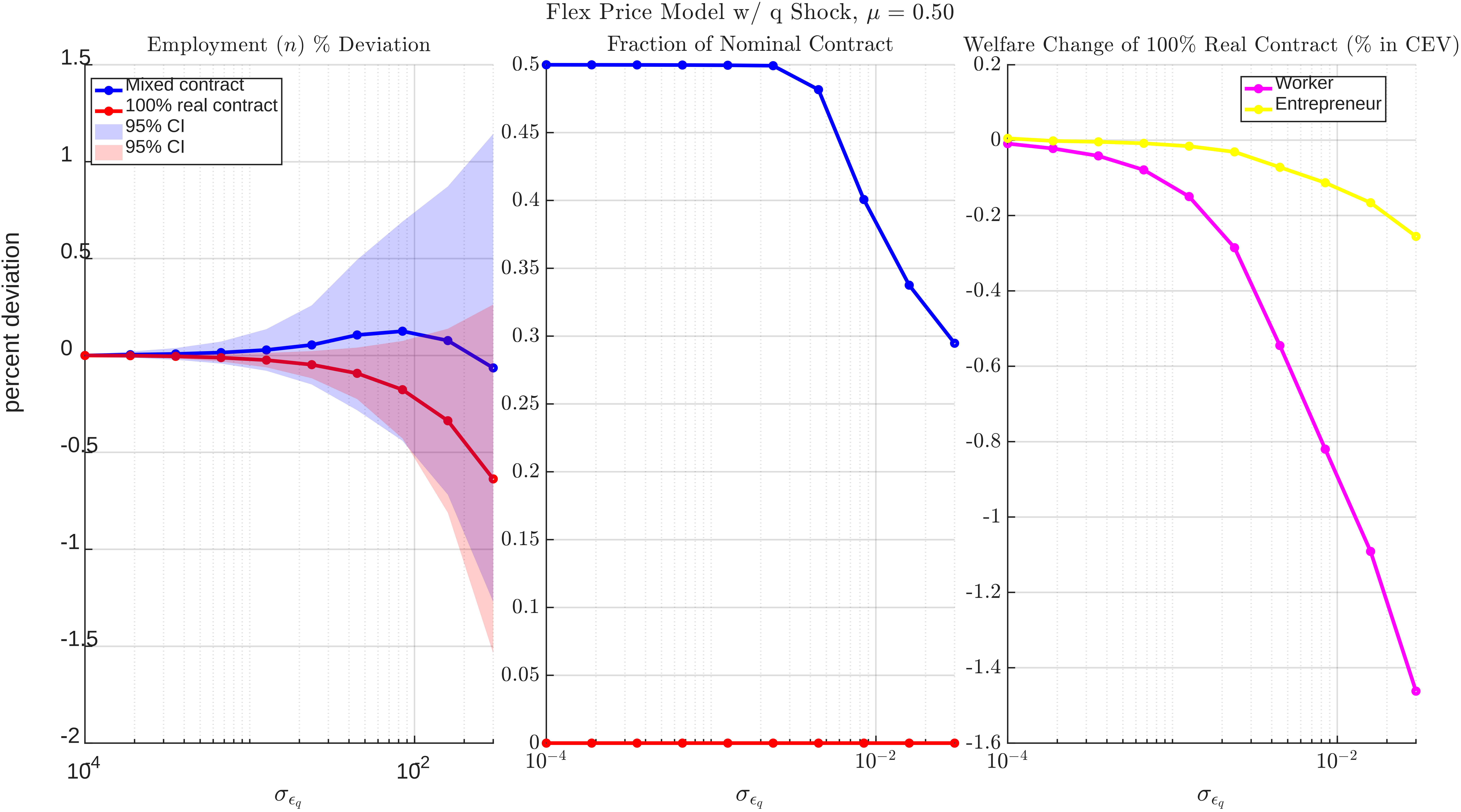
Fig 10: Large Productivity or Fiscal Shocks



Wage is more nominal than government bond

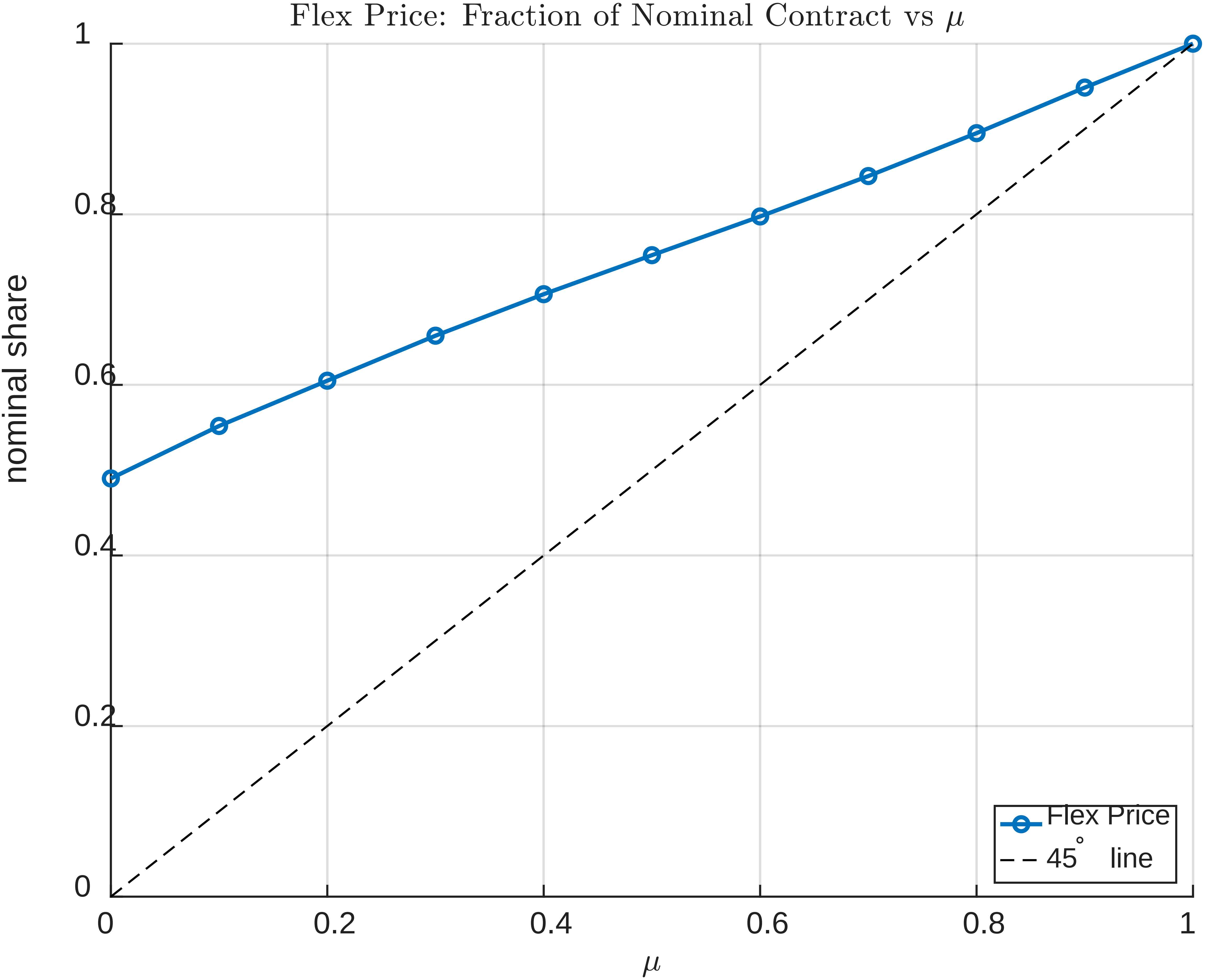


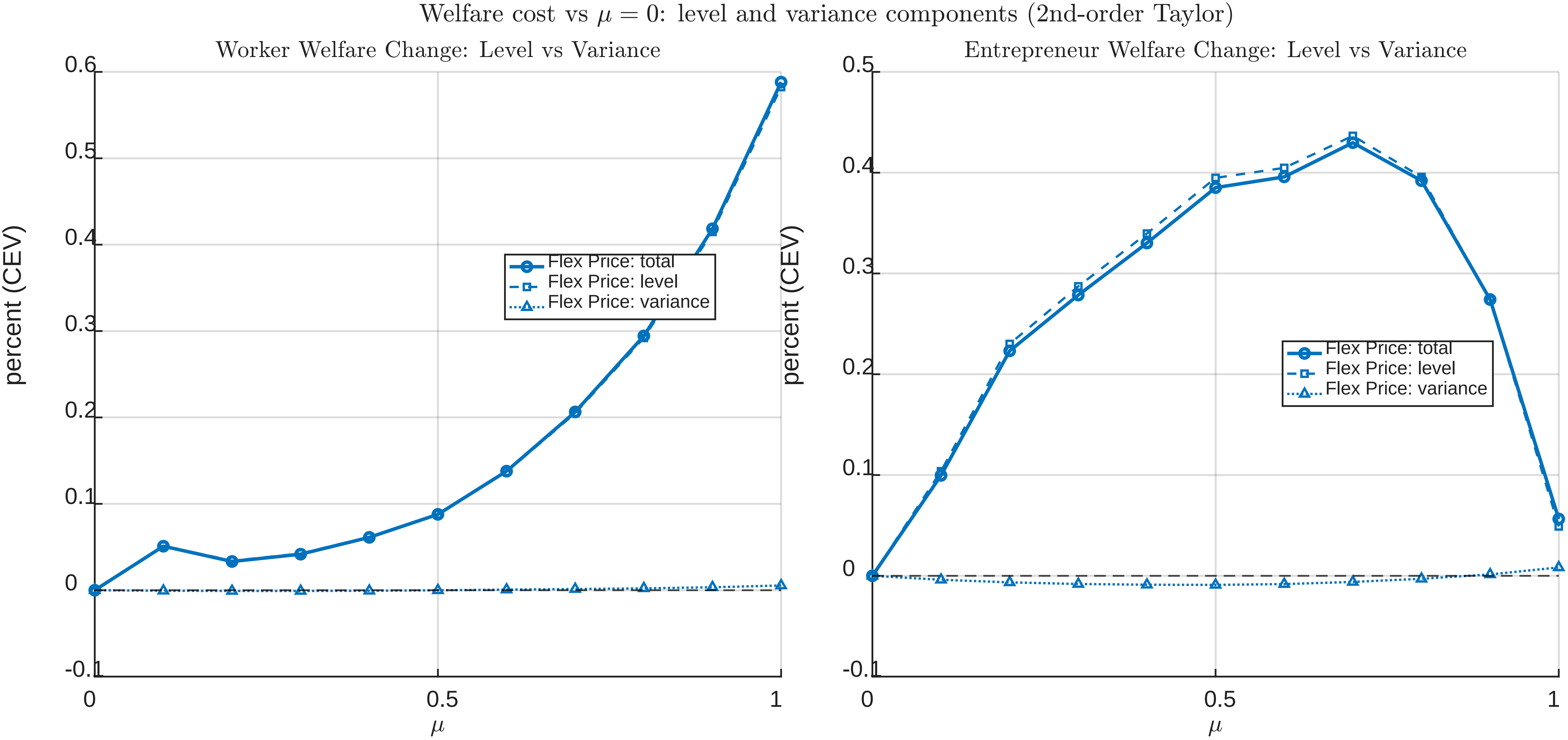
Flex Price Model w/ A Shock, $\mu = 0.50$
Employment (n) % Deviation
percent deviation
Mixed contract
100% real contract
95% CI
95% CI
σ_{ϵ_A}
Fraction of Nominal Contract
σ_{ϵ_A}
Welfare Change of 100% Real Contract (% in CEV)
Worker
Entrepreneur
σ_{ϵ_A}


Flex Price Model w/ q Shock, $\mu = 0.50$
Employment (n) % Deviation
Mixed contract
100% real contract
95% CI
95% CI
percent deviation
σ_{ϵ_q}
Fraction of Nominal Contract
σ_{ϵ_q}
Welfare Change of 100% Real Contract (% in CEV)
Worker
Entrepreneur
σ_{ϵ_q}


Flex Price: $(\sigma_{\epsilon_q} = 0.01, \sigma_{\epsilon_g} = 0.01, \sigma_{\epsilon_A} = 0.01)$, $\mu = 0.50$
n (employment)
Mixed contract
percent deviation
$w_m^-/\pi + w_y^-$ (total wage)
p_x
Fraction of Nominal Contract
percent
C^e (entrepreneur cons)
λ (liquidity constraint)
$(1-\mu+\mu/\pi)\,b^-$ (government bond)
Y (real GDP)
$\pi - 1$ (inflation rate)
absolute percent deviation
i (nominal interest rate)
G (government purchase)
ϵ_A (productivity shock)

Flex Price: $(\sigma_{\epsilon_q} = 0.01, \sigma_{\epsilon_g} = 0.01, \sigma_{\epsilon_A} = 0.01)$, $\mu = 0.50$
n (employment)
Mixed contract
$w_m^-/\pi + w_y^-$ (total wage)
p_x
Fraction of Nominal Contract
C^e (entrepreneur cons)
λ (liquidity constraint)
$(1-\mu+\mu/\pi)\, b^-$ (government bond)
Y (real GDP)
$\pi - 1$ (inflation rate)
i (nominal interest rate)
G (government purchase)
ϵ_q (monetary policy shock)
percent deviation
percent
absolute percent deviation


Flex Price: Fraction of Nominal Contract vs μ
nominal share
μ
0
0.2
0.4
0.6
0.8
1
Flex Price
45° line



Welfare cost vs $\mu = 0$: level and variance components (2nd-order Taylor)
Worker Welfare Change: Level vs Variance
Entrepreneur Welfare Change: Level vs Variance
percent (CEV)
μ
Flex Price: total
Flex Price: level
Flex Price: variance
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.5
1

Conclusion

Wage contract is largely sticky in nominal term when

(i) Fraction of nominal government bond is large relative to indexed fraction, or

(ii) Productivity and fiscal shocks are significant relative to monetary policy shock

In such economy,

(i) sticky nominal wage helps expand employment by mitigating the shortage of outside liquidity

(ii) utility of workers and entrepreneurs are significantly higher with endogenous wage indexation than with exogenous 100% indexation