Online Summer School
Macro, Money, and Finance
Problem Set 3

June 1, 2026

Please submit your solutions to the dropbox link by 6/7/2026 23:59 pm (EDT).

1 Fire Sales

In this exercise you will solve the model from Lecture 04 numerically, under the assumption of log utility with agents' deaths.

1. Our goal is to construct functions $q(\eta), \iota(\eta), \kappa(\eta)$ and $\sigma^q(\eta)$ on the $[0, 1]$ grid. Slides 46-47 provide the parameter values, and slide 43-45 provides the set of equations and the algorithm.
   (a) Solve the model at the boundaries: for $\eta = 0$ and $\eta = 1$.
   (b) Create a uniform grid for $\eta \in [0.0001, 0.9999] = \{\eta_1 = 0.0001, \eta_2, \ldots, \eta_N = 0.9999\}$.
   (c) Using the implicit method with the one-step Newton's algorithm, solve the system of equations on slide 43 for $\eta_1, \eta_2, \ldots$ and so on.
   (d) Stop once you reach $\kappa \geq 1$. From here on, set $\kappa = 1$, solve for q and σ^q.
   (e) Verify your solution by plotting $q(\eta)$ and $\sigma^q(\eta)$ and comparing it with the graph on slide 46. Do your functions converge to the boundary solution for $\eta = 1$ that you obtained in (a) as $\eta \to 1$?
   (f) Plot the remaining variables: $\iota(\eta), \kappa(\eta)$.
   (g) We can also look at the experts' balance sheet: derive expression for the scaled version of issued debt: $\frac{D_t^e}{q_t K_t}$ and plot it against η.
   (h) Repeat exercises (a)–(f) using the deep_macrofin package. For installation instructions and documentation, visit the website https://rotmanfinhub.github.io/deep-macrofin/
   Hints: A step-by-step tutorial video is also available on the summer school website, which covers both this question and the question 2(c) below. You may find deep learning is much slower in a one-state variable setting than finite differences. However, this tool could be very helpful for more involved models with multiple state-variables.

2. Recall from the lecture that drift and volatility of η in the general case are given by:

$$\mu_t^\eta = (1-\eta_t)\Big[(\varsigma_t^e - \sigma - \sigma_t^q)(\sigma_t^\eta + \sigma + \sigma_t^q) - (\varsigma_t^h - \sigma - \sigma_t^q)\Big(-\tfrac{\eta_t}{1-\eta_t}\sigma_t^\eta + \sigma + \sigma_t^q\Big)$$
$$- \Big(\tfrac{C_t^e}{N_t^e} - \tfrac{C_t^h}{N_t^h}\Big) + \frac{\rho_d^h \zeta(1-\eta_t) - \rho_d^e(1-\zeta)\eta_t}{\eta_t(1-\eta_t)}\Big],$$

$$\sigma_t^\eta = \frac{\kappa_t - \eta_t}{\eta_t}(\sigma + \sigma_t^q).$$

(a) Which terms in the above equations can we simplify/substitute because of log utility and why? Perform these substitutions and derive the drift and volatility of η under log utility.

(b) Verify your solution by plotting $\eta\mu^\eta(\eta)$ and $\eta\sigma^\eta(\eta)$ and comparing them with the graph on slide 47.

(c) Using your solution from part 1(h), compute the solution to part (b). Compare the resulting solution with your answer from part (b) by plotting them together and assessing the numerical accuracy.