New Keynesian Model: (Un)Conventional Monetary Policy

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- What is the welfare-maximizing policy mix?
- We will extend the one-sector money model step by step

Recall: One Sector Monetary Model

- Households solve:

$$\max_{c_t^H,\iota_t,\theta_t^K} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(c_t^H)dt\right] \quad \text{s.t.}$$

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H}dt + (1-\theta_t^K)dr_t^{\mathcal{B}} + \theta_t^K dr_t^K(\iota_t)$$

$$dr_t^{\mathcal{B}} = (i_t - \pi_t)dt - \sigma_t^{\mathcal{P}} dZ_t$$

$$dr_t^K(\iota_t) = \left(\frac{a-\iota_t-\tau_t^K}{q_t^K} + \mu_t^{q^K} + \Phi(\iota_t) - \delta\right)dt + \tilde{\sigma}_t d\tilde{Z}_t + \sigma_t^{q^K} dZ_t$$

$$d\tilde{\sigma}_t^2 = -\psi(\tilde{\sigma}_t^2 - \tilde{\sigma}_{ss}^2)dt + \sigma\tilde{\sigma}_t^2 dZ_t$$

Introducing Price Stickiness

- Standard setup with monopolistic firms
- Households produce a common input good y_t
- Monopolistic producers $j \in [0, 1]$ create its varieties Y_t^j
- Final good producer bundles varieties into consumption good:

$$Y_t = \left(\int_0^1 \left(Y_t^j \right)^{\frac{\varepsilon - 1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

- Demand function:

$$Y_t^j = \left(\frac{P_t^j}{\mathcal{P}_t} \right)^{-\varepsilon} Y_t$$

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- Households choose capital utilization level υ_t and produce input good $a\upsilon_t k_t$
- Sell it to monopolistic firms at price p_t

$$dr_t^K(\iota_t, \upsilon_t) = \left(\frac{p_t a \upsilon_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} + \mu_t^{q^K} + \Phi(\iota_t) - \delta \right) dt + \tilde{\sigma}_t d\tilde{Z}_t + \sigma_t^{q^K} dZ_t$$

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- $\mathfrak{d}_t$ are dividends from monopolistic firms
- Utilization has a convex utility cost $b(\upsilon_t)$: $\log(c_t) - b(\upsilon_t)$

Introducing Price Stickiness

- Continuum of monopolistic firms with linear technology $Y_t^j = y_t^j$
- Adjust prices $dP_t^j = \pi_t^j P_t^j dt$ subject to a Rotemberg cost $\frac{\kappa}{2}\left(\pi_t^j\right)^2 Y_t dt$

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- Objective function:

$$\max_{\pi_t^j} \int_0^\infty \Xi_t^H \left[\left(\frac{P_t^j}{\mathcal{P}_t}\right)^{1-\varepsilon} - p_t(1-\tau_t)\left(\frac{P_t^j}{\mathcal{P}_t}\right)^{-\varepsilon} - \frac{\kappa}{2}\left(\pi_t^j\right)^2 - T_t \right] Y_t dt$$

Introducing Price Stickiness

- Consider a symmetric equilibrium ($P_t^j = \mathcal{P}_t$)
- Derive the New Keynesian Phillips Curve:

$$\pi_t = \frac{\varepsilon}{\kappa Y_t} \mathbb{E}_t \int_t^{\infty} e^{-\int_t^s r_\tau^f d\tau} Y_s \left(mc_s - mc^f \right) ds$$

- $mc_s = p_s(1 - \tau_s)$ is the real marginal cost
- $mc^f = \frac{\varepsilon - 1}{\varepsilon}$ is the flex-price real marginal cost

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- $\mathcal{P}_t$ drifts with π_t and does not load on dZ_t ($\sigma_t^{\mathcal{P}} = 0$): $d\mathcal{P}_t = \pi_t \mathcal{P}_t dt$

Introducing Intermediaries

- Households issue risky claims on capital and offload fraction χ_t of risk
- Intermediaries hold these claims and diversify idiosyncratic risk to $\varphi \in (0, 1)$

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$$\max_{c_t^I, \theta_t^{\mathcal{B}}, \theta_t^{D,I}, \theta_t^{x,I}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log(c_t^I) dt\right] \qquad \text{s.t.}$$

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$$1 = \theta_t^{\mathcal{B}} + \theta_t^{D,I} + \theta_t^{x,I}$$

$$dr_t^{\mathcal{B}} = (i_t - \pi_t) dt \qquad dr_t^D = (i_t^D - \pi_t) dt$$

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- For comparison: $dr_t^{x,H} = r_t^x dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$

Introducing Reserves and Long-term bonds

- Relabel short-term bonds as "reserves" $\mathcal{R}_t$, only allow intermediaries to hold them

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- Long-term bonds L_t are perpetual, pay fixed nominal interest i^L and have nominal price P_t^L; all agents can hold them:

$$dr_t^L = \frac{i^L}{P_t^L}dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right]dt + \sigma_t^{P^L} dZ_t$$

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- Nominal wealth is now $\mathcal{B}_t = \mathcal{R}_t + P_t^L L_t$

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- Households use deposits for transactions related to capital with velocity ν_t:

$$\theta_t^K = \nu_t \theta_t^{D,H}$$

- Higher velocity $\Longrightarrow$ higher transaction cost $\mathfrak{t}(\nu_t)$

$$dr_t^K(\iota_t, \upsilon_t, \nu_t) = \left[\frac{p_t a \upsilon_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}(\nu_t) + \mu_t^{q^K} + \Phi(\iota_t) - \delta\right] dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$$

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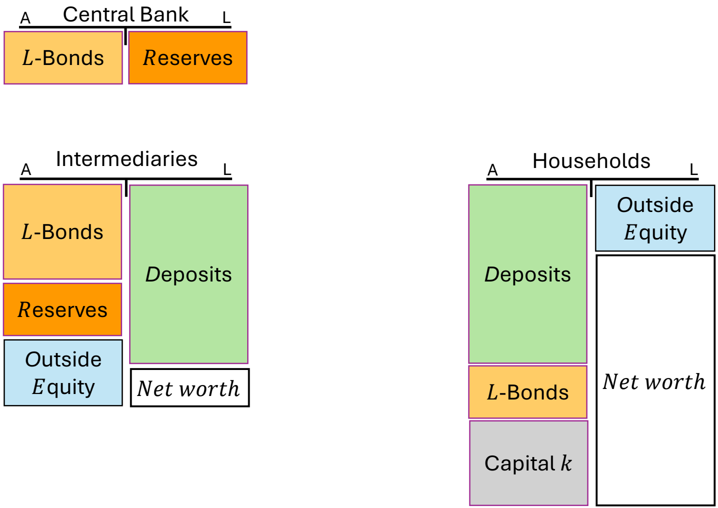
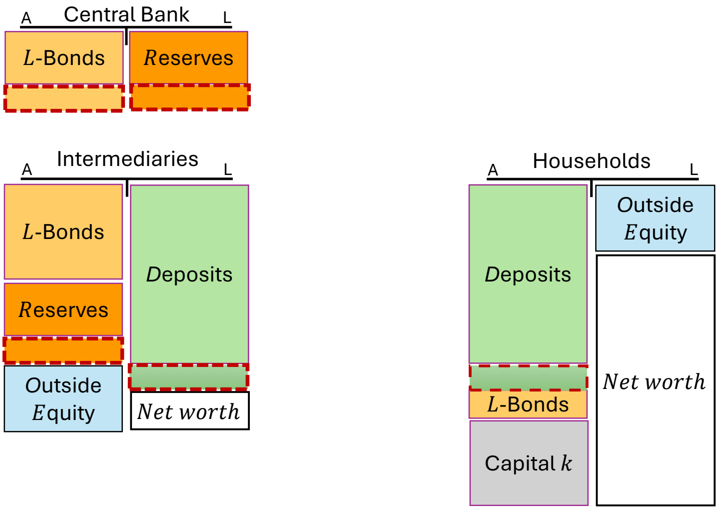
- Prevents agents from hedging each other against aggregate risk

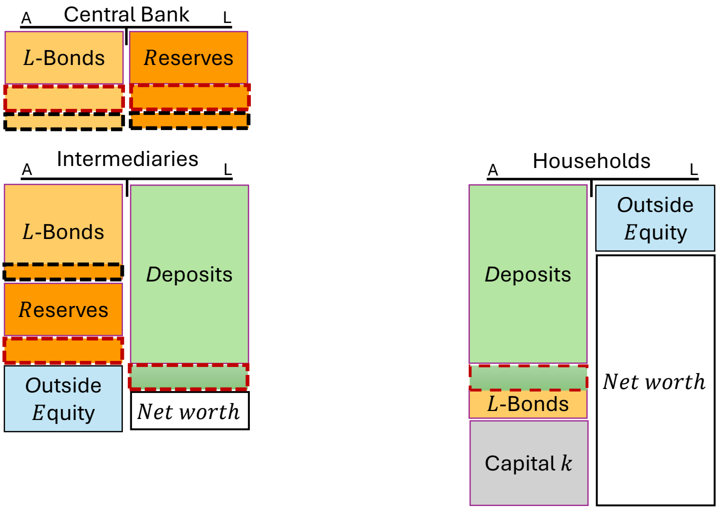
Balance Sheets



Balance Sheet Management: L-Bond purchases from Households



Balance Sheet Management: L-Bond purchases from HH & Banks



Constrained Efficiency

- Suppose the government raises taxes over time (flow/dt taxes) and in response to aggregate shocks (loading on dZ_t), but not to idios. shocks (not on $d\tilde{Z}_t$)

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- Suppose the government raises taxes over time (flow/dt taxes) and in response to aggregate shocks (loading on dZ_t), but not to idios. shocks (not on $d\tilde{Z}_t$)
- Behaves as a planner that can freely set:
  - Capital utilization rate υ_t
  - Capital investment rate ι_t
  - Distribution of wealth/consumption across sectors $\eta_t = N_t^I / N_t$,
  - Distribution of wealth across assets $\vartheta_t = \mathcal{B}_t / (\mathcal{P}_t N_t) = (\mathcal{R}_t + P_t^L L_t) / (\mathcal{P}_t N_t)$
  - Distribution of idiosyncratic risk exposure χ_t

Constrained Efficiency

- Planner's objective:

$$\max_{\{\iota_t,\upsilon_t,\vartheta_t,\eta_t,\chi_t\}_{t=0}^{\infty}} \lambda\mathbb{E}\int_0^{\infty} e^{-\rho t}\log(\tilde{\eta}_t^I\eta_t c_t K_t)dt + (1-\lambda)\mathbb{E}\int_0^{\infty} e^{-\rho t}\left(\log(\tilde{\eta}_t^H(1-\eta_t)c_t K_t) - b(\upsilon_t)\right)dt$$

$$\text{s.t. } c_t = a\upsilon_t - \iota_t = \rho\frac{q_t^K}{1-\vartheta_t}, \qquad q_t^K = (1+\phi\iota_t)$$

$$\frac{d\tilde{\eta}_t^I}{\tilde{\eta}_t^I} = \chi_t\frac{1-\vartheta_t}{\eta_t}\varphi\tilde{\sigma}_t d\tilde{Z}_t, \qquad \frac{d\tilde{\eta}_t^H}{\tilde{\eta}_t^H} = (1-\chi_t)\frac{1-\vartheta_t}{1-\eta_t}\tilde{\sigma}_t d\tilde{Z}_t$$

Constrained Efficiency

- Planner's objective can be simplified to a static one

$$\max_{\upsilon_t,\iota_t,\eta_t,\chi_t,\vartheta_t} W_t = \overbrace{\log(a\upsilon_t - \iota_t) - (1-\lambda)b(\upsilon_t) + \frac{1}{\rho}\left(\frac{1}{\phi}\log(1+\phi\iota_t) - \delta\right)}^{\text{aggregate efficiency at } t}$$
$$\underbrace{+ \lambda\log(\eta_t) + (1-\lambda)\log(1-\eta_t) - \frac{\tilde{\sigma}_t^2}{2\rho}\left[\lambda\frac{\chi_t^2}{\eta_t^2}\varphi^2 + (1-\lambda)\frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right](1-\vartheta_t)^2}_{\text{distributional efficiency at } t}$$

- Constrained efficient allocation $\upsilon^*(\tilde{\sigma})$, $\iota^*(\tilde{\sigma})$, $\eta^*(\tilde{\sigma})$, $\vartheta^*(\tilde{\sigma})$, $\chi^*(\tilde{\sigma})$

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\begin{aligned}
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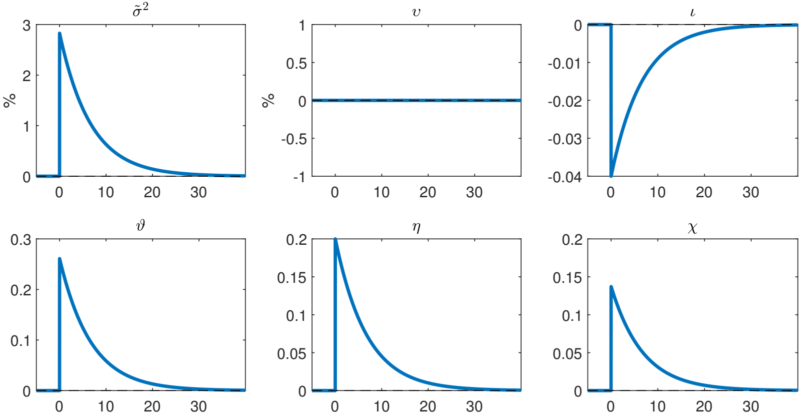
- Constrained efficient allocation $\upsilon^*(\tilde{\sigma})$, $\iota^*(\tilde{\sigma})$, $\eta^*(\tilde{\sigma})$, $\vartheta^*(\tilde{\sigma})$, $\chi^*(\tilde{\sigma})$
- Optimal allocation: $1 > \chi^*(\tilde{\sigma}) > \eta^*(\tilde{\sigma}) > \lambda$

IRF Planner's Solution after $\tilde{\sigma}_t$-Shock



Laissez-faire

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- Goods market clearing + Tobin's q:

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- Output gap $\hat{Y}_t \equiv Y_t / Y_t^* = \frac{a\upsilon_t K_t}{a\upsilon_t^* K_t^*}$:

$$\log \hat{Y}_t = \underbrace{\left(\log \upsilon_t - \log \upsilon_t^*\right)}_{\text{instantaneous}} + \underbrace{\left(\log K_t - \log K_t^*\right)}_{\text{dynamic}}$$

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- Sticky prices and no policy response $\implies$ either static or dynamic output gap
- Policy needs to move nominal wealth in response to an aggregate shock (see also Li and Merkel (2025))

Laissez-faire

- Distributional efficiency requires that $\tilde{\sigma}_t^2 \uparrow \Longrightarrow \vartheta_t \uparrow$ and $\eta_t \uparrow$ ($\sigma_t^{\eta} > 0$, $\sigma_t^{\vartheta} > 0$)

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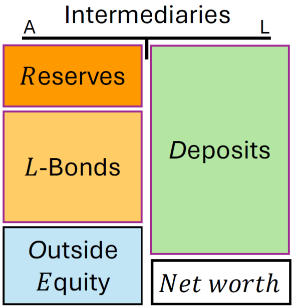

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- Higher $\tilde{\sigma}_t^2 \implies$ higher ϑ_t but lower η_t
- Intermediaries are short in nominal assets and long in capital $\implies$ their net worth share decreases

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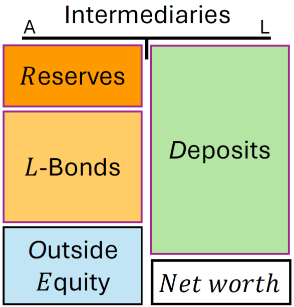

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- Tight link between wealth alloc. across assets (ϑ_t) and agents (η_t) in CE

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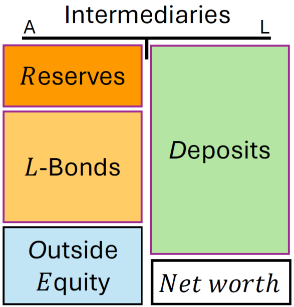

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- Policy needs to redistribute wealth in response to an aggregate shock

Realistic Government

- **Central bank:**
  - Sets interest rates $\underline{i}_t$ and i_t, reserve requirements $\underline{\theta}_t^R$
  - Issues reserves $\frac{d\mathcal{R}_t}{\mathcal{R}_t} = \mu_t^{\mathcal{R}} dt + \sigma_t^{\mathcal{R}} dZ_t$
  - Holds bonds $\frac{dL_t^{CB}}{L_t^{CB}} = \mu_t^{L,CB} dt + \sigma_t^{L,CB} dZ_t$

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  - Holds bonds $\frac{dL_t^{CB}}{L_t^{CB}} = \mu_t^{L,CB} dt + \sigma_t^{L,CB} dZ_t$
- **Fiscal authority:**
  - Issues long-term bonds $dL_t^F = \mu_t^{L,F} L_t^F dt$ paying interest i^L, nominal price P_t^L
  - Levies a range of 'flow' taxes (intermediation, wealth, capital)
- Motivation: bonds are issued at auctions, CB bond purchases/sales are OMO

Realistic Government

- **Central bank:**
  - Sets interest rates $\underline{i}_t$ and i_t, reserve requirements $\underline{\theta}_t^R$
  - Issues reserves $\frac{d\mathcal{R}_t}{\mathcal{R}_t} = \mu_t^{\mathcal{R}} dt + \sigma_t^{\mathcal{R}} dZ_t$
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- Motivation: bonds are issued at auctions, CB bond purchases/sales are OMO
- Long-term bond holdings of private agents are $L_t^I + L_t^H = L_t = L_t^F - L_t^{CB}$
- Distribution across sectors $\alpha_t = L_t^I / L_t$ is endogenous

App

Policy

- Focus on interest rate and balance sheet policy
- Fiscal policy operates in the background
- Balance sheet policy controls the share of long-term bonds in nominal wealth:

$$\vartheta_t^L = \frac{P_t^L L_t}{\mathcal{R}_t + P_t^L L_t} = \frac{P_t^L L_t}{\mathcal{B}_t}$$

- Interest rate controls sensitivity of bond price to aggregate shocks:

$$\sigma_t^{P^L} = \frac{\partial \log P_t^L}{\partial \log \tilde{\sigma}_t^2}\sigma, \qquad P_t^L = \mathbb{E}_t \int_t^\infty e^{-\int_t^\tau \left(i_s + \sigma_s^{P^L}\left(\sigma_s^\eta - \sigma_s^\vartheta + \sigma_s^\mathcal{B}\right)\right) ds} i^L d\tau$$

Passive Balance Sheet Management

- Suppose $\vartheta_t^L = \vartheta^L \in (0, 1)$

Passive Balance Sheet Management

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- **Proposition 1.** Fiscal + interest rate policy can implement aggregate efficiency:

$$a v_t^* = \rho + (1+\rho\phi)\iota_t^* + \rho \frac{\mathcal{B}_t}{\mathcal{P}_t K_t}$$

$$\mathcal{B}_t = \mathcal{R}_t + P_t^L L_t \qquad \sigma_t^{\mathcal{B}} = \vartheta^L \sigma_t^{P^L} = \underbrace{\frac{P_t^L L_t}{\mathcal{R}_t + P_t^L L_t}}_{\vartheta^L} \underbrace{\frac{\partial \log P_t^L}{\partial \log \tilde{\sigma}_t^2}}_{\text{interest rate } i_t} \sigma$$

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- Larger CB balance sheet (smaller ϑ^L) $\Longrightarrow$ more aggressive interest rate policy
- Preparatory role of balance sheet management

Passive Balance Sheet Management

- Can we also implement distributional efficiency?

Passive Balance Sheet Management

- Can we also implement distributional efficiency?
- From intermediaries' balance sheet:

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*)\sigma_t^{\vartheta,*} + \underbrace{\alpha_t \vartheta_t^* \vartheta^L}_{L_t^I/N_t} \sigma_t^{P^L} + \underbrace{(\chi_t^* - \eta_t^* - \vartheta_t^* \chi_t^*)}_{(OE_t^I - N_t^I)/N_t} \vartheta^L \sigma_t^{P^L}$$

- Direct bond revaluation: $\frac{L_t^I}{N_t} \sigma_t^{P^L}$
- Indirect aggregate wealth effect: $\frac{OE_t^I - N_t^I}{N_t} \vartheta^L \sigma_t^{P^L}$
  - For a fixed $\vartheta_t = \frac{\mathcal{B}/\mathcal{P}_t}{\mathcal{B}/\mathcal{P}_t + q_t^K K_t}$, $\mathcal{B}_t \uparrow \implies q_t^K \uparrow$
  - Capital price increase benefits levered intermediaries

App

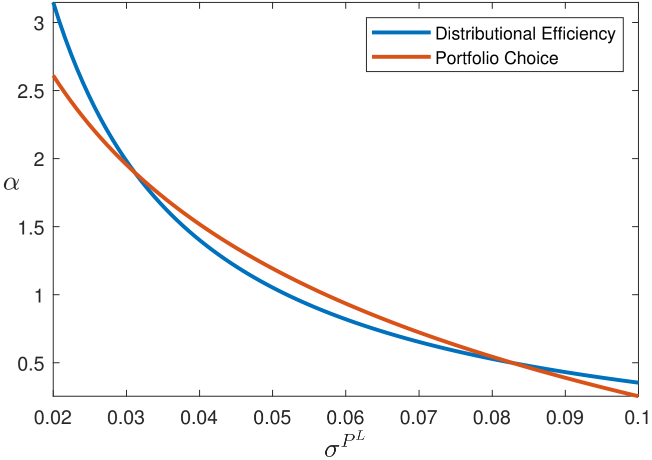
Passive Balance Sheet Management

- Can we also implement distributional efficiency?
- From intermediaries' balance sheet:

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*)\sigma_t^{\vartheta,*} + \alpha_t \vartheta_t^* \vartheta^L \sigma_t^{P^L} + (\chi_t^* - \eta_t^* - \vartheta_t^* \chi_t^*)\vartheta^L \sigma_t^{P^L}$$

- Challenge: endogenous bond distribution α_t
- Bonds are anti-hedge for intermediaries
- Intermediaries scale down on bonds if they get more volatile (if $\sigma_t^{P^L} \uparrow$)

Distributional Efficiency Fixing ϑ^L



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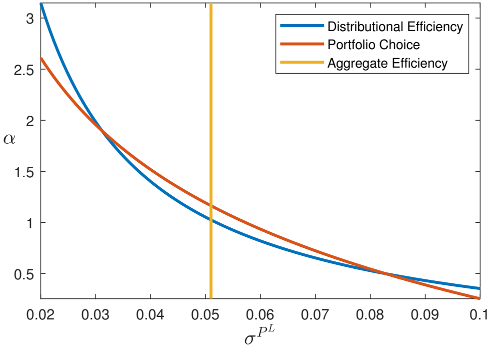
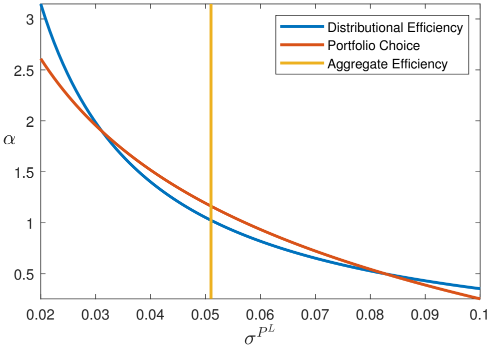
Aggregate and Distributional Efficiency Fixing ϑ^L

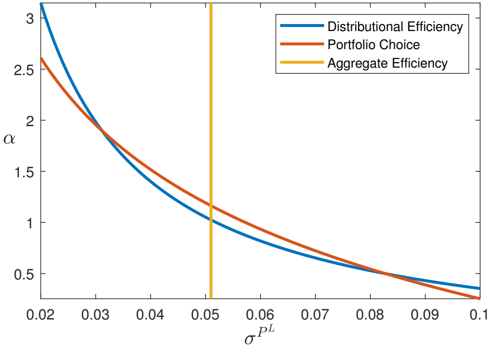


Aggregate and Distributional Efficiency Fixing ϑ^L



- Requires active balance sheet management!

Aggregate and Distributional Efficiency Fixing ϑ^L



- Requires active balance sheet management!
- Existence and uniqueness of an optimal policy mix under certain conditions
  - Intermediate degree of bond market segmentation

App

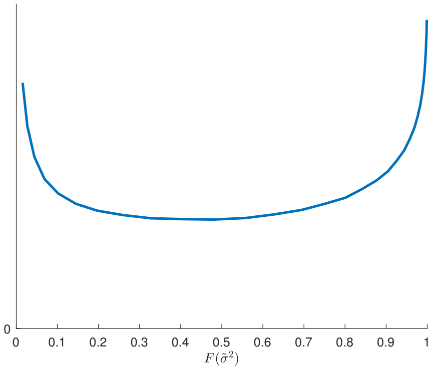
Passive balance sheet management: Welfare Loss

- Suppose pick ϑ^L such that $\sigma^{\eta}_{ss} = \sigma^{\eta,*}_{ss}$ and $\sigma^{\upsilon}_t = 0$
- $\implies$ efficient responses to shocks starting from the stochastic steady state

Passive balance sheet management: Welfare Loss

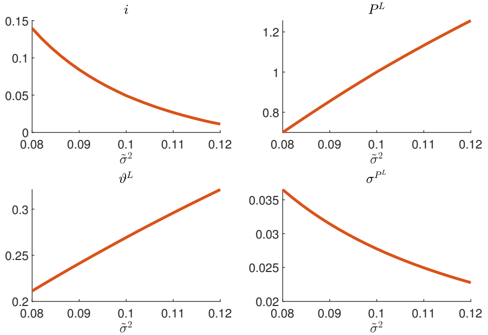
- Suppose pick ϑ^L such that $\sigma^{\eta}_{ss} = \sigma^{\eta,*}_{ss}$ and $\sigma^{\upsilon}_t = 0$
- $\implies$ efficient responses to shocks starting from the stochastic steady state

Figure: Welfare Loss



Optimal Policy Over the Cycle

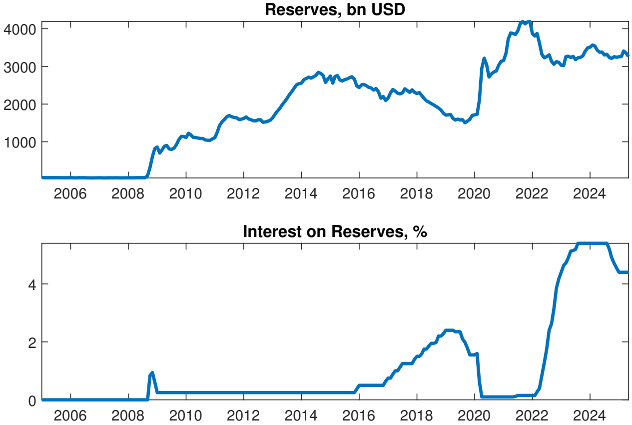
- **Proposition 2.** Let $\tilde{\sigma}_t^2$ be small. Then, an increase in $\tilde{\sigma}_t^2$ leads to:
  - a cut in the interest rate i_t
  - a milder interest rate policy going forward (lower $\sigma_t^{P^L}$)
  - a rebalancing towards long-term bonds (higher ϑ_t^L)



Summary

- Macrofinance model to study optimal policy mix
  - Role for aggregate and distributional efficiencies
- Preparatory role of balance sheet policies: efficient exposure to future shocks
- Larger CB balance sheet requires more aggressive interest rate policy subsequently
- Joint aggregate and distributional efficiency requires active balance sheet management over the cycle

Motivation



Back

Household's Problem

$$\max_{c_t^H, \upsilon_t, \iota_t, \nu_t, \theta_t^{D,H}, \theta_t^{L,H}, \theta_t^K, \chi_t} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log(c_t^H) - b(\upsilon_t)\right) dt\right] \qquad \text{s.t.}$$

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H} dt + \theta_t^{D,H} dr_t^D + \theta_t^{L,H} dr_t^L + \theta_t^K \left(dr_t^K(\upsilon_t, \iota_t, \nu_t) - \chi_t dr_t^{x,H}\right) + \tau_t^H dt$$

$$1 = \theta_t^{D,H} + \theta_t^{L,H} + \theta_t^K(1 - \chi_t) \qquad \nu_t \theta_t^{D,H} = \theta_t^K$$

- $dr_t^K = \left[\frac{p_t a \upsilon_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}_t(\nu_t) + \mu_t^{q^K} + g(\iota_t)\right] dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^{x,H} = r_t^x dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^D = i_t^D dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[i_t^D - \pi_t\right] dt$
- $dr_t^L = \frac{i^L}{P_t^L} dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right] dt + \sigma_t^{P^L} dZ_t$

Back

Intermediary's Problem

$$\max_{c_t^I, \theta_t^{\mathcal{R}}, \theta_t^{L,I}, \theta_t^{D,I}, \theta_t^{x,I}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log(c_t^I) dt\right] \qquad \text{s.t.}$$

$$\frac{dn_t^I}{n_t^I} = -\frac{c_t^I}{n_t^I} dt + \theta_t^{\mathcal{R}} dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) + \theta_t^{D,I} dr_t^D + \theta_t^{L,I} dr_t^L + \theta_t^{x,I} dr_t^{x,I} + \tau_t^I dt$$

$$1 = \theta_t^{\mathcal{R}} + \theta_t^{D,I} + \theta_t^{L,I} + \theta_t^{x,I} \qquad \theta_t^{\mathcal{R}} \geq \underline{\theta}_t^{\mathcal{R}}$$

- $dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) = i(\theta_t^{\mathcal{R}}) dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[\frac{\underline{\theta}_t^{\mathcal{R}} \underline{i}_t + (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}}) i_t}{\theta_t^{\mathcal{R}}} - \pi_t\right] dt$
- $dr_t^{x,I} = (r_t^x + \tau_t^x)\, dt + \sigma_t^{q^K} dZ_t + \varphi \tilde{\sigma}_t d\tilde{Z}_t$
- $dr_t^D = i_t^D dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[i_t^D - \pi_t\right] dt$
- $dr_t^L = \frac{i^L}{P_t^L} dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right] dt + \sigma_t^{P^L} dZ_t$

Back

Monopolistic Firms

- Monopolistic producers add variety to a common good produced by HH:
  - Linear technology: $Y_t^j = y_t^j$, set prices P_t^j s.t. Rotemberg frictions:

$$\int_0^\infty \Xi_t^H \left[\left(\frac{P_t^j}{P_t} \right)^{1-\varepsilon} - p_t(1-\tau^F) \left(\frac{P_t^j}{P_t} \right)^{-\varepsilon} - \frac{\kappa}{2} \left(\pi_t^j \right)^2 - T_t^F \right] Y_t dt$$

- Perfectly competitive final good producers
  - Bundle varieties into consumption good using CES aggregator
- NKPC:

$$\frac{\mathbb{E}\left[d\pi_t\right]}{dt} = \left(r_t^{f,H} - \frac{\mathbb{E}\left[dY_t\right]}{Y_t dt} + \varsigma_t^{C,H} \sigma_t^Y \right) \pi_t - \frac{\varepsilon}{\kappa} \left(p_t(1-\tau) - \frac{\varepsilon - 1}{\varepsilon} \right)$$

$$\pi_t = \frac{\varepsilon}{\kappa Y_t} \mathbb{E}_t \int_t^\infty e^{-\int_t^s r_\tau^f d\tau} Y_s \left(p_s(1-\tau) - \frac{\varepsilon - 1}{\varepsilon} \right) ds$$

Back

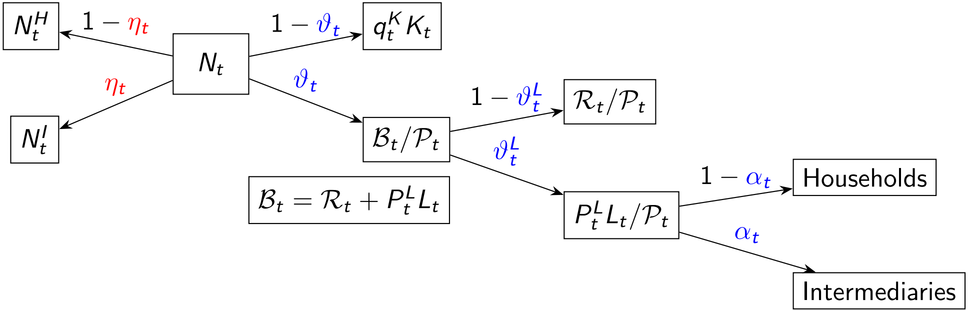
Constrained Efficiency

- **Proposition 1.** Under some assumption on $\{\lambda, \varphi\}$, there exists a unique solution to the planner's problem for $\eta^*(\tilde{\sigma}_t) > \lambda$ and the constrained efficient allocation has the following properties:
  - Capital utilization $\upsilon^*(\tilde{\sigma}_t)$ is constant ($\mu_t^{\upsilon,*} = \sigma_t^{\upsilon,*} = 0$)
  - Intermediaries' wealth and risk shares $\eta^*(\tilde{\sigma}_t)$ and $\chi^*(\tilde{\sigma}_t)$ are increasing in $\tilde{\sigma}_t$
  - Nominal wealth share $\vartheta^*(\tilde{\sigma}_t)$ is increasing in $\tilde{\sigma}_t$
  - Investment rate $\iota^*(\tilde{\sigma}_t)$ is decreasing in $\tilde{\sigma}_t$

App

Net Worth and Risk Distributions

- **Net worth distribution:**



- **Idiosyncratic risk distribution:** χ_t held by Intermediaries

  $1-\chi_t$ held by Households

Equilibrium

- Key variables: $\tilde{\sigma}_t$, η_t, υ_t, ϑ_t, P_t^L, π_t
- Markovian equilibrium with state variables $S \equiv \{\tilde{\sigma}, \eta, \upsilon\}$:
  - Laws of motion for S:

$$d\tilde{\sigma}_t^2 = -b_s(\tilde{\sigma}_t^2 - \tilde{\sigma}_{ss}^2)dt + \sigma\tilde{\sigma}_t^2 dZ_t$$
$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ_t$$
$$\frac{d\upsilon_t}{\upsilon_t} = \mu_t^\upsilon dt + \sigma_t^\upsilon dZ_t$$

  - Policy variables $\underline{i}(S)$, $i(S)$, $\vartheta^L(S)$, $\underline{\theta}^{\mathcal{R}}(S)$, $\tau^I(S)$, $\tau^x(S)$, $\tau^K(S)$
  - Mappings $\vartheta(S), P^L(S), \pi(S)$

satisfying agents' optimality and market clearing

App

Efficient Consumption & Risk Allocation Only: Implementation

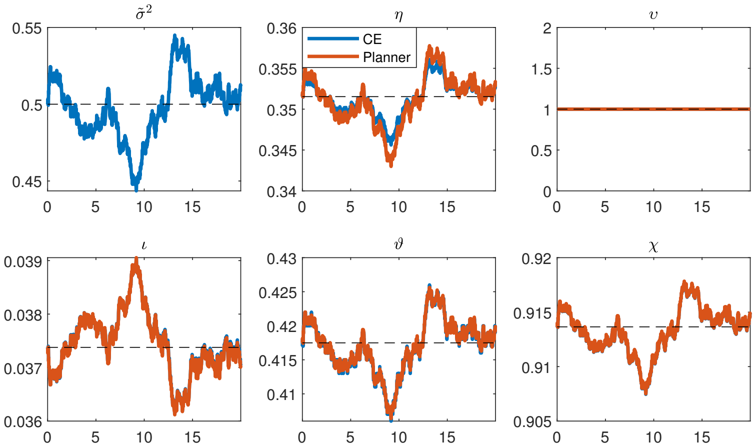
$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*)\sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^*(\alpha_t - \chi_t^*))\vartheta_t^L \sigma_t^{P^L}$$

$$\frac{\sigma_t^{\eta,*}}{1-\eta_t^*}\sigma_t^{P^L} = \nu_t^2 \mathfrak{t}'(\nu_t)$$

$$\nu_t \left[\chi_t^* - \eta_t^* + \vartheta_t^*(1-\chi_t^*) - (1-\alpha_t)\vartheta_t^L \vartheta_t^*\right] = 1 - \vartheta_t^*$$

Back

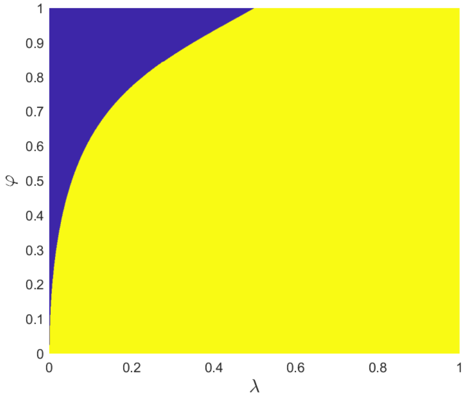
Passive balance sheet management



Back

Constrained Efficiency: Properties

$$6\lambda(1-\lambda)(1-\varphi^2)(1-\lambda+\lambda\varphi^2)-(1-2\lambda)\varphi^2 \geq 0$$



Back

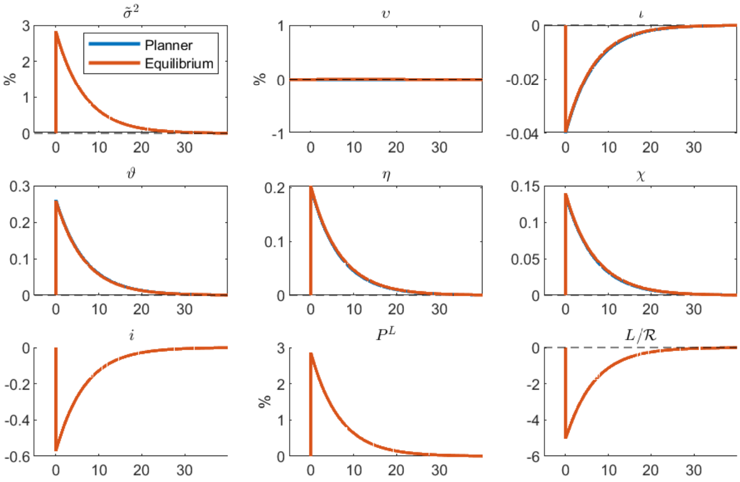
Consolidated Government

$$\mu_t^{\mathcal{R}}\mathcal{R}_t + P_t^L\mu_t^L L_t + \mathcal{P}_t\tau_t^K K_t = \underline{i}_t\underline{\mathcal{R}}_t + i_t(\mathcal{R}_t - \underline{\mathcal{R}}_t) + i^L L_t - \sigma_t^{P^L}\sigma_t^L P_t^L L_t$$

$$\sigma_t^{\mathcal{R}}\mathcal{R}_t + P_t^L\sigma_t^L L_t = 0$$

Back

IRF under Full Efficiency



Back

Production Efficiency Only: Implementation

- ϑ_t is an equilibrium 'mapping'

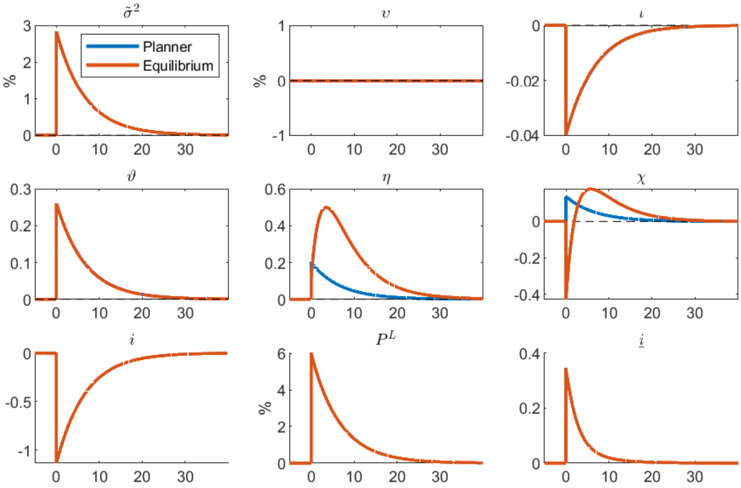
  $\Rightarrow$ implement ϑ_t^* by appropriate capital taxes τ_t^K along the equilibrium path
- υ_t is a state variable $\Rightarrow$ need to ensure $\mu_t^\upsilon = \sigma_t^\upsilon = 0 \; \forall t$
  - Drift is targeted by $\underline{i}_t$
  - Volatility loading is targeted by i_t and ϑ_t^L
- From goods market clearing:

$$a\upsilon_t = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \iota_t = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \frac{q_t^K - 1}{\phi} = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} + \frac{q_t^{\mathcal{B}}(1 - \vartheta_t)}{\phi \vartheta_t} - \frac{1}{\phi}$$

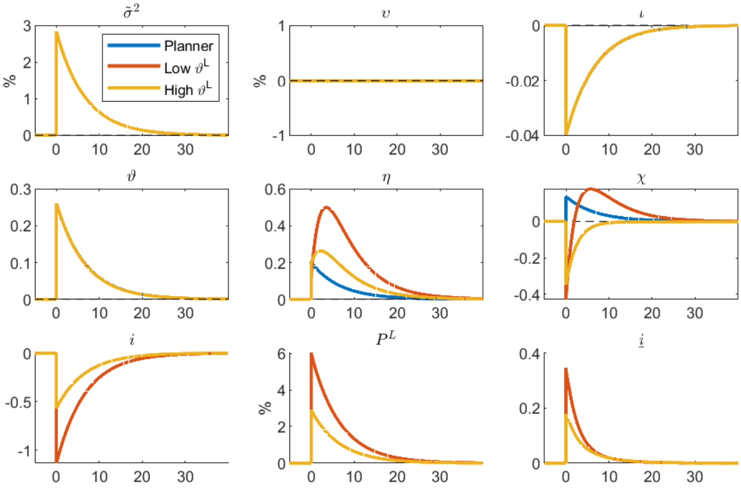
$$q_t^{\mathcal{B}} = \frac{\mathcal{B}_t}{\mathcal{P}_t K_t} \qquad \mathcal{B}_t = \mathcal{R}_t + P_t^L L_t$$

back

IRF under Production Efficiency

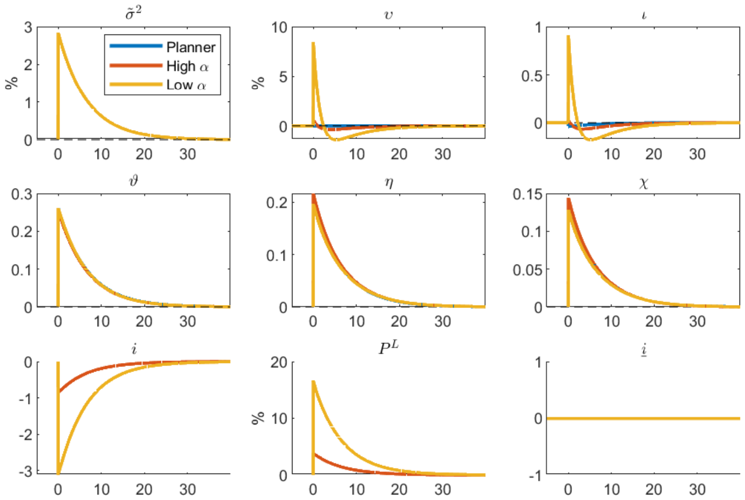


IRF under Production Efficiency: Equivalence



Intro FE

IRF under Allocative Efficiency: Multiplicity



Back