The Resilience Trade-off of Capital Regulation

(Princeton Initiative 2025)

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Objectives

1. Study the optimal capital regulation in a macrofinance model with
   * Heterogeneous agents with idiosyncratic + aggregate shocks and macro growth.
   * Regulatory constraint on capital.
   * Risk dynamics and resilience trade-off.

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   * Decomposing welfare into distribution, capital allocation, growth effects.
   * Asymmetry of over- vs under-regulation.

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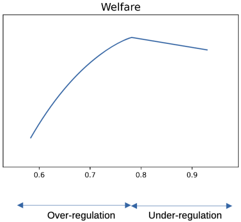
2. Welfare analysis
   * Decomposing welfare into distribution, capital allocation, growth effects.
   * Asymmetry of over- vs under-regulation.
3. Calibrate the model using a machine learning based estimation technique.
   * Build a surrogate model that maps the structural parameters to model moments
   * Balances numerical accuracy on equilibrium solution with computational efficiency on estimation. Easily generalizes to models with non-trivial equilibrium selection.

Results

- Resilience trade-off: tighter regulation implies
  * Amplification less pronounced.
  * Recovery slower and more dispersed.



- Welfare analysis:
  * Asymmetric welfare costs of over- vs under-regulation → Gradualism.
  * Welfare gain from current to optimal regulation is 2.5%.
  * Decomposition results: Growth ≥ Distribution > Capital allocation.

Literature review (Selected)

- Quantitative macro models with bank capital regulation
  [Abad et al., 2025], [Begenau, 2020], [Begenau and Landvoigt, 2022], , [Davydiuk, 2017], [Corbae and D'Erasmo, 2021], [Elenev et al., 2021], [Mendicino et al., 2018], ...
  - *This paper:* Asymmetric welfare costs, net worth trap, distributional implications.
- Dynamic banking and capital structure models
  [Gersbach and Rochet, 2017], [Hugonnier and Morellec, 2017], [Sundaresan and Wang, 2023], ...
  - *This paper:* Macro growth and asset price implications.
- Financial accelerator and intermediary asset pricing
  [Brunnermeier and Sannikov, 2014], [Krishnamurthy and Li, 2025], ...
  - *This paper:* Studies optimal bank capital regulation.
- I-theory and money.
  [Brunnermeier and Sannikov, 2016], [Li and Merkel, 2024], ...
  - *This paper:* Capital regulation.
- *Resilience and dispersion of recovery (Novel in this paper).*

Table of Contents

Model: Economic Environment

- Continuous time $t \in [0, \infty)$. One perishable consumption good.
- Three types of agents (sectors): **intermediaries** (I), **households** (h): sector a and sector b.
    1. Households operate firms, produce consumption goods, and issue outside equity. Friction: can only issue on capital b, and constrained up to $\bar{\chi}$.
    2. Intermediaries purchase risky claims from firms, lend to households via safe deposits.
    3. Intermediaries are better risk managers of idiosyncratic risk than households.

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  2. Intermediaries purchase risky claims from firms, lend to households via safe deposits.
  3. Intermediaries are better risk managers of idiosyncratic risk than households.
- AK production technology with two types of capital: technology a and b.

$$\frac{dk_t^j}{k_t^j} = (\Phi(\iota_t) - \delta)\, dt + \boldsymbol{\sigma}^{\boldsymbol{T}} \boldsymbol{d}\boldsymbol{Z_t} + \tilde{\sigma} d\tilde{Z}_t; \quad j \in \{a, b\}; \qquad \boldsymbol{\sigma}^{\boldsymbol{j}\,T} := [\sigma^a 1_{j=a} \quad \sigma^b 1_{j=b}]$$

where $\boldsymbol{Z_t} ;= [Z_t^a \quad Z_t^b]$, and $\tilde{Z}_t$ are aggregate and idiosyncratic Brownian shocks.

- Total output is given by (Leontiff)
  $Y_t = AK_t = a(1-\kappa)K_t + a\kappa K_t; \quad K_t = \int k_t^a di + \int k_t^b di$

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta)\, dt + \left((1-\kappa)\boldsymbol{\sigma^a} \mathbf{1}^a + \kappa \boldsymbol{\sigma^b} \mathbf{1}^b\right)^T \boldsymbol{d}\boldsymbol{Z_t}$$

Model: Economic Environment

- Govt. issues outside money whose nominal value is B_t, pays interest i_t, and collects taxes. The bond supply dynamics is

$$\frac{dB_t}{B_t} = \mu_t^B dt$$

- The government budget constraint is

$$\mu_t^B B_t = i_t B_t + \tau_t \mathcal{P}_t K_t$$

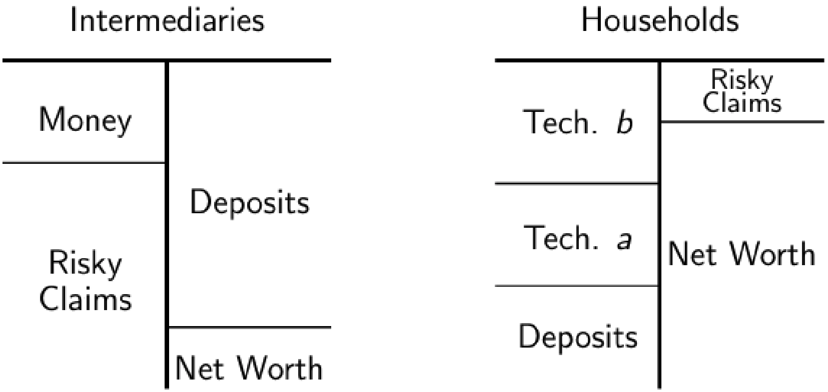
where $\mathcal{P}_t$ is the price level, and τ_t denotes taxes.

- The real price of bonds (per unit of K) is denoted by $q_t^B = \frac{B_t}{\mathcal{P}_t K_t}$,
The real price of capital is $q_t^K K_t$. Total wealth: $N_t = q_t^B K_t + q_t^K K_t$.

- The government budget constraint can be written as

$$\underbrace{\left(\mu_t^B - i_t\right)}_{\check{\mu}_t^B} q_t^B = \tau_t$$

Balance Sheet



Households

- Operates firms, holds capital, deposits, and issues outside equity χ_t.
  Constraint: $\chi_t \leq \bar{\chi} < 1$.

- Maximize lifetime utility subject to net worth constraints, short-selling constraints on capital and deposits.

  Choose portfolio allocations (capital, deposit, and outside equity), investment rate ι_t^h.

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  Choose portfolio allocations (capital, deposit, and outside equity), investment rate ι_t^h.

Formally, their problem is

$$\max_{\{c_t^h, \iota_t^h\}, \theta_t^a, \theta_t^b, \theta_t^D, \theta_t^{h,OE}} E_0 \left[\int_0^\infty e^{-\rho t} \log c_t^h dt \right]$$

subject to wealth dynamics

$$\frac{dn_t^h}{n_t^h} = -\frac{c_t^h}{n_t^h} dt + \theta_t^{h,D} dr_t^D + \theta_t^a dR_t^a(\iota_t) + \theta_t^b dR_t^b(\iota_t) + \theta_t^{h,OE} dR_t^{h,OE}$$

$$1 = \theta_t^{h,D} + \theta_t^a + \theta_t^b + \theta_t^{h,OE}$$

$$\frac{\chi_t}{\kappa} := -\frac{\theta_t^{h,OE}}{\theta_t^b} \leq \frac{\bar{\chi}}{\kappa}$$

Intermediaries

- Does not produce but holds risky claims of firms, holds money and safe deposits (B for both - abuse of notation), and issues safe deposits.
- Maximize lifetime utility subject to wealth constraints, short-selling constraints on capital and deposits.

  Choose portfolio allocations (share in risky claims, deposit and money (D for both - abuse of notation)), investment rate.
- Faces a leverage constraint ℓ on risky claims imposed by the regulator.

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Formally, their problem is

$$\max_{\{c_t^I, \iota_t\}, \theta_t^{I,B}, \theta_t^{I,D}, \theta_t^{I,OE}} E_0 \left[\int_0^\infty e^{-\rho t} \log c_t^I \, dt \right]$$

subject to wealth dynamics written in N_t numeraire

$$\frac{d\hat{n}_t^I}{\hat{n}_t^I} = -\frac{\hat{c}_t^I}{\hat{n}_t^I} dt + d\hat{R}_t^B + \theta_t^{I,OE} \left(dR_t^{I,OE} - d\hat{R}_t^B \right)$$

s.t. $\theta_t^{I,OE} < 1/(1-\ell)$, where ℓ is max leverage (regulatory constraint).

FOCs

Markov Equilibrium

We solve for a Markov equilibrium in the wealth share of intermediaries $\eta_t = N_t^I / N_t$. The equilibrium is standard with the following market clearing conditions.

1. Goods market: $AK_t - \iota_t K_t = C_t^h + C_t^I$, where the capital letters denote aggretate quantities (e.g., $C_t^h = \int c_t^h dh$). Rewrite as

$$(A - \iota_t)(1 - \vartheta_t) = q_t^K \left[(1 - \eta_t) \frac{C_t^h}{N_t^h} + \eta_t \frac{C_t^I}{N_t^I} \right]$$

2. Capital market:
   - Technology a: $(1 - \kappa) q_t^K K_t = \theta_t^a N_t^h$
   - Technology b: $\kappa q_t^k K_t = \theta_t^b N_t^h$
3. Risky claims market: $\eta_t \theta_t^{I,OE} + (1 - \eta_t) \theta_t^{h,OE} = 0$
4. Deposit market: $\eta_t \theta_t^{I,D} + (1 - \eta_t) \theta_t^{h,D} = 0$

Model Dynamics

Recall that $\eta_t = \frac{N_t^I}{N_t}$.

$$\frac{d\eta_t}{\eta_t} = \underbrace{\left[-\rho + \hat{r}_t^B + \theta_t^{I,OE}\left(\hat{r}_t^{OE} - \hat{r}_t^B\right)\right]}_{\mu_t^\eta} dt + \boldsymbol{\sigma_t^{\eta}}^T \boldsymbol{dZ_t}$$

The equilibrium functions depend on the share of nominal wealth ϑ_t that follows

$$\frac{d\vartheta_t}{\vartheta_t} = \mu_t^\vartheta dt + \boldsymbol{\sigma_t^{\vartheta}}^T \boldsymbol{dZ_t}$$

where the drift of ϑ_t is the money valuation equation

$$\boxed{\mu_t^\vartheta = \rho + \check{\mu}_t^B - \left[\eta_t(\boldsymbol{\sigma_t^{\eta}}^T \boldsymbol{\sigma_t^{\eta}}) + (1-\eta_t)(\boldsymbol{\sigma_t^{1-\eta}}^T \boldsymbol{\sigma_t^{1-\eta}}) + \eta_t(\tilde{\sigma}_t^\eta)^2 + (1-\eta_t)(\tilde{\sigma}_t^{1-\eta})^2\right]}$$

Table of Contents

Model solution

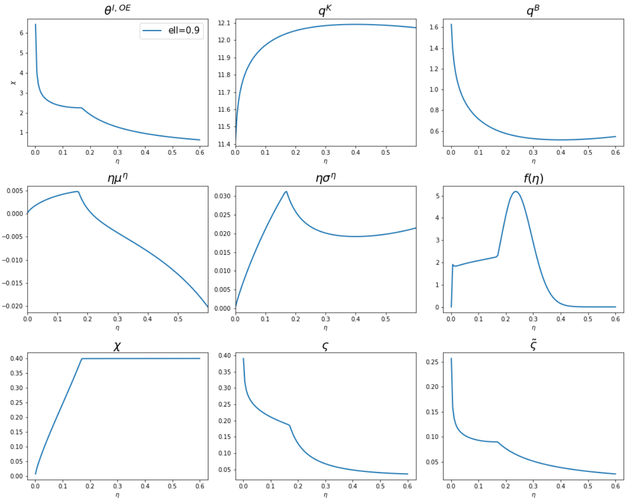


Figure: Parameters: $a = 0.5; \rho = 0.05; \delta = 0.03; \sigma = 0.1; \tilde{\sigma} = 0.4; \bar{\chi} = 0.4; \varphi = 0.2, \phi = 2$

Model solution

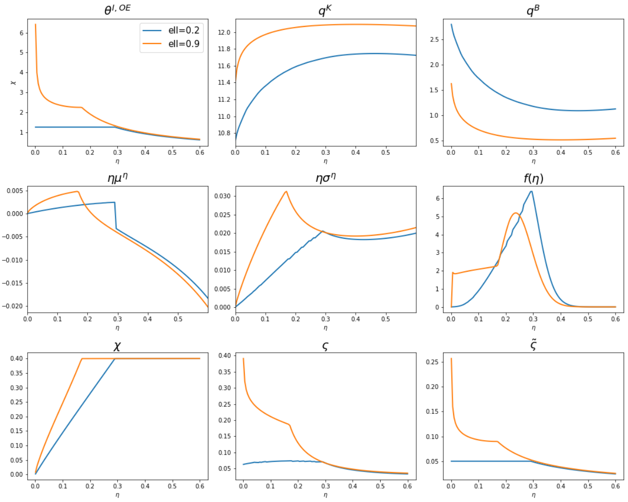
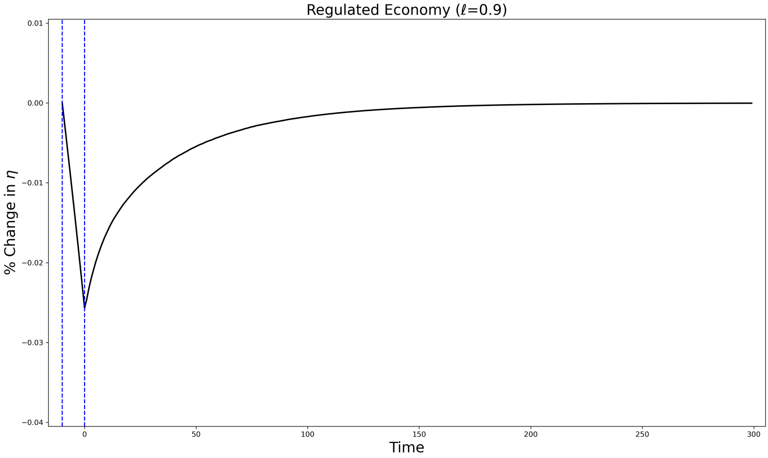


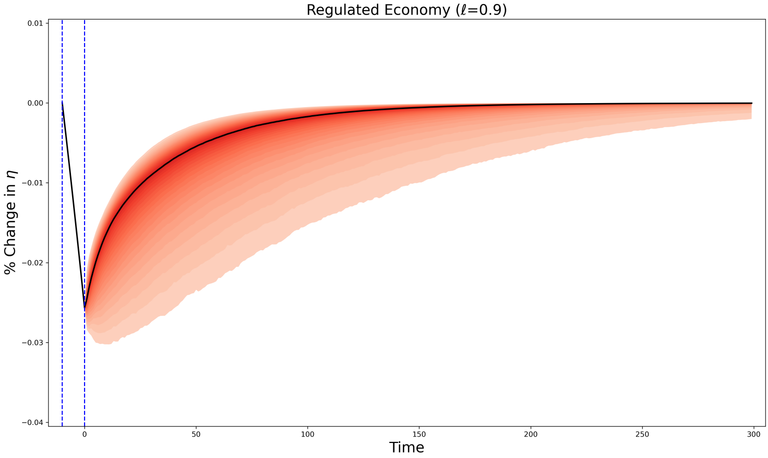
Figure: Parameters: $a = 0.5; \rho = 0.05; \delta = 0.03; \sigma = 0.1; \tilde{\sigma} = 0.4; \bar{\chi} = 0.4; \varphi = 0.2, \phi = 2$

Resilience



- Lax regulation: $\uparrow$ amplification
- Tight regulation: $\downarrow$ amplification

Resilience



- Lax regulation: ↑ amplification, **less dispersed recovery**.
- Tight regulation: ↓ amplification, **more dispersed recovery**.

Welfare

Total welfare: $W = \int_0^{1/2} W^I(i)di + \int_{1/2}^1 W^h(i)di.$

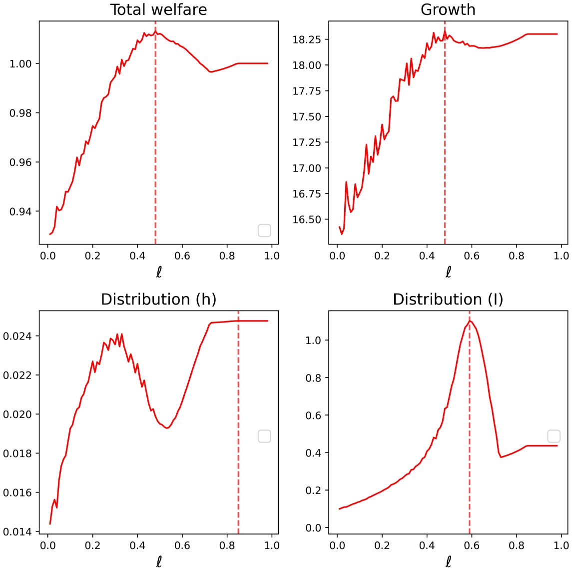
Welfare of agent i is

$$W(i) = \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log c_t^i dt\right]$$
$$= \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\underbrace{\log \check{\eta}_t{}^i}_{\text{Distribution}} + \underbrace{\log A(\kappa)}_{\text{Capital allocation}} + \underbrace{\frac{\Phi(\iota_t)-\delta}{\rho}}_{\text{Growth}} - \underbrace{\frac{(\tilde{\sigma}_t^{\eta,i})^2}{2\rho}}_{\text{Idio. risk}}\right) dt\right]$$

- $\check{\eta}_t{}^i$ is the net worth share of sector i.
- $A(\kappa)$ is the productivity net of investment.
- $\Phi(\iota_t) - \delta$ is the growth rate of economy.
- σ, ρ are exogenous volatility and discount rate parameters.

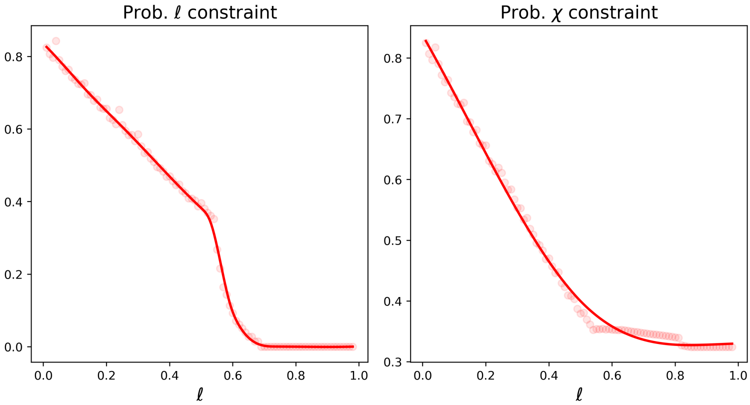
- We convert the welfare into permanent consumption equivalent units. Computation.

Welfare



Total welfare
1.00
0.98
0.96
0.94
0.0
0.2
0.4
0.6
0.8
1.0
ℓ
Growth
18.25
18.00
17.75
17.50
17.25
17.00
16.75
16.50
0.0
0.2
0.4
0.6
0.8
1.0
ℓ
Distribution (h)
0.024
0.022
0.020
0.018
0.016
0.014
0.0
0.2
0.4
0.6
0.8
1.0
ℓ
Distribution (l)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
ℓ

Welfare



Prob. ℓ constraint
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
ℓ
Prob. χ constraint
0.8
0.7
0.6
0.5
0.4
0.3
0.0
0.2
0.4
0.6
0.8
1.0
ℓ

Table of Contents

ML Calibration Strategy [WIP]

Parameter	**Variable definition**
$\sigma^a(\sigma^b)$	Fundamental vol. tech. a (tech. b)
$\tilde{\sigma}$	Idiosyncratic vol.
κ	Capital share in tech. a
$\bar{\chi}$	Equity constraint
φ	Risk management
ϕ	Investment friction
ρ	Discount rate
Data target moment	**Value**
GDP growth volatility	2%
Vol. Investment/GDP rate	5%
GDP growth rate	2%
Equity risk premium	6%
Risk-free rate	2%
Bank Leverage	10

Table: Estimated parameters (top panel) and data target moments (bottom panel).

ML Calibration Strategy [WIP]

- Let $\Psi \in \Omega^{\Psi}$ denote the structural parameters to be estimated.
- Let $\varphi(\Psi) = (\varphi_1(\Psi), ..., \varphi_N(\Psi))$ denote the corresponding model moments.
- Let $\tilde{\varphi} = (\tilde{\varphi_1}, ..., \tilde{\varphi_N})$ denote the data moments.

ML Calibration Strategy [WIP]

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Overview

Step-1: Use an ML model to learn the mapping $\widehat{\varphi}(\Psi; \Theta)$, where Θ are the ML model parameters. The model is trained on a set of simulated model moments $\varphi(\Psi)$ for different values of Ψ.

Step-2: Use the learned ML model to predict the model moments $\widehat{\varphi}(\Psi; \Theta^*)$ for any given parameter Ψ and optimal ML model parameters Θ^*.

Step-3: Use the predicted model moments to estimate the parameters Ψ by minimizing the loss function.

$$\hat{\Psi} = \arg\min \sum_{i}^{N} \left(\frac{\tilde{\varphi}_i - \widehat{\varphi}_i(\Psi; \Theta^*)}{\tilde{\varphi}_i} \right)^2 \tag{1}$$

ML Calibration Strategy [WIP]

We follow an iterative procedure to alleviate the curse of dimensionality in Step 1.

Draw parameters $\Psi^0 \sim \mathcal{U}(\Omega^\Psi)$. Set $\Psi = \Psi^0$.
while Loss $>$ tolerance **do**
    Simulate model, learn $\widehat{\varphi}(\Psi; \Theta^*)$ via ML model.
    Compute train/val losses, pick best Θ^*.
    Gradient Descent on (1) to find Ψ^*.
    Obtain perturbed parameter set Ψ^j by perturbing Ψ^*, i.e., $\Psi^j = \Psi^* + \epsilon$, where ϵ is $N(0, \sigma^2)$, where σ is equal to perturbation range.
    Draw new parameters Ψ^j from a normal distribution $\mathcal{N}(\overline{\Psi}_i, 1)$, where $\overline{\Psi}_i$ is the mean parameter value that generates model moments with the smallest $||\widehat{\varphi}(\Psi; \Theta^*)$ - $\tilde{\varphi}^j||$.
    Append the new parameter set $\Psi \leftarrow \Psi \cup \Psi^j$
    $i \leftarrow i + 1$
end while

Table of Contents

Conclusion

- Studied optimal capital regulation with macro growth and financial stability implications.
- Highlighted the resilience trade-off of tight regulation.
  - Amplification: small due to low risk taking.
  - Recovery: slow and dispersed due to net worth trap.
- Welfare analysis reveals
  - Non-linear effect: tighter regulation increases welfare but then suddenly drops off sharply $\rightarrow$ argues for gradual experimentation.
  - Welfare decomposition reveals that growth effects are important.
  - Welfare gains from current to optimal regulation is 2.5%.

Table of Contents

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Household problem

The SDF process for households in the N_t numeraire is given by

$$\frac{d\hat{\xi}_t^h}{\hat{\xi}_t^h} = -\hat{r}_t^h dt - \boldsymbol{\varsigma_t^h}^T \boldsymbol{dZ}_t - \tilde{\varsigma}_t{}^h d\tilde{Z}_t$$

Note, the prices of risk $\boldsymbol{\zeta_t}, \tilde{\zeta}_t$ are in units of N_t.

Household problem

The SDF process for households in the N_t numeraire is given by

$$\frac{d\hat{\xi}_t^h}{\hat{\xi}_t^h} = -\hat{r}_t^h dt - \boldsymbol{\varsigma_t}^{\boldsymbol{h}T} \boldsymbol{dZ}_t - \tilde{\varsigma}_t{}^h d\tilde{Z}_t$$

Note, the prices of risk $\boldsymbol{\zeta_t}, \tilde{\zeta}_t$ are in units of N_t. The Hamiltonian is given by

$$\begin{aligned}\mathcal{H}_t^h = {} & e^{-\rho t} \log c_t^h - \frac{c_t^h}{n_t^h} + \hat{\xi}_t^h \hat{n}_t^h \left[\hat{r}_t^D + \theta_t^a \left(\hat{r}_t^a - \hat{r}_t^D \right) + \theta_t^b \left(\hat{r}_t^b - \hat{r}_t^D - \frac{\chi_t}{\kappa} \left(\hat{r}_t^{OE} - \hat{r}_t^D \right) \right) \right] \\ & - \hat{\xi}_t^h \hat{n}_t^h \boldsymbol{\varsigma_t}^{\boldsymbol{h}T} \left[\hat{r}_t^D + \theta_t^a \left(\hat{\sigma}_t^a - \hat{\sigma}_t^D \right) + \theta_t^b \left(\hat{\boldsymbol{\sigma}}_t^{bT} \mathbf{1}^b - \hat{\sigma}_t^D \right) \left(1 - \frac{\chi_t}{\kappa} \right) \right] \\ & - \hat{\xi}_t^h \hat{n}_t^h \tilde{\varsigma}_t^h \left[\theta_t^a + \theta_t^b \left(1 - \frac{\chi_t}{\kappa} \right) \right] \tilde{\sigma} + \lambda_t^h \hat{\xi}_t^h \hat{n}_t^h \theta_t^b \frac{(\chi_t - \bar{\chi}_t)}{\kappa}\end{aligned}$$

Back

Household problem

The first order conditions are given by

$$[\theta_t^a]: \quad \hat{r}_t^a - \hat{r}_t^D = \varsigma_t^h \left(\hat{\boldsymbol{\sigma}}_t^a \mathbf{1}^a - \hat{\sigma}_t^D\right) + \tilde{\varsigma}_t^h \tilde{\sigma}$$

$$[\theta_t^b]: \quad \hat{r}_t^b - \hat{r}_t^D = \frac{\chi_t}{\kappa}\left(\hat{r}_t^{OE} - r_t^D\right) + \varsigma_t^h \left(\hat{\boldsymbol{\sigma}}_t^b \mathbf{1}^b - \hat{\sigma}_t^D\right)\left(1 - \frac{\chi_t}{\kappa}\right) + \tilde{\varsigma}_t^h \left(1 - \frac{\chi_t}{\kappa}\right)\tilde{\sigma}$$

$$[\chi_t]: \quad \hat{r}_t^{OE} - \hat{r}_t^D = \varsigma_t^h \left(\hat{\boldsymbol{\sigma}}_t^b \mathbf{1}^b - \hat{\sigma}_t^D\right) + \tilde{\varsigma}_t^h \tilde{\sigma} - \lambda_t^h$$

$$[\lambda_t]: \quad 0 = \lambda_t^h (\bar{\chi} - \chi_t)$$

We can combine the FOC for θ_t^b and χ_t to get the following relation

$$\hat{r}_t^b - \hat{r}_t^D = \varsigma_t^h \left(\hat{\boldsymbol{\sigma}}_t^b \mathbf{1}^b - \sigma_t^B\right) + \tilde{\varsigma}_t^h \tilde{\sigma} - \lambda_t^h \frac{\chi_t}{\kappa}$$

With log utility, we have $\frac{c_t^h}{n_t^h} = \rho$.

Back

Expert problem

The hamiltonian is given by

$$\mathcal{H}_t^I = e^{-\rho t} \log c_t^I - \frac{c_t^I}{n_t^I} + \hat{\xi}_t^I \hat{n}_t^I \left[\hat{r}_t^B + \theta_t^{I,OE} \left(\hat{r}_t^{OE} - \hat{r}_t^B \right) \right]$$
$$- \hat{\xi}_t^I \hat{n}_t^I \boldsymbol{\varsigma_t^{I}}^T \left[\hat{r}_t^B + \theta_t^{I,OE} \left(\hat{\boldsymbol{\sigma}}_t^b \mathbf{1}^b - \hat{\sigma}_t^B \right) \right] - \hat{\xi}_t^I \hat{n}_t^I \tilde{\varsigma}_t^I \tilde{\sigma}$$

The first order conditions are given by

$$[\theta_t^{I,OE}] : \qquad \hat{r}_t^{OE} - \hat{r}_t^B = \boldsymbol{\varsigma}_t^{IT} \left(\hat{\boldsymbol{\sigma}}_t^b \mathbf{1}^b - \hat{\sigma}_t^B \right) + \tilde{\varsigma}_t^I \tilde{\sigma} \varphi$$

As before, with log utility, we have $\frac{c_t^I}{n_t^I} = \rho$.

Back

Equilibrium

We solve for a Markov equilibrium in wealth share of intermediaries defined as $\eta_t := \frac{N_t^I}{N_t}$, where $N_t^I = \int \hat{n}_t^I di$ is the total wealth of intermediaries. The equilibrium is defined as follows

Definition: Markov Equilibrium A Markov equilibrium is a collection of prices $\{\hat{r}_t^D, \hat{r}_t^{OE}, \hat{r}_t^B, \hat{r}_t^a, \hat{r}_t^b\}$ and policies $\frac{c_t^h}{n_t^h}, \frac{c_t^I}{n_t^I}, \theta_t^a, \theta_t^b, \theta_t^{h,OE}, \theta_t^{I,OE}, \theta_t^D, \theta_t^B\}$ such that

1. The policies satisfy the first order conditions for households and intermediaries and is consistent with the wealth dynamics of households and intermediaries.
2. The processes are consistent with the government budget constraint.
3. The following markets clear
   3.1 Goods market: $AK_t - \iota_t K_t = C_t^h + C_t^I$, where the capital letters denote aggretate quantities (e.g., $C_t^h = \int c_t^h dh$). This can be rewritten as
   $$(A - \iota_t)(1 - \vartheta_t) = q_t^K \left[(1 - \eta_t) \frac{C_t^h}{N_t^h} + \eta_t \frac{C_t^I}{N_t^I} \right].$$
   3.2 Capital market:
   - Technology a: $(1 - \kappa) q_t^K K_t = \theta_t^a N_t^h$
   - Technology b: $\kappa q_t^k K_t = \theta_t^b N_t^h$

   3.3 Risky claims market: $\eta_t \theta_t^{h,OE} + (1 - \eta_t) \theta_t^{I,OE} = 0$
   3.4 Deposit market: $(1 - \eta_t) \theta_t^{h,D} + \eta_t \theta_t^{I,D} = 0$

Appendix: Welfare

The permanent consumption-equivalent welfare for the economy m for agent j is computed as

$$\begin{aligned} V^{j,m} &= \exp\left\{\rho\left[\left(\int v^{j,m}(\eta)d\nu^m(\eta) - \int v^{j,0}(\eta)d\nu^0(\eta)\right)\right]\right\} \\ &\quad + \exp\left\{\rho\left[\left(\int q^m(\eta)d\nu^m(\eta) - \int q^0(\eta)d\nu^0(\eta)\right)\right]\right\} \\ &= \exp\left\{\rho\left[\left(W^{j,m} - W^{j,0}\right) + \left(\int q^m(\eta)d\nu^m(\eta) - \int q^0(\eta)d\nu^0(\eta)\right)\right]\right\} \end{aligned}$$

Back