Macrofinance

Lecture 09: New Keynesian Money Model

Guest Lecture: Sebastian Merkel

Princeton University

Summer, 2025

Course Overview

This Lecture

Questions:

- Modeling questions:
  - How to incorporate New Keynesian (NK) price setting frictions into continuous-time macrofinance models?
  - What are implications of adding them to safe asset framework?
- Broader economic questions:
  - What are implications of risk (premium) shocks for aggregate economic activity?
  - How do these shocks transmit to the real economy?
  - How can (monetary) policy affect this transmission?

Outline

Outline

Simplified Money Model from Lectures 06 & 07: Setup

- Continuous time, infinite horizon, one consumption good
- Continuum of households
  - operate capital subject to idiosyncratic risk, linear production technology
  - can trade capital and government bonds
  - *simplifying assumptions*: no real investment or capital growth; no monetary frictions
- Government
  - taxes capital
  - issues nominal bonds
  - *simplifying assumption*: no government spending
- Financial friction: incomplete markets
  - agents cannot trade idiosyncratic risk
- Aggregate fluctuations: idiosyncratic shock volatility ($\tilde{\sigma}_t$) and productivity (a_t)
  - *simplifying assumption*: MIT shocks only

Simplified Money Model: Formal Details

- Preferences ($i \in [0,1]$ agent index):

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right]$$

- Each agent manages capital k_t^i
  - output flow: $y_t^i dt = a_t k_t^i dt$
  - capital tax by government: $\tau_t k_t^i dt$
  - capital evolution: $dk_t^i = \underbrace{k_t^i d\Delta_t^{k,i}}_{\text{trading}} + \underbrace{k_t^i \tilde{\sigma}_t d\tilde{Z}_t^i}_{\text{idio. shocks}}$

- Aggregates and market clearing
  - normalize $K_t := \int k_t^i di = 1$
  - goods market clearing $C_t := \int c_t^i di = \int y_t^i di =: Y_t = 1$

- Government:
  - budget constraint $\overbrace{i_t \mathcal{B}_t}^{\text{interest payments}} = \overbrace{\mathcal{P}_t \tau_t}^{\text{prim. surpluses}} + \overbrace{\mu_t^{\mathcal{B}} \mathcal{B}_t}^{\text{bond issuance}} \Rightarrow i_t = \mu_t^{\mathcal{B}} + \underbrace{\frac{\tau_t}{\mathcal{B}_t / \mathcal{P}_t}}_{=: \check{s}_t}$

Notation: Assets Values

- Assets in positive net supply: capital & bonds
  - capital: aggregate supply $K_t = 1$, value q_t^K
  - bonds: real value of bond stock $q_t^B := \frac{\mathcal{B}_t}{\mathcal{P}_t}$
- Also define total wealth (per unit of capital) $q_t := q_t^B + q_t^K$
- Share of bond wealth

$$\vartheta_t := \frac{\mathcal{B}_t/\mathcal{P}_t}{q_t^K + \mathcal{B}_t/\mathcal{P}_t} = \frac{q_t^B}{q_t}$$

- In equilibrium:
  - all households choose identical portfolios
  - ϑ_t is also individual portfolio weight in bonds

Model Solution

Recall from previous lectures:

- Model solution conditional on ϑ_t and exogenous a_t

$$C_t = \rho q_t \qquad q_t = \frac{a_t}{\rho} \qquad q_t^B = \vartheta_t \frac{a_t}{\rho} \qquad q_t^K = (1-\vartheta_t)\frac{a_t}{\rho}$$

- Implied price level and inflation dynamics

$$\mathcal{P}_t = \mathcal{B}_t / q_t^B \qquad \Rightarrow \qquad \frac{d\mathcal{P}_t}{\mathcal{P}_t} = (i_t - \check{s}_t)dt + \frac{d(1/q_t^B)}{1/q_t^B}$$

- Endogenous dynamics of ϑ_t: unique solution to *government liabilities valuation equation*

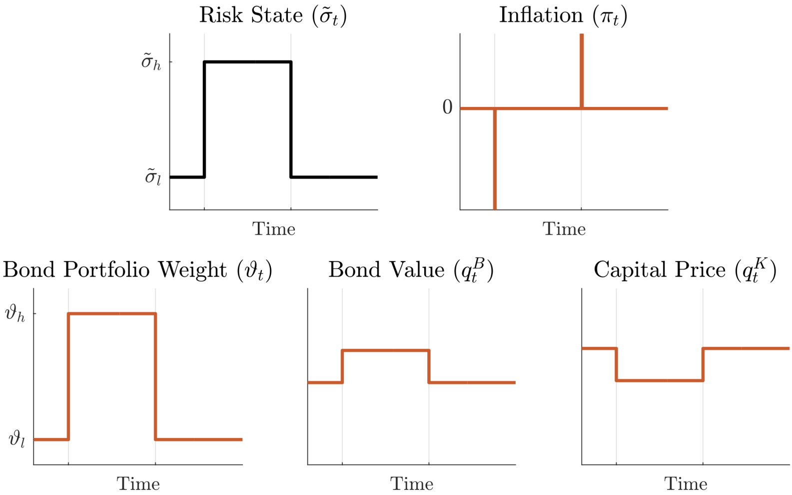
$$d\vartheta_t = (\rho - \check{s}_t - (1-\vartheta_t)^2 \tilde{\sigma}_t^2)\vartheta_t dt$$

that satisfies $\liminf_{t\to\infty} \vartheta_t > 0$

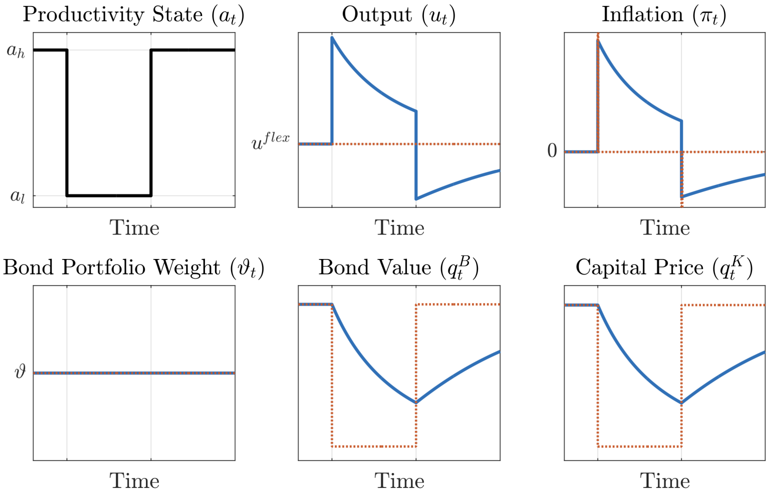
  - In steady state (constant $\tilde{\sigma}$, $\check{s}$): $\vartheta = \frac{\tilde{\sigma} - \sqrt{\rho - \check{s}}}{\tilde{\sigma}}$

Outline

Safe Asset Demand Shock: $\tilde{\sigma} \uparrow$ ($\Rightarrow$ Negative Aggregate Demand Shock)



Negative Aggregate Supply Shock: $a_t \downarrow$



Outline

Accommodating Price Setting Frictions

- Goal: add price setting frictions to generate sticky nominal goods price dynamics
- Need two features to accommodate price setting frictions
  1. elastic short-term supply (within *dt*-period)
     - at "wrong" prices, goods demand may be excessive or insufficient
     - markets can only clear if supply can adjust within the period
     - → introduce variable capital utilization
  2. individual price-setting firms cannot face perfectly elastic demand
     - Walrasian market: each agent faces a flat demand curve (price taker)
     - no meaningful price setting problem: $p + \varepsilon$: no demand, $p - \varepsilon$: infinite demand
     - → introduce differentiated goods and monopolistic competition (but eliminate other distortions this creates with subsidy & profit redistribution)
- Will first introduce these features in flexible price environment

Extended Model: Supply Side

- Final goods producers: combine differentiated goods using CES technology

$$Y_t = \left(\int (y_t(j))^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \qquad \Rightarrow \text{demand for variety } j: \quad y_t(j) = (p_t(j))^{-\epsilon} Y_t$$

  - final goods market clearing: $C_t = Y_t$

- Intermediate goods firm j: produces differentiated good j with capital services
  - rent capital services $\hat{k}_t(j)$ from households at unit price p_t^R
  - production function: $y_t(j) = a_t \hat{k}_t(j)$
  - time-t profits (τ^f: output subsidy)

$$\varpi_t(j) = (1 + \tau^f) p_t(j) y_t(j) - p_t^R \hat{k}_t(j)$$

- Household i: creates capital services $\hat{k}_t^i$ by utilizing capital
  - household preferences:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \left(\log c_t^i - \frac{(u_t^i)^{1+\varphi}}{1+\varphi} \right) dt \right]$$

  - produces capital services $u_t^i k_t^i dt$
  - market clearing for capital services: $\int \hat{k}_t(j) dj = \int \hat{k}_t^i di$

Distribution of Firm Profits and Firm Objective

- Aggregate firm profits net of subsidies

$$\omega_t := \int \varpi_t(j)dj - \tau^f \int p_t(j)y_t(j)dj = \int \varpi_t(j)dj - \tau^f Y_t$$

- Profits net of subsidies are distributed to households *in proportion to capital holdings*
  - reasoning: separate intermediate goods firms are fiction to keep model tractable
  - ultimately, all cash flows from production should accrue to productive factors (here capital)
- Implications for firm objective: (will matter only later with sticky prices)
  - Cash flow $\omega_t k_t^i$ distributed to agent i has idiosyncratic risk
  - Household i would like firms to take this into account in its choices

  $\Rightarrow$ Firm j should maximize present value of profits

$$\mathbb{E}\left[\int_0^\infty \xi_t^{**} \varpi_t(j)dt\right]$$

  with *weighted average SDF* $\xi_t^{**} = \int \eta_t^i \xi_t^i di = e^{-\rho t}/C_t$

Solving the Model: Firm Problem

Firm price setting problem: choose real prices $\{p_t(j)\}$ and capital service demand $\{\hat{k}_t(j)\}$ to maximize

$$\mathbb{E}\left[\int_0^\infty \xi_t^{**}\left((1+\tau^f)p_t(j)a_t\hat{k}_t(j) - p_t^R\hat{k}_t(j)\right)dt\right]$$

subject to the demand curve of final goods producers

$$a_t\hat{k}_t(j) = (p_t(j))^{-\epsilon}\,Y_t$$

- optimal choice: constant markup over unit marginal cost

$$p_t(j) = \frac{1}{1+\tau^f}\frac{\epsilon}{\epsilon-1}\frac{p_t^R}{a_t}$$

- in equilibrium: $p_t(j)=1$ for all j, so this determines p_t^R
- if $\tau^f = \frac{1}{\epsilon-1}$, $p_t^R = a_t$ and $\omega_t = 0$ (assume this from now on)

Solving the Model: Household Problem

Household problem: choose $\{c_t^i, u_t^i, \theta_t^i\}$ to maximize

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log c_t^i - \frac{(u_t^i)^{1+\varphi}}{1+\varphi}\right)dt\right]$$

subject to

$$dn_t^i = -c_t^i dt + n_t^i\left(\theta_t^i dr_t^B + (1-\theta_t^i)dr_t^{K,i}(u_t^i)\right)$$

- return on capital:

$$dr_t^{K,i}(u_t^i) = \left(\frac{p_t^R u_t^i + \omega_t - \tau_t}{q_t^K} + \mu_t^{q,K}\right)dt + \tilde{\sigma}_t d\tilde{Z}_t^i$$

- optimal choices:

$$c_t^i = \rho n_t^i \qquad \text{(consumption)}$$

$$(u_t^i)^\varphi = \frac{p_t^R k_t^i}{c_t^i} \qquad \text{(utilization effort)}$$

$$\frac{\mathbb{E}_t[dr_t^K]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = (1-\theta_t^i)\tilde{\sigma}_t^2 \qquad \text{(portfolio choice)}$$

Model Solution

- Production side:
  - all firms choose same $p_t(j)$, all households same u_t^i
    $\Rightarrow$ aggregate supply: $Y_t = a_t u_t$
  - combine optimal u_t with goods market clearing:

$$u_t^{\varphi} = \frac{p_t^R}{a_t u_t} \Rightarrow u_t = \left(\frac{p_t^R}{a_t}\right)^{\frac{1}{1+\varphi}}$$

  - using $p_t = a_t$:

$$\boxed{u_t = u^* := 1}$$

- Asset pricing:
  - in equilibrium: returns on capital and bonds as in baseline model
  - therefore: portfolio choice implies same government liability valuation equation as before

$$\boxed{d\vartheta_t = (\rho - \check{s}_t - (1-\vartheta_t)^2 \tilde{\sigma}_t^2)\vartheta_t dt}$$

- *Conclusion*: identical equilibrium as in baseline model

Aside: Relationship to Textbook Model with Labor

- Turn off idiosyncratic risk, $\tilde{\sigma}_t = 0$, so that all households are the same
- Can relabel things:
  - utilization $u_t \rightarrow$ labor ℓ_t
  - rental price $p_t^R \rightarrow$ wage w_t
  - make capital non-tradeable
- Then this is the flexible-price version of a standard New Keynesian textbook model
  (e.g. Gali 2015)
- Why the (unconventional) capital formulation?
  - closer to other models you have seen in this lecture
  - with $\tilde{\sigma}_t > 0$: gains from u_t scale with wealth, preserves linear aggregation
    (relies on two features: (1) i's productivity is $u_t^i k_t^i$; (ii) capital is tradeable)

Outline

Price Adjustment Costs

- Denote by $\mathcal{P}_t^j = p_t(j)\mathcal{P}_t$ firm j's *nominal price*, suppose

$$d\mathcal{P}_t^j = \pi_t^j dt \qquad (*)$$

  and the firm faces *Rotemberg price adjustment costs* $-\frac{\psi}{2}(\pi_t^j)^2 Y_t$

- Firm problem: choose $\{\pi_t^j, \hat{k}_t(j)\}$ to maximize

$$\mathbb{E}\left[\int_0^\infty \xi_t^{**}\left((1+\tau^f)p_t(j)a_t\hat{k}_t(j) - p_t^R\hat{k}_t(j) - \frac{\psi}{2}(\pi_t^j)^2 Y_t\right)dt\right]$$

  subject to demand and $(*)$

- Rebate price adjustment costs to households (instead of resource cost):

$$\omega_t = \int \varpi_t(j)dj - \tau^f Y_t + \frac{\psi}{2}\int (\pi_t^j)^2 dj\, Y_t$$

Discussion

- Why do we rebate price adjustment costs?
  - makes model solution *simpler*: no output effect from inflation
  - makes analysis *cleaner*: only one model aspect is affected, firm price setting
  - ultimately, this is not a good model of the costs of inflation anyway
- Would it be tractable without the rebate?
  - yes, but get an additional productivity wedge from adjustment costs
- What about Calvo frictions?
  - also possible but adds additional state variable: one-dim. summary statistic of price dispersion

Solving the Firm Problem

- Hamiltonian of firm j

$$H_t^j = \xi_t^{**}\left((1+\tau^f)\frac{\mathcal{P}_t^j}{\mathcal{P}_t}a_t\hat{k}_t^j - p_t^R\hat{k}_t^j - \frac{\psi}{2}\left(\pi_t^j\right)^2 Y_t\right) + \lambda_t^j\pi_t^j\mathcal{P}_t^j$$

$$= \underbrace{\xi_t^{**}Y_t}_{=e^{-\rho t}}\left(\left(\frac{\epsilon}{\epsilon-1}\frac{\mathcal{P}_t^j}{\mathcal{P}_t} - \frac{p_t^R}{a_t}\right)\left(\frac{\mathcal{P}_t^j}{\mathcal{P}_t}\right)^{-\varepsilon} - \frac{\psi}{2}\left(\pi_t^j\right)^2\right) + \lambda_t^j\pi_t^j\mathcal{P}_t^j$$

- First-order condition for π_t^j

$$\pi_t^j = \frac{\lambda_t^j\mathcal{P}_t^j}{\psi e^{-\rho t}}$$

- Costate equation

$$d\lambda_t^j = -\left(\frac{e^{-\rho t}}{\mathcal{P}_t}\left(\frac{\mathcal{P}_t^j}{\mathcal{P}_t}\right)^{-\epsilon}\epsilon\left(\frac{p_t^R}{a_t}\frac{\mathcal{P}_t}{\mathcal{P}_t^j} - 1\right) + \lambda_t^j\pi_t^j\right)dt$$

Solving the Firm Problem Continued

- In symmetric equilibrium, $\mathcal{P}_t^j = \mathcal{P}_t$, $\pi_t^j = \pi_t$, $\lambda_t^j = \lambda_t$, therefore:

$$\pi_t = \frac{\lambda_t \mathcal{P}_t}{\psi e^{-\rho t}}$$

$$d\lambda_t = -\left(\frac{e^{-\rho t}}{\mathcal{P}_t}\epsilon\left(\frac{p_t^R}{a_t} - 1\right) + \lambda_t \pi_t\right) dt$$

- Combining the two yields the *New Keynesian Phillips Curve* ($\kappa := \epsilon/\psi$)

$$\boxed{d\pi_t = \left(\rho\pi_t - \kappa\left(\frac{p_t^R}{a_t} - 1\right)\right) dt}$$

- *Remarks*:
  - this equation replaces $p_t^R = a_t$ from the flexible price model
  - the slope κ is inversely related to the degree of price flexibility (flexible prices: $\kappa \to \infty$)

Other Parts of the Model

- Price adjustment costs do not have a direct impact on
  - aggregate output and goods market clearing (due to rebate)
  - the household problem
  - the government budget constraint
- Most other model equations remain as in the flexible price model:

$$q_t = \frac{a_t u_t}{\rho} \qquad \text{(aggregate wealth)}$$

$$p_t^R = a_t u_t^{1+\varphi} \qquad \text{(optimal utilization)}$$

$$d\vartheta_t = (\rho - \check{s}_t - (1-\vartheta_t)^2 \tilde{\sigma}_t^2)\vartheta_t dt \qquad \text{(portfolio choice)}$$

- Key change: q_t^B becomes *"slow-moving" state variable*

$$dq_t^B = d(\mathcal{B}_t/\mathcal{P}_t) = (\mu_t^{\mathcal{B}} - \pi_t)dt$$

  - backward-looking drift dynamics, no reaction to shocks
  - difference to flexible prices where $\mathcal{P}_t$ and q_t^B are forward-looking "jump variables"

Equilibrium Definition

- Let X_t be an exogenous Markov process and $\tilde{\sigma}_t = \tilde{\sigma}(X_t)$, $a_t = a(X_t)$
- Let policy variables i_t, $\check{s}_t$ be given by feedback rules

$$i_t = i\left(\mathbf{A}_t\right), \qquad \check{s}_t = \check{s}\left(\mathbf{A}_t\right),$$

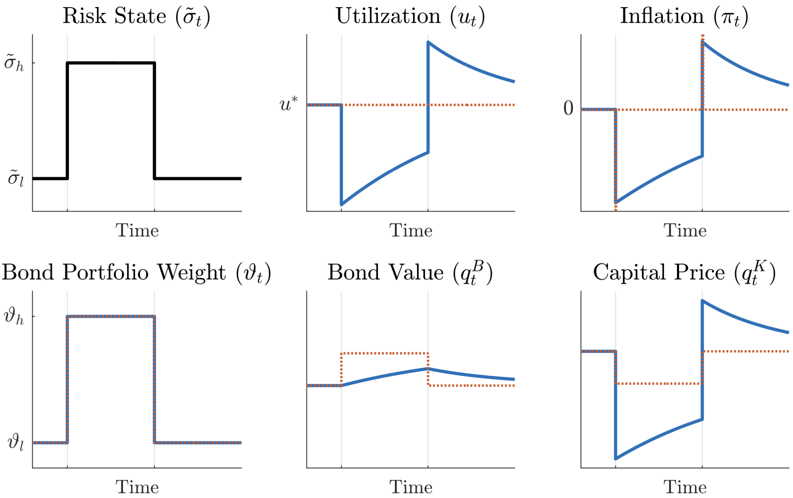
  where $\mathbf{A}_t$ is vector of (possibly endogenous) aggregates (e.g. $\mathbf{A}_t = (X_t, \pi_t)$)
- A *Markov equilibrium* consists of functions

$$(X, q^B) \mapsto \Big(C(X, q^B), u(X, q^B), q(X, q^B), \vartheta(X, q^B), p^R(X, q^B), i(X, q^B), \check{s}(X, q^B)\Big)$$

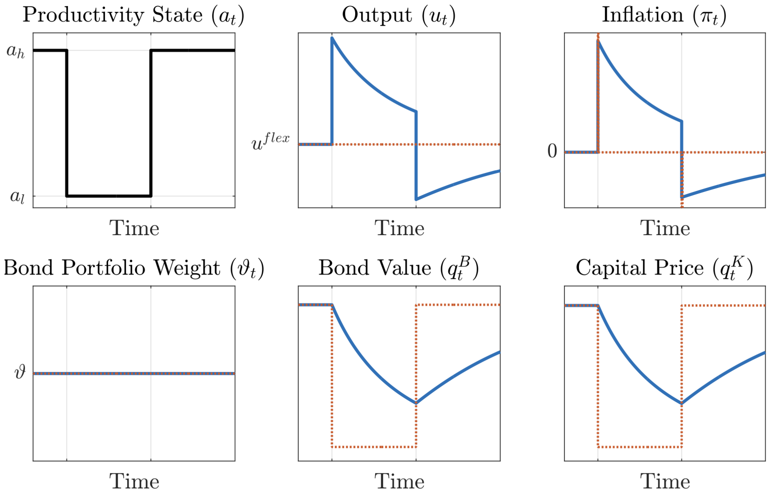
  such that
  - households & firms maximize
  - $i_t = i(X_t, q_t^B)$ & $\check{s}_t = \check{s}(X_t, q_t^B)$ satisfy the feedback rules
  - markets clear
  - q_t^B evolves as on the previous slide with $\mu_t^{\mathcal{B}} = i_t - \check{s}_t$

Outline

Positive Safe Asset Demand Shock $\tilde{\sigma} \uparrow$



Negative Supply Shock $a \downarrow$



Transmission Preliminaries I: Separation of Portfolio Choice

- Government liabilities valuation equation in integral form:

$$\vartheta_t = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \vartheta_s \left((1-\vartheta_s)^2 \tilde{\sigma}_s^2 + \check{s}_s \right) ds \right].$$

  - depends only on fiscal instrument $\check{s}_t$ and idiosyncratic risk $\tilde{\sigma}_t$
  - not on aggregate output or price setting frictions

- *Separation:* if $\check{s}_t$ is function of $(\tilde{\sigma}_t, \vartheta_t)$ only, then $\vartheta_t = \vartheta(\tilde{\sigma}_t)$ does not depend on bond valuation state q_t^B

  → portfolios adjust "fast" (as under flexible prices)

- *Remark*: separation condition satisfied for conventional linear fiscal reaction rules

$$S_t/Y_t = \alpha + \beta q_t^B/Y_t \qquad \Rightarrow \qquad \check{s}_t = \alpha \frac{\rho}{\vartheta_t} + \beta$$

- *Flight to Safety:* Unless $\check{s}$-policy leans strongly against it, rise in σ_t leads to increase in ϑ_t

Transmission Preliminaries II: Asset Valuations & Demand

- Goods market clearing relates real activity to *level of asset valuations*

$$a_t u_t = \rho q_t = \rho(q_t^B + q_t^K)$$

- Portfolio choice (ϑ_t) determines *relative asset valuations*

$$q_t = q_t^B + q_t^K = \frac{1}{\vartheta_t} q_t^B$$

- Combining the previous:

$$\boxed{a_t u_t = \rho \frac{q_t^B}{\vartheta_t}}$$

Shock Transmission: Impact Effect

$$\underbrace{a_t u_t}_{\text{supply}} = \underbrace{\rho \frac{q_t^B}{\vartheta_t}}_{\text{demand}}$$

	risk shock $\tilde{\sigma}_t \uparrow$		productivity shock $a_t \downarrow$	
	flexible prices	sticky prices	flexible prices	sticky prices
portfolio choice	$\vartheta_t \uparrow$	$\vartheta_t \uparrow$	$\vartheta_t \rightarrow$	$\vartheta_t \rightarrow$
demand for given q_t^B	$\downarrow$	$\downarrow$	$\rightarrow$	$\rightarrow$
supply for given u_t	$\rightarrow$	$\rightarrow$	$\downarrow$	$\downarrow$
equilibrium adjustment	$u_t \rightarrow$, $q_t^B \uparrow$	$u_t \downarrow$, $q_t^B \rightarrow$	$u_t \rightarrow$, $q_t^B \downarrow$	$u_t \uparrow$, $q_t^B \rightarrow$
required price adjustment	$\mathcal{P}_t \downarrow$	$\mathcal{P}_t \rightarrow$	$\mathcal{P}_t \uparrow$	$\mathcal{P}_t \rightarrow$

Shock Transmission: Adjustment Dynamics with Stickiness

- After shock, gradual inflation/deflation slowly adjusts q_t^B towards flexible-price value ("Pigou effect") (Pigou, 1943; Patinkin, 1956)
- Dynamics guided by two equations
  - Bond value evolution (backward-looking):

$$dq_t^B = (\underbrace{i_t - \check{s}_t}_{=\mu_t^{\mathcal{B}}} - \pi_t) q_t^B dt$$

  - Phillips curve (forward-looking):

$$d\pi_t = \left(\rho\pi_t - \kappa\left(\left(\frac{\rho}{a_t}\frac{q_t^B}{\vartheta_t}\right)^{1+\varphi} - 1\right)\right) dt$$

- In particular: Phillips curve slope (κ) affects speed of adjustment but not impact effect

Closed-Form Solution under Simplifying Assumptions

Make the following simplifying assumptions:

1. Replace dynamic Phillips curve with static Phillips curve

$$\pi_t = \kappa\left(\left(\frac{\rho}{a_t}\frac{q_t^B}{\vartheta_t}\right)^{1+\varphi} - 1\right)$$

2. Assume $i_t = i$, $\check{s}_t = \check{s}$, $a_t = a$, $\vartheta_t = \vartheta$ are constant after the shock ($\Rightarrow \mu^{\mathcal{B}} = i - \check{s}$ is constant)

Then get ODE for q_t^B that can be solved in closed form:

$$q_t^B = \left(\frac{\alpha (q_0^B)^{1+\varphi}}{\beta (q_0^B)^{1+\varphi}\left(1 - e^{-\alpha t}\right) + \alpha e^{-\alpha t}}\right)^{\frac{1}{1+\varphi}},$$

where $\alpha := (1+\varphi)(\mu^{\mathcal{B}} + \kappa)$, $\quad \beta := (1+\varphi)\kappa\left(\frac{\rho}{\vartheta a}\right)^{1+\varphi}$

Outline

Intertemporal Substitution versus Portfolio Choice

- Standard NK story: intertemporal substitution drives aggregate demand
  - key equation: IS equation (in terms of wealth-capital ratio q_t)

$$\mathbb{E}_t[dq_t] = (i_t - \pi_t - r_t^*)\, q_t dt$$

  - relates *level* of wealth to *level of interest rate*
  - usual interpretation: future q_T fixed (e.g., by "anchored beliefs"), q_0 adjusts
  - if $i_t - \pi_t > r_t^*$ for a while: q_0 falls (demand recession)

- This model: portfolio demand for nominal safe assets drives aggregate demand
  - recall: $a_t u_t = \rho q_t^B / \vartheta_t$ fully determined by ϑ_t and safe asset state q_t^B
  - portfolio choice determines *relative* asset values ϑ_t from *excess return* & *excess risk* of capital
  - "level component" in $q_t = q_t^B / \vartheta_t$ is backward-looking state variable q_t^B

Conclusion: Portfolio choice and flight to safety are key for impact (demand) effect of shocks

Interest Rate Policy Ineffectiveness

- How does i_t affect aggregate demand?
  1. Portfolio separation: portfolio demand for safe assets (ϑ_t) unaffected by i_t
  2. Safe asset value q_t^B is slow-moving state: affected by i_t only gradually over time
     - here (due to zero duration): higher $i_t \Rightarrow$ higher $\mu_t^{\mathcal{B}}$
     - in particular: rate hikes are inflationary ("Neo-Fisherian" prediction)

$\Rightarrow$ Impact effect of shock on aggregate demand unaffected by interest rate policy

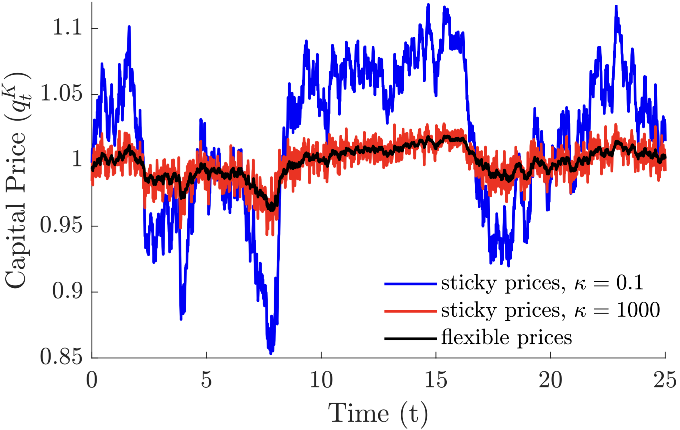
- *Conclusion*: interest rate policy cannot eliminate aggregate demand recession

(in contrast to standard NK models)

Capital Price Overshooting & Flight-to-Safety Volatility

- Portfolio separation: ϑ_t rises as fast as under flexible prices for $\tilde{\sigma}_t$-shock
- Stickiness of bond value: q_t^B unaffected by shock, whereas $q_t^{B,flex} \uparrow$
- Consequence: capital price *overshoots* relative to flexible price response
  - $q_t^K = (1-\vartheta_t)/\vartheta_t \cdot q_t^B$ falls by more under sticky prices
- Corrects major shortcoming of flexible price model (Brunnermeier, Merkel, Sannikov 2024)
  - in that model: bond market (q^B) more volatile than stock market (q^K)
  - here: *any* degree of price stickiness shifts *all* relative volatility into q^K fluctuations
- Reminiscent of Dornbusch's (1976) overshooting model
  - original: sticky domestic price $\rightarrow$ volatile exchange rate
  - here: sticky bond value $\rightarrow$ volatile capital price

Illustration: Overshooting & Flight-to-Safety Volatility



Outline

An Economy without Nominal Bonds

- Consider economy with $\mathcal{B}_t \equiv 0 \Rightarrow \vartheta_t = q_t^B \equiv 0$
- Goods market clearing equation (note $q_t = q_t^K$)

$$a_t u_t = \rho q_t$$

- Effects of shocks depend on effects on return on capital:

$$\underbrace{\rho + \mu_t^q}_{=\mathbb{E}_t[dr_t^K]/dt} = \underbrace{i_t - \pi_t}_{=r_t^f} + \underbrace{\tilde{\sigma}_t^2}_{=\text{risk premium}}$$

  - risk shock: $\tilde{\sigma}_t \uparrow \quad \Rightarrow \quad$ risk premium $\uparrow \quad \Rightarrow \quad q_t \downarrow$
  - productivity shock: $a_t \downarrow \quad \Rightarrow \quad$ risk premium $\rightarrow \quad \Rightarrow \quad q_t \rightarrow$
  - $r_t^f = i_t - \pi_t$ effectively controlled by monetary policy

Shock Transmission & Power of Monetary Policy

Aggregate demand in model without bonds is *purely forward-looking* and follows IS equation logic:

1. no sticky bond value state & no nominal anchor
2. previous equation

$$\mu_t^q = i_t - \pi_t - (\rho - \tilde{\sigma}_t^2)$$

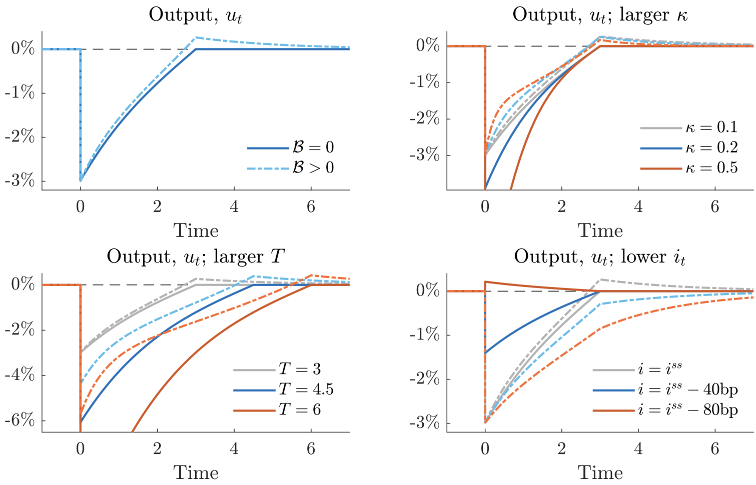
   determines level of asset values to as function of level of returns
   $\rightarrow$ conventional IS equation logic restored

Conclusions & Power of Monetary Policy

Conclusions from such models (e.g. Basu, Bundick 2017; Caballero, Simsek 2020):

1. if insufficient reduction in $r_t^f = i_t - \pi_t$, also these models predict shortfall in demand
2. but this is not necessary: sufficient reduction in r_t^f can prevent demand shortfall
   - e.g. natural rate policy $i_t = r_t^* := \rho - \tilde{\sigma}_t^2$ & appropriate equilibrium selection
   - leads to divine coincidence: $\pi_t = 0$, $u_t = u^*$
   - more generally: only task of policy is *expectations management*
3. corollary: demand recessions are (mostly) a problem at ZLB

Illustration: $\mathcal{B}_t > 0$ versus $\mathcal{B}_t \equiv 0$



A Cleaner Comparison: Two Units of Account

- Issue with previous comparison: $\mathcal{B} = 0$ economy also has no safe assets
- Affects model dynamics even in the absence of price setting frictions
- Cleaner (but artificial!) comparison to highlight what matters: two units of account
  - good prices are quoted in "goods dollars", subject to price setting frictions
  - bonds are quoted in "bond dollars", adjust flexibly
- Also that model behaves close to $\mathcal{B} = 0$ economy

  (exchange rate between two units of accounts does most of the adjustment)

$\Rightarrow$ What really matters: *safe asset is denominated in sticky unit of account*

Outline

How Can Policy Stabilize Aggregate Demand on Impact of a $\tilde{\sigma}$-Shock?

1. Manage *safe asset demand* by distorting portfolio choice
   - use policy instrument $\check{s}_t$ (by adjusting taxes)
   - mitigates flight to safety, but not optimal (in richer model)
     (safe asset services more valuable when $\tilde{\sigma}$ is large, higher ϑ beneficial)

2. Manage *safe asset supply* by introducing safe asset whose value is not (fully) sticky
   a. lump-sum transfers (or taxes, if negative)
      - PV of lump-sum transfers acts as implicit safe asset
      - use dynamic adjustments of transfers to absorb variations in safe asset demand
      - issue: works in theory but difficult in practice
   b. long-term bonds
      - i-policy affects (flexible) nominal bond price through expected future rates
      - *but*: cannot control i_t and q_t^B independently, insufficient to prevent demand recession

      → generates interesting policy problem

Model Extension with Long-term Bonds

- In baseline model: bonds have infinitesimal duration
  - there is no relative price between "money" (sticky unit of pricing) and nominal bonds
- Modified model: bonds are long-term with geometric maturity structure
  - nominal face value $\mathcal{B}_t$ as before
  - each period: government must make payments $\lambda \mathcal{B}_t dt$, $\lambda > 0$
  - $\mathcal{P}_t^B$ is the nominal price of one nominal unit of bonds
  - note: $\lambda \to \infty$: short-term bonds, $\lambda \to 0$: perpetuities
- *Key result*: all model equations are as before except
  - $q_t^B = \mathcal{P}_t^B q_t^{B,0}$ and only $q_t^{B,0} := \mathcal{B}_t / \mathcal{P}_t$ is a state due to stickiness
  - $P_t^B = \frac{\lambda}{\lambda + i_t}$
  - i_t is the long-term interest rate (but fully controlled by controlling short rate i_t^0)

Interest Rate Policy Ineffectiveness Revisited

- Two effects of higher i_t:
  1. lower bond price $\mathcal{P}_t^B$: reduces safe asset supply q_t^B immediately
  2. higher debt growth rate $\mu_t^{\mathcal{B}}$: raises safe asset supply q_t^B gradually (without need for deflation)
- First effect appears to overturn interest rate policy ineffectiveness:
  - i-policy can control q_t^B on shock impact
  - e.g. can completely eliminate output gap without any fiscal support
- *But*: interest rate policy still unable to eliminate sticky price distortions
  - second effect: lower i_t shifts deflation pressures into the future
  - i-policy cannot control level and dynamics of q_t^B independently

Outline

Benchmark: Constrained Efficient Allocation

Proposition: Representation of Welfare Objective

For any welfare weights $\{\lambda^i\}_{i\in[0,1]} \geqslant 0$, maximizing $\int \lambda^i V_0^i di$ is equivalent to maximizing

$$\text{const.} + \int_0^\infty e^{-\rho t}\left(\log\left(a_t\right) + \log\left(u_t\right) - \frac{u_t^{1+\varphi}}{1+\varphi} - \frac{\left(\left(1-\vartheta_t\right)\tilde{\sigma}_t\right)^2}{2\rho}\right) dt$$

Benchmark: Constrained Efficient Allocation

Proposition: Representation of Welfare Objective

For any welfare weights $\{\lambda^i\}_{i\in[0,1]} \geqslant 0$, maximizing $\int \lambda^i V_0^i di$ is equivalent to maximizing

$$\text{const.} + \int_0^\infty e^{-\rho t} \left(\log(a_t) + \log(u_t) - \frac{u_t^{1+\varphi}}{1+\varphi} - \frac{((1-\vartheta_t)\,\tilde{\sigma}_t)^2}{2\rho} \right) dt$$

Optimal allocation if planner can control u_t and ϑ_t directly but faces constraint $\check{s}_t \leqslant \bar{s}$

- $u_t = u^* = 1$ constant
- $\vartheta_t = \vartheta^*(\tilde{\sigma}_t)$ function of $\tilde{\sigma}_t$ only, $\vartheta^{*\prime} > 0$ (note: can be implemented with $\check{s}_t$-instrument)

Optimal Interest Rate Policy with Long-term Bonds

Proposition: Optimal Monetary Policy

Suppose $\tilde{\sigma}_t$ evolves deterministically and let a path $\{\check{s}_t\}$ for fiscal policy be given.

i There is precisely one initial state $q_0^{B,0} = q_0^{B,0*}$ such that interest rate policy can implement $u_t = u^*$ for all $t \geqslant 0$

ii If $q_0^{B,0} > q_0^{B,0*}$, then the optimal interest rate policy is such that $u_t > u^*$ for all $t \geqslant 0$ and $u_t \searrow u^*$.

iii If $q_0^{B,0} < q_0^{B,0*}$, then the optimal interest rate policy is such that $u_t < u^*$ for all $t \geqslant 0$ and $u_t \nearrow u^*$.

Optimal Interest Rate Policy with Long-term Bonds

Proposition: Optimal Monetary Policy

Suppose $\tilde{\sigma}_t$ evolves deterministically and let a path $\{\check{s}_t\}$ for fiscal policy be given.

i. There is precisely one initial state $q_0^{B,0} = q_0^{B,0*}$ such that interest rate policy can implement $u_t = u^*$ for all $t \geqslant 0$
ii. If $q_0^{B,0} > q_0^{B,0*}$, then the optimal interest rate policy is such that $u_t > u^*$ for all $t \geqslant 0$ and $u_t \searrow u^*$.
iii. If $q_0^{B,0} < q_0^{B,0*}$, then the optimal interest rate policy is such that $u_t < u^*$ for all $t \geqslant 0$ and $u_t \nearrow u^*$.

- In sum: monetary policy underreacts relative to full output gap stabilization
- Intuition for underreaction:
  - unless initial state $q_0^{B,0}$ is exactly right, (inefficient) inflation/deflation required at some point
  - concave objective $\longrightarrow$ smooth resulting distortions over time

Thank You!