Eco529: Modern Macro, Money, and International Finance

Lecture 08: Multi-Sector, Banks & I Theory

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Course Overview

Key Takeaways

- Risk Sharing via Inflation Risk (Redistribution)
- Real vs. Nominal Debt/Cashless vs. Cash
- Intertemporal Unit of Account
    - State-contingent Monetary Policy if $\sigma^{\mathcal{B}} \neq 0$
- Equivalence of Capital vs. Risk Allocation Setting (κ vs. χ)
- Liquidity and Disinflationary Spiral
- Policy
    - Fiscal Policy
    - (Redistributive) Monetary Policy
      - "Stealth Recapitalization" of Bottleneck Sector (Intermediaries)
    - Macroprudential Policy
- Technical Takeaways
    - Two Sector Money Models

The Big Roadmap: Towards the I Theory of Money

- One sector model with idio risk - "The I Theory without I" **Lecture 06-07**
  (steady state focus)
  - Store of Value
    Insurance Role of Money within a Sector
  - Medium of Exchange Role
  - Fiscal Theory of the Price Level
  - Time-varying Idiosyncratic Risk and Safe Asset
- 2 Sector/Type Model with Money and Idiosyncratic Risk **Today**
  - Equivalence btw Expert Producers and Intermediaries
  - Real Debt vs. Nominal Debt/Money
    Implicit insurance role of money *across sectors*
  - Banking, I Theory, Redistributive Monetary Policy
- Price/Wage Rigidities (New Keynesian) **Next Lecture**
- Welfare Analysis
  - Optimal Monetary Policy and Macroprudential Policy
- International Monetary Model

“Money and Banking” (in Macro-finance)

- Money store of value/safe asset/Gov. bond
- Banking “diversifier”
  holds risky assets, issues inside money

Watch “Money and Banking”
YouTube Video Channel: “markus.economicus”

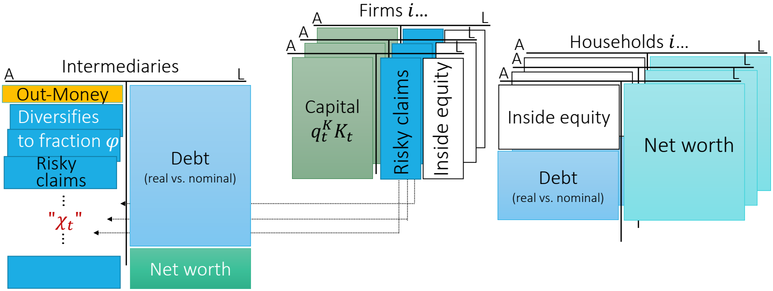
“Money and Banking” (in Macrofinance)

- Money store of value/safe asset/Gov. bond
- Banking “diversifier”
  holds risky assets, issues inside money
- Amplification/endogenous risk dynamics
  - Value of capital declines due to fire-sales Liquidity spiral
    - Flight to safety
  - Value of bond/money rises Disinflation spiral a la Fisher
    - Demand for bond/money rises – less idiosyncratic risk is diversified
    - Supply for inside money declines – less creation by intermediaries
      - Endogenous money multiplier = f(capitalization of critical sector)
    - ~~Paradox of Thrift~~
    - Paradox of Prudence (in risk terms)
- Monetary Policy (redistributive)

Overview

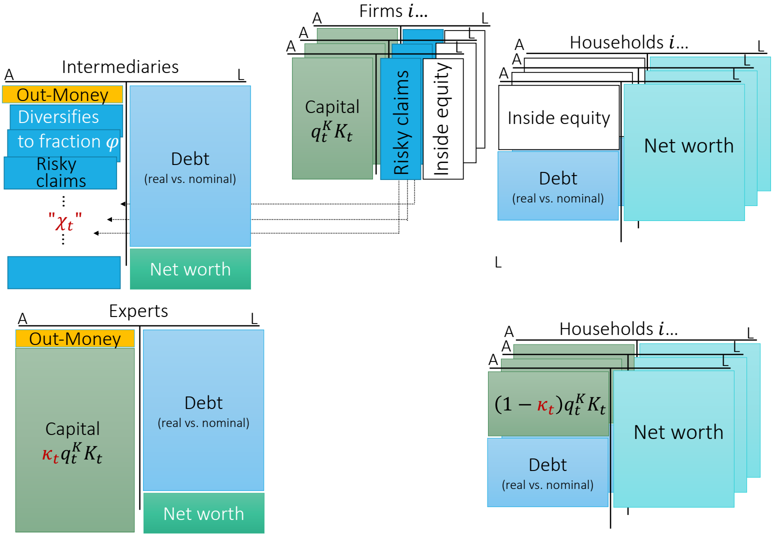
- Intro
- Equivalence btw Experts Producers and Intermediaries
- Real vs. Nominal Debt: Unit of Account in Incomplete Markets Setting
- I Theory of Money:
  - Liquidity and Deflationary Spiral
  - Banks as Diversifiers $\Rightarrow \tilde{\sigma}$ is a Function of Banks' Capitalization η_t
- Policy with Long-Dated Bonds

Intermediaries



- Frictions
  - Household cannot diversify idio risk
  - Limited risky claims issuance

Equivalence



- $a^e = a^h$
- $\tilde{\sigma}^e < \tilde{\sigma}^h$

Equivalence

- Why equivalence btw. intermediaries χ-risk allocation model and experts κ-capital allocation model?

  Poll: Why are both settings equivalent?
  a) Since $a^e = a^h$.
  b) Intermediary sector does not produce any output.
  c) Risk χ and capital allocation κ are fundamentally different.

- Next: Contrast Real Debt with Nominal Debt/Money Model
  - Solve generic model and highlight the differences btw both settings.

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Model with Intermediary Sector

Intermediary sector

- Hold equity up to $\bar{\chi} \leq 1$
- Consumption rate: c_t^I
- Diversify idio risk to $\varphi\tilde{\sigma}$

- Objective: $\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(c_t^I)\mathrm{d}t\right]$

Household Sector

- Output: $y_t^h = a^h k_t^h$
- Consumption rate: c_t^h
- Investment rate: ι_t^h

$$\frac{\mathrm{d}k_t^{h,\tilde{i}}}{k_t^{h,\tilde{i}}} = \left(\Phi(\iota_t^{h,\tilde{i}}) - \delta\right)\mathrm{d}t + \sigma\mathrm{d}Z_t + \tilde{\sigma}^h d\tilde{Z}_t^{\tilde{i}} + \mathrm{d}\Delta_t^{k,\tilde{i},h}$$

- Objective: $\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(c_t^h)\mathrm{d}t\right]$

Friction: Can only issue debt

2 Models

1. Real debt issuance only (and money has no value)
2. Nominal debt issuance

- Bond/Money supply (nominal) $\frac{d\mathcal{B}_t}{\mathcal{B}_t} = \mu_t^{\mathcal{B}}\mathrm{d}t + \sigma_t^{\mathcal{B}}\mathrm{d}Z_t$
- "Seigniorage" distribution as in previous lecture

  (no fiscal impact – per period balanced budget)

Solving Macro Models Step-by-Step

0. Postulate aggregates, price processes and obtain return processes
1. For given $\check{\rho}^i := C^i/N^i$-ratio and $\xi^i = SDF^i$ processes for each i **finance block**
   Toolbox 1: Martingale approach, HJB vs. Stochastic Maximum Principle Approach
   Fisher separation theorem
   - a. Real investment ι + Goods market clearing *(static)*
   - b. Portfolio choice θ + asset market clearing or
     Asset allocation κ & risk allocation χ
     Toolbox 2: "Price-taking" social planner approach
     Toolbox 3: Change in numeraire to total wealth (including SDF)
     - "Gov. Liability Evaluation/FTPL equation" ϑ
2. Evolution of state variable η (and K) **forward equation**
3. Solve $\check{\rho}^i := C^i/N^i$-ratio and $\xi^i = SDF^i$ processes **backward equation**
   - a. Investment opportunities ω and K_t and $\tilde{\eta}^{\tilde{i}}$-descaled v_t^i-process
   - b. Derive C^i/N^i-ratio and ς^i price of risk
   - c. Derive BSDEs
   - d. Separating risk aversion from intertemporal substitution
4. Numerical model solution
   - a. Inner loop: For given $\check{\rho}^i := C^i/N^i$s and ς^is solve ODE for $q(\eta)$
   - b. Outer loop: Transform BSDE into PDE and iterate functions $v^i(\eta, t)$
5. KFE: Stationary distribution, fan charts

0. Postulate Aggregates and Processes

- Assets: capital and bonds
  - q_t^K Capital price
  - $q_t^{\mathcal{B}} := \frac{\mathcal{B}_t}{\mathcal{P}_t} / K_t$ value of the bonds per unit of capital
  - $\vartheta_t := \frac{\frac{\mathcal{B}_t}{\mathcal{P}_t}}{q_t^K K_t + \frac{\mathcal{B}_t}{\mathcal{P}_t}} = \frac{q_t^{\mathcal{B}}}{q_t^K + q_t^{\mathcal{B}}}$ share of bond wealth
  - Postulate Ito price processes $\mathrm{d}q_t^K / q_t^K = \mu_t^{q,K} \mathrm{d}t + \sigma_t^{q,K} \mathrm{d}Z_t, \mathrm{d}q_t^{\mathcal{B}} / q_t^{\mathcal{B}} = \mu_t^{q,\mathcal{B}} \mathrm{d}t + \sigma_t^{q,\mathcal{B}} \mathrm{d}Z_t, \mathrm{d}\vartheta_t / \vartheta_t = \mu_t^{\vartheta} \mathrm{d}t + \sigma_t^{\vartheta} \mathrm{d}Z_t$
  - SDF for each $\tilde{i}$ agent: $\mathrm{d}\xi_t^{\tilde{i}} / \xi_t^{\tilde{i}} = -r_t^i dt - \varsigma_t^{\tilde{i}} \mathrm{d}Z_t - \tilde{\varsigma}_t^{\tilde{i}} \mathrm{d}\tilde{Z}_t^{\tilde{i}}$
- Aggregate resource constraints:
  - Output: $C_t + \iota_t K_t + \mathcal{g} K_t = a K_t$
  - Capital: $\int k_t^{\tilde{i}} \mathrm{d}\Delta k_t^{k,\tilde{i}} \mathrm{d}\tilde{i} = 0$
- Markets: Walrasian goods, bonds, and capital markets

Poll: Why is the drift $-r_t^i$ and not simply $-r_t^f$?

a) With only nominal debt a real risk-free rate might not be in asset span.

b) Negative drift of the SDF in N_t-numeraire is not risk-free rate.

1. Optimal ι + Goods Market

Recall Equilibrium

- Price of physical capital

$$q_t^K = (1-\vartheta_t)\frac{1+\phi\check{a}}{(1-\vartheta_t)+\phi\rho}$$

- Price of nominal capital

$$q_t^{\mathcal{B}} = \vartheta_t\frac{1+\phi\check{a}}{(1-\vartheta_t)+\phi\rho}$$

- Optimal investment rate

$$\iota_t = \frac{(1-\vartheta_t)\check{a}-\rho}{(1-\vartheta_t)+\phi\rho}$$

- Moneyless equilibrium with $q_t^{\mathcal{B}} = 0 \Rightarrow \vartheta_t = 0 \Rightarrow q_t^K = \frac{1+\phi\check{a}}{1+\phi\rho}$

1. Portfolio Choice: Price-taking Planner κ, χ Allocation

- Objective (in total net worth N_t-numeraire):

$$\max_{\{\boldsymbol{\kappa}_t, \boldsymbol{\chi}_t, \tilde{\boldsymbol{\chi}}_t\}} \mathbb{E}[\mathrm{d}r_t^N(\boldsymbol{\kappa}_t)/dt] - \boldsymbol{\varsigma}_t \boldsymbol{\sigma}(\boldsymbol{\kappa}_t, \boldsymbol{\chi}_t) - \tilde{\boldsymbol{\varsigma}}_t \tilde{\boldsymbol{\sigma}}_t(\boldsymbol{\kappa}_t, \tilde{\boldsymbol{\chi}}_t)$$

- In our model(s):
  - $\kappa = 0$ (households manage all physical capital)
  - $\tilde{\chi}_t = \chi_t$
  - $\mathbb{E}[\mathrm{d}r_t^N(\kappa_t)/dt] = 0$

Poll: Why is $\mathbb{E}[\mathrm{d}r_t^N(\kappa_t)/dt] = 0$?

a) Because capital is not reallocated, i.e. $\kappa = 0$ all the time.

b) In the N_t-numeraire return of total wealth $\mathrm{d}r_t^N = 0$

1. Portfolio Choice: Price-taking Planner κ, χ Allocation

- Objective:

$$\max_{\{\boldsymbol{\kappa}_t, \boldsymbol{\chi}_t, \tilde{\boldsymbol{\chi}}_t\}} \mathbb{E}[\mathrm{d}r_t^N(\boldsymbol{\kappa}_t)/dt] - \boldsymbol{\varsigma}_t \boldsymbol{\sigma}(\boldsymbol{\kappa}_t, \boldsymbol{\chi}_t) - \tilde{\boldsymbol{\varsigma}}_t \tilde{\boldsymbol{\sigma}}_t(\boldsymbol{\kappa}_t, \tilde{\boldsymbol{\chi}}_t)$$

- In our model(s):
  - $\kappa = 0$ (households manage all physical capital)
  - $\tilde{\chi}_t = \chi_t$
  - $\mathbb{E}[\mathrm{d}r_t^N(\kappa_t)/dt] = 0$
  - $\boldsymbol{\sigma} = (\chi_t \sigma_t^{xK}, (1 - \chi_t)\sigma_t^{xK})$,
    - where σ_t^{xK} = Risk of the excess return of capital beyond benchmark asset
  - $\tilde{\boldsymbol{\sigma}} = (\chi_t \varphi \tilde{\sigma}, (1 - \chi_t)\tilde{\sigma})$, $\varphi < 1$

1. Portfolio Choice: Price-taking Planner κ, χ Allocation

- Minimize weighted average cost of financing

$$\min_{\chi_t \le \bar{\chi}} (\varsigma_t^I \chi_t + \varsigma_t^h (1-\chi_t))\sigma_t^{xK} + (\tilde{\varsigma}_t^I \varphi \chi_t + \tilde{\varsigma}_t^h (1-\chi_t))\tilde{\sigma}$$

- FOC: (equality if $\chi_t < \bar{\chi}$)

$$\varsigma_t^I \sigma_t^{xK} + \tilde{\varsigma}_t^I \varphi \tilde{\sigma} \le \varsigma_t^h \sigma_t^{xK} + \tilde{\varsigma}_t^h \tilde{\sigma}$$

- Real debt model: $\sigma_t^{xk} = \sigma + \sigma_t^{q^K}$ (recall q_t^K is constant)
- Nominal debt model: $\sigma_t^{xK} = (-\sigma_t^{\vartheta} + \sigma_t^{\mathcal{B}})/(1-\vartheta_t)$
  - Risk of capital $\sigma + \sigma_t^{q^K} + \vartheta_t \sigma_t^{\mathcal{B}}/(1-\vartheta_t) - \sigma_t^N$ (in N_t-numeraire)
  - Risk of bond/money $\sigma + \sigma_t^{q^{\mathcal{B}}} - \sigma_t^{\mathcal{B}} - \sigma_t^N$(in N_t-numeraire)

1. "Benchmark Asset Evaluation (FTPL) Equation"

- In N_t-numeraire η^i_t takes on role of sector networth N^i_t
- Return on individual agent's networth return (in N_t-numeraire)

$$\underbrace{\frac{d\eta^i_t}{\eta^i_t}}_{\text{sector share}} + \underbrace{\frac{\mathrm{d}\tilde{\eta}^i_t}{\tilde{\eta}^i_t}}_{\text{within sector share}} + \underbrace{\rho \mathrm{d}t}_{\text{consumption}}$$

- Martingale condition relative to benchmark asset is

$$\mu^{\eta^i}_t + \rho - r^{bm}_t = \varsigma^i_t(\sigma^{\eta^i}_t - \sigma^{bm}_t) + \tilde{\varsigma}^i_t \tilde{\sigma}^{\tilde{i}}_t$$

- Take η^i_t-weighted sum (across 2 types $i = I, h$ here)

$$\rho - r^{bm}_t = \eta_t \varsigma^I_t(\sigma^{\eta}_t - \sigma^{bm}_t) + (1-\eta_t)\varsigma^h_t \left(-\tfrac{\eta_t}{1-\eta_t}\sigma^{\eta}_t - \sigma^{bm}_t\right) + \eta_t \tilde{\varsigma}^I_t \tilde{\sigma}^{\tilde{\eta}^{\tilde{I}}}_t + (1-\eta_t)\tilde{\varsigma}^h_t \tilde{\sigma}^{\tilde{\eta}^{\tilde{h}}}_t$$

- For log utility: $\varsigma^I_t = \sigma^{\eta}_t, \varsigma^h_t = -\tfrac{\eta_t}{1-\eta_t}\sigma^{\eta}_t, \tilde{\varsigma}^I_t = \tilde{\sigma}^{\tilde{\eta}^{\tilde{I}}}_t, \tilde{\varsigma}^h_t = \tilde{\sigma}^{\tilde{\eta}^{\tilde{h}}}_t$:

$$\rho - r^{bm}_t = \eta_t(\sigma^{\eta}_t)^2 + (1-\eta_t)\left(-\tfrac{\eta_t}{1-\eta_t}\sigma^{\eta}_t\right)^2 + \eta_t(\tilde{\sigma}^{\tilde{\eta}^{\tilde{I}}}_t)^2 + (1-\eta_t)(\tilde{\sigma}^{\tilde{\eta}^{\tilde{h}}}_t)^2$$

1. "Benchmark Asset Evaluation (FTPL) Equation"

- Real debt = benchmark asset bm
  - Redundant equation for allocation just useful for deriving risk-free rate in c-numeraire r_t^f (expressed in N_t-numeraire)
- Nominal debt/money = benchmark asset bm
  - Money evaluation equation (bubble) [FTPL Equation]
  - Replace: $r_t^{bm} = \mu_t^{\vartheta/\mathcal{B}} := \mu_t^{\vartheta} - \mu_t^{\mathcal{B}} - \sigma_t^{\mathcal{B}}(\sigma_t^{\vartheta} - \sigma_t^{\mathcal{B}})$ (and $\sigma_t^{bm} = \sigma_t^{\vartheta}$)

$$\underbrace{\rho - \mu_t^{\vartheta/\mathcal{B}}}_{\text{excess return of } N_t} = \underbrace{\eta_t(\sigma_t^{\eta})^2 + (1-\eta_t)\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^{\eta}\right)^2 + \eta_t(\tilde{\sigma}_t^{\tilde{\eta}^l})^2 + (1-\eta_t)(\tilde{\sigma}_t^{\tilde{\eta}^h})^2}_{\text{(required) "net worth weighted risk premium" (for holding risk in excess of money risk)}}$$

1. "Benchmark Asset Evaluation (FTPL) Equation"

- Nominal debt/money = benchmark asset *bm*
  - Gov. liability evaluation equation (bubble) [FTPL Equation]
  - Replace: $r_t^{bm} = \mu_t^{\vartheta/\mathcal{B}} := \mu_t^{\vartheta} - \mu_t^{\mathcal{B}} - \sigma_t^{\mathcal{B}}(\sigma_t^{\vartheta} - \sigma_t^{\mathcal{B}})$ (and $\sigma_t^{bm} = \sigma_t^{\vartheta}$)

$$\rho - \mu_t^{\vartheta/\mathcal{B}} = \eta_t(\sigma_t^{\eta})^2 + (1-\eta_t)\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^{\eta}\right)^2 + \eta_t(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}})^2 + (1-\eta_t)(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}})^2$$

  - Integrate:

$$\vartheta_t = \mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}\left(\eta_s(\sigma_s^{\eta})^2 + (1-\eta_s)\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^{\eta}\right)^2 + \eta_s(\tilde{\sigma}_s^{\tilde{\eta}^{\tilde{I}}})^2 + (1-\eta_s)(\tilde{\sigma}_s^{\tilde{\eta}^{\tilde{h}}})^2\right)\vartheta_s \mathrm{d}s\right]$$

2. η-Evolution: Drift μ_t^η (in N_t-numeraire)

- Take difference from two earlier equations

$$\mu_t^\eta + \rho - r_t^{bm} = \varsigma_t^I(\sigma_t^\eta - \sigma_t^{bm}) + \tilde{\varsigma}_t^I \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}$$

$$\rho - r_t^{bm} = \eta_t \varsigma_t^I(\sigma_t^I - \sigma_t^{bm}) + (1-\eta_t)\varsigma_t^h\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^\eta - \sigma_t^{bm}\right) + \eta_t \tilde{\varsigma}_t^I \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}} + (1-\eta_t)\tilde{\varsigma}_t^h \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}$$

- $\mu_t^\eta = (1-\eta_t)\left[\varsigma_t^I(\sigma_t^I - \sigma_t^{bm}) - \varsigma_t^h\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^\eta - \sigma_t^{bm}\right) + \tilde{\varsigma}_t^I \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}} - \tilde{\varsigma}_t^h \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}\right]$
- Real Debt: $\sigma_t^{bm} = -\sigma_t^N = -\sigma$ (Recall $\sigma_t^{q^K} = 0$)
- Nominal Debt/Money $\sigma_t^{bm} = \sigma_t^\vartheta - \sigma^{\mathcal{B}}$

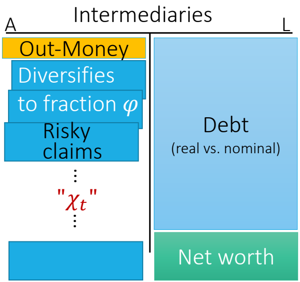
2. η-Evolution: η-Aggregate Risk

- $\sigma_t^{\eta} = \sigma_t^{r^{bm}} + (1-\theta_t^I)(\sigma_t^{r^K} - \sigma_t^{r^{bm}})$
  - where portfolio share $1-\theta_t^I = \frac{\chi_t}{\eta_t}(1-\vartheta_t)$
- Real Debt
  - Note $\sigma_t^{r^k} = 0$ given $N_t = q_t^K K_t$ - Numeraire
  - $\sigma_t^{\eta} = \frac{\chi_t - \eta_t}{\eta_t}\sigma$ (recall $\vartheta_t = 0$)
  - No amplification since q^K is constant
  - Imperfect aggregate risk-sharing for $\chi_t \neq \eta_t$

2. Inflation Risk allows Perfect Risk Sharing

- Nominal Debt
  - Note: $\sigma_t^{r^K} = \sigma_t^{1-\vartheta} = -\frac{\vartheta_t}{1-\vartheta_t}\sigma_t^{\vartheta}$
  - $\sigma_t^{\eta} = \sigma_t^{\vartheta} - \sigma^{\mathcal{B}} + \frac{\chi_t}{\eta_t}(1-\vartheta_t)\left(-\frac{\vartheta_t}{1-\vartheta_t}\sigma_t^{\vartheta} - \sigma_t^{\vartheta} + \sigma^{\mathcal{B}}\right)$
  - Use $\sigma_t^{\vartheta} = \frac{\vartheta'(\eta_t)}{\vartheta(\eta_t)}\eta_t\sigma_t^{\eta}$ and solve for $\eta_t\sigma_t^{\eta}$ yields

$$\eta_t\sigma_t^{\eta} = \frac{(\chi_t - \eta_t)\sigma_t^{\mathcal{B}}}{1 - \frac{\chi_t-\eta_t}{\eta_t}\left(\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\right)}$$



- Intermediaries' balance sheet
  perfectly hedges agg. risk for $\sigma^{\mathcal{B}} = 0$

Proposition:

Aggregate risk is perfectly shared for $\sigma^{\mathcal{B}} = 0$!

- Via inflation risk
- Stable inflation (targeting) would ruin risk-sharing
  - Example: Brexit uncertainty. Use inflation reaction to share risks within UK.

2. Within Type $\tilde{\eta}$-Risk

- Within intermediary sector

$$\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}} = (1 - \theta_t^I)\varphi\tilde{\sigma} = \frac{\chi_t}{\eta_t}(1 - \vartheta_t)\varphi\tilde{\sigma}$$

- Within household sector

$$\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}} = (1 - \theta_t^h)\tilde{\sigma} = \frac{1 - \chi_t}{1 - \eta_t}(1 - \vartheta_t)\tilde{\sigma}$$

Solving for χ_t

- Recall planner condition: (equality if $\chi_t < \bar{\chi}$)

Price of Risks	Real Debt	Nominal Debt with $\sigma^{\mathcal{B}} = 0$
$\varsigma_t^I = \sigma_t^\eta$	$= \frac{\chi_t - \eta_t}{\eta_t}\sigma$	$= 0$
$\varsigma_t^h = -\frac{\eta_t}{1-\eta_t}\sigma_t^\eta$	$= \frac{\chi_t - \eta_t}{1-\eta_t}\sigma$	$= 0$
$\tilde{\varsigma}_t^I = -\frac{\chi_t}{\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}$	$= \frac{\chi_t}{\eta_t}\varphi\tilde{\sigma}$	$= \frac{\chi_t}{\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}$
$\tilde{\varsigma}_t^h = -\frac{1-\chi_t}{1-\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}$	$= \frac{1-\chi_t}{1-\eta_t}\tilde{\sigma}$	$= \frac{1-\chi_t}{1-\eta_t}(1-\vartheta_t)\tilde{\sigma}$

Solving for χ_t

- Real debt:

$$\chi_t = \min\{\frac{\eta_t(\sigma^2 + \tilde{\sigma}^2)}{\sigma^2 + [(1-\eta_t)\varphi^2 + \eta_t]\tilde{\sigma}^2}, \bar{\chi}\}$$

- Nominal debt:

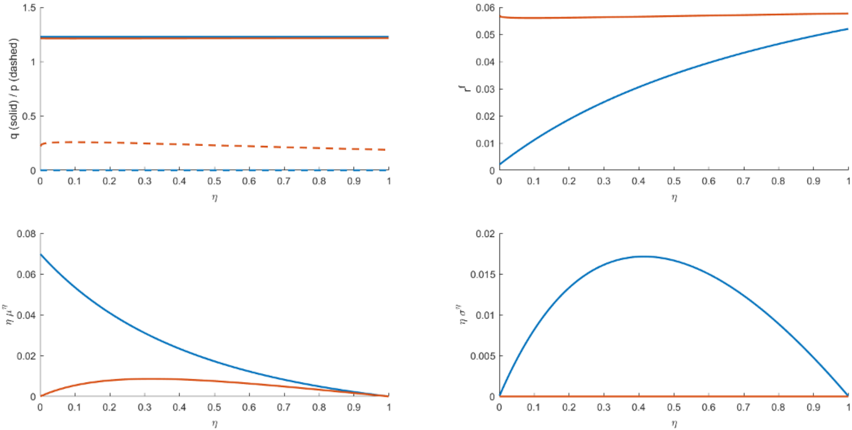
$$\chi_t = \min\{\frac{\eta_t}{(1-\eta_t)\varphi^2 + \eta_t}, \bar{\chi}\}$$

Solution

	Real Debt	Nominal Debt with $\sigma^{\mathcal{B}} = 0$
χ_t	$\min\{\frac{\eta_t(\sigma^2+\tilde{\sigma}^2)}{\sigma^2+[(1-\eta_t)\varphi^2+\eta_t]\tilde{\sigma}^2}, \bar{\chi}\}$	$\chi_t = \min\{\frac{\eta_t}{(1-\eta_t)\varphi^2+\eta_t}, \bar{\chi}\}$
μ_t^η	$(1-\eta_t)\left(\frac{\chi_t^2\varphi^2}{\eta_t^2} - \frac{(1-\chi_t)^2}{(1-\eta)^2}\right)\tilde{\sigma}^2$	$(1-\eta_t)(1-\vartheta_t)^2\left(\frac{\chi_t^2\varphi^2}{\eta_t^2} - \frac{(1-\chi_t)^2}{(1-\eta)^2}\right)\tilde{\sigma}^2$
σ_t^η	$\frac{\chi_t-\eta_t}{\eta_t}\sigma$	0
q_t^K	$\frac{1+\phi a}{1+\phi\rho}$	$(1-\vartheta_t)\frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$
$q_t^{\mathcal{B}}$	0	$\vartheta_t\frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$
ϑ_t	0	$\rho - \mu_t^\vartheta + \mu_t^{\mathcal{B}} = (1-\vartheta_t)^2\left(\eta_t\frac{\chi_t^2\varphi^2}{\eta_t^2} - (1-\eta_t)\frac{(1-\chi_t)^2}{(1-\eta)^2}\right)\tilde{\sigma}^2$
ι_t	$\frac{a-\rho}{1+\phi\rho}$	$\frac{(1-\vartheta_t)a-\rho}{(1-\vartheta_t)+\phi\rho}$

Example: Nominal Debt/Money with $\bar{\chi} = 1$

- $a = 0.15, \rho = 0.03, \sigma = 0.1, \phi = 2, \delta = 0.03, \tilde{\sigma}^e = 0.2, \tilde{\sigma}^h = 0.3, \varphi = 2/3, \bar{\chi} = 1$
  Blue: real debt model, Red: nominal model

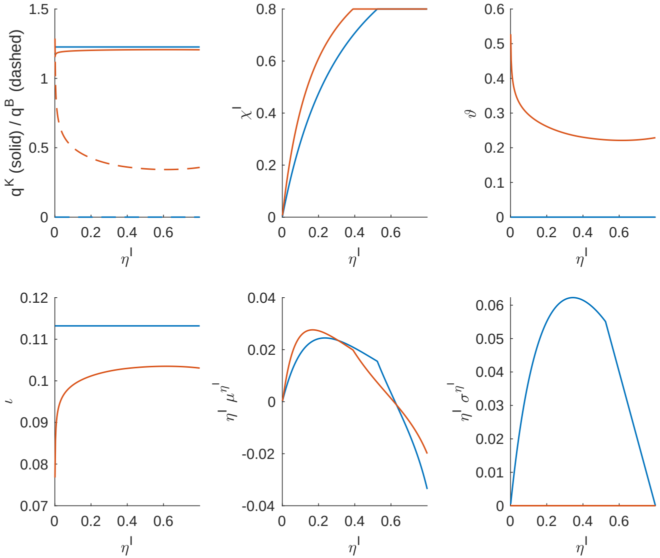


Contrasting Real with Nominal Debt

- Real debt model
  - Changes in η are absorbed by risk-free rate moves
  - Aggregate risk
  - $\iota(\eta)$ and $q^K(\eta)$ are constant
- Nominal debt/money model
  - Inflation risk completes markets
  - Perfect aggregate risk sharing
    - Banks balance sheet is perfectly hedged!!!
  - Risk-free rate is high
  - $\iota(\eta)$ and $q^K(\eta)$ are functions of η
- **Remark:**
  Both Settings: Real Debt and Money/Nominal Debt converge in the long-run to the "I Theory without I" steady state model of Lecture 06 if $\bar{\chi} = 1$.

Example: Nominal Debt with Limit on Risk Offloading

- $\rho = 0.05, a = .15, \delta = .03, \phi = 2, \tilde{\sigma} = 0.5, \varphi = 0.4, \mu^{\mathcal{B}} = .01, \sigma^{\mathcal{B}} = 0, \bar{\chi} = .8$



Combining Nominal & Real Debt

- Adding real debt to money model does not alter the equilibrium, since
  - Markets are complete w.r.t. to aggregate risk
    (perfect aggregate risk sharing)
  - Markets are incomplete w.r.t. to idiosyncratic risk only
  - Real debt is a redundant asset
- Note: Result relies on absence of price stickiness

ϑ Minimized at Stochastic Steady State

- Claim: $\vartheta(\eta)$ and average idiosyncratic risk exposure, $X(\eta)$, is minimized at the stochastic steady state of η.
  - Intuition: at steady state both sectors earn same risk premia + idiosyncratic seems well spread out ... less desire to hold money to self-insure
- With $\sigma_t^{\mathcal{B}} = 0, \forall t$ for steady state s,t, $\chi = \bar{\chi}$
  - $\sigma_t^{\eta} = 0$, (perfect risk sharing with nominal debt)
  - $\mu_t^{\eta} = (\tilde{\sigma}_t^{l})^2 - \eta_t(\tilde{\sigma}_t^{l})^2 - (1-\eta_t)(\tilde{\sigma}_t^{h})^2 = (1-\eta_t)(1-\vartheta_t)^2 \underbrace{\left(\frac{\chi_t^2\varphi^2}{\eta_t^2} - \frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right)}_{-dX/d\eta} \tilde{\sigma}^2$
- Gov. liability evaluation (FTPL) equation

$$\rho - \mu_t^{\vartheta/\mathcal{B}} = \underbrace{(1-\vartheta_t)^2 \overbrace{\left(\eta_t \frac{\chi_t^2\varphi^2}{\eta_t^2} - (1-\eta_t)\frac{(1-\chi_t)^2}{(1-\eta)^2}\right)\tilde{\sigma}^2}^{X(\eta):=}}_{\eta_t(\tilde{\sigma}_t^{l})^2 + (1-\eta_t)(\tilde{\sigma}_t^{h})^2}$$

where $\chi_t = \min\{\frac{\eta_t}{(1-\eta_t)\varphi^2+\eta_t}, \bar{\chi}\}$

Cashless/Bondless Limit with Discontinuity

- Removing cash/nominal gov. bonds (comparative static)
  - $\mathcal{B} > 0$ vs. $\mathcal{B} = 0$
    - Price flexibility $\Rightarrow$ Neutrality of money
  - Discontinuity at $\lim_{\mathcal{B}\to 0}$
  - Remark:
    - Different from Woodford (2003) – medium of exchange role of money
      CIA becomes relevant for fewer and fewer goods
- Inflation on nominal claims (bond/cash)
  - Change $\mu^{\mathcal{B}}$ and subsidize capital
  - Continuous process

Overview

- Intro
- Equivalence btw Experts Producers and Intermediaries
- Real vs. Nominal Debt: Unit of Account in Incomplete Markets Setting
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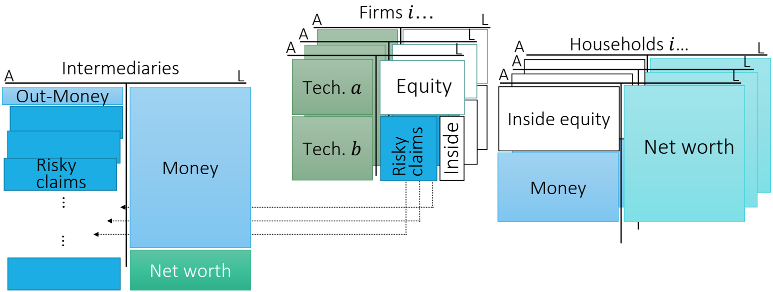
- Policy with Long-Dated Bonds

I Theory of Money

- Aim: intermediary sector is not perfectly hedged (connection to nominal debt in previous slides)
- Idiosyncratic risk that HH have to bear is time-varying $\tilde{\sigma}(\eta)$ (connection to nominal debt in previous slides)
- Needed: Intermediaries' aggregate risk $\neq$ aggregate risk of economy

Technology	**a**	**b**
Capital share (Leontief)	$1-\bar{\kappa}$	$\bar{\kappa}$
Risk	$\frac{dk_t^{a,\tilde{i}}}{k_t^{a,\tilde{i}}} = (\cdot)\mathrm{d}t + \sigma^a \mathrm{d}Z_t + \tilde{\sigma}\mathrm{d}\tilde{Z}_t^{\tilde{i}}$	$\frac{dk_t^{b,\tilde{i}}}{k_t^{b,\tilde{i}}} = (\cdot)\mathrm{d}t + \sigma^b \mathrm{d}Z_t + \tilde{\sigma}\mathrm{d}\tilde{Z}_t^{\tilde{i}}$
Intermediaries	No	Yes, reduce $\tilde{\sigma}$ to $\varphi\tilde{\sigma}$
Excess risk	$-\bar{\kappa}(\sigma^b - \sigma^a) - \frac{\sigma^{\vartheta}-\sigma^{\mathcal{B}}}{1-\vartheta}$	$(1-\bar{\kappa})\underbrace{(\sigma^b - \sigma^a)}_{=\sigma} - \frac{\sigma^{\vartheta}-\sigma^{\mathcal{B}}}{1-\vartheta}$

I Theory: Balance Sheets



- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits

0. Postulate Aggregates and Processes

- Total output: $Y_t = [A^a_t(1-\bar{\kappa}) + A^b_t\bar{\kappa}]K_t$
- Aggregate capital evolution: $\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta)\mathrm{d}t + \underbrace{[(1-\bar{\kappa})\sigma^a + \bar{\kappa}\sigma^b]}_{=\sigma^K}\mathrm{d}Z_t$
- Return process (for $x \in \{a, b\}$):

$$dr^x_t(\iota_t) = \left\{\frac{A^x_t - \iota_t}{q^K_t} + \Phi(\iota_t) - \delta + \mu^{q^K}_t + \sigma^x\sigma^{q^K}_t + \frac{q^{\mathcal{B}}_t}{q^K_t}\left[\mu^{\mathcal{B}}_t + (\sigma^{q^{\mathcal{B}}}_t - \sigma^{\mathcal{B}}_t)\sigma^{\mathcal{B}}_t\right]\right\}\mathrm{d}t$$
$$+ \left(\sigma^x + \sigma^{q^K}_t + \frac{q^{\mathcal{B}}_t}{q^K_t}\sigma^{\mathcal{B}}_t\right)dZ_t + \tilde{\sigma}\mathrm{d}\tilde{Z}^{\tilde{i}}_t,$$

- Outside equity:

$$\mathrm{d}r^{OE,I}_t = r^{OE}_t\mathrm{d}t + \left(\sigma^b + \sigma^{q^K}_t + \frac{q^{\mathcal{B}}_t}{q^K_t}\sigma^{\mathcal{B}}_t\right)\mathrm{d}Z_t + \varphi\tilde{\sigma}\mathrm{d}\tilde{Z}^{\tilde{i}}_t$$

- Household return: $\mathrm{d}r^{OE,h}_t = \mathrm{d}r^{OE,I}_t + (1-\varphi)\tilde{\sigma}\mathrm{d}\tilde{Z}^{\tilde{i}}_t$

Overview: The Role of Each Model Ingredient

- $\bar{\chi}$ – avoid degenerated distribution (households dying out)
- φ
  - if $\varphi = 1$ intermediaries would die out,
  - if $\varphi = 0$ don't earn risk premium (except for aggregate risk)
- $\sigma^b > \sigma^a$ – avoid perfect hedging for intermediaries
  - except $\sigma^{\mathcal{B}} \neq 0$ – for example risk-free asset is in zero net supply (like AER paper/handbook chapter)
- Fraction $\bar{\kappa}$ of K has aggregate risk of $\sigma = \sigma^b - \sigma^a$, rest has risk of zero (it's exogenous) (allocation does not determine total risk in aggregate economy) (To keep it clean (taste choice): price-taking planner's choice is less involved)

1. Portfolio Choice: Price-taking Planner's Allocation

- Minimize weighted average cost of financing

$$\min_{\chi_t \leq \bar{\chi}} (1-\bar{\chi})\varsigma_t^h \sigma_t^{xK^a} + (\varsigma_t^I \chi_t + \varsigma_t^h(\kappa - \chi_t))\sigma_t^{xK^b} + (\tilde{\varsigma}_t^I \varphi \chi_t + \tilde{\varsigma}_t^h (1-\chi_t))\tilde{\sigma}$$

- FOC: (equality if $\chi_t < \bar{\chi}$)

$$\varsigma_t^I \sigma_t^{xK^b} + \tilde{\varsigma}_t^I \varphi \tilde{\sigma} \leq \varsigma_t^h \sigma_t^{xK^b} + \tilde{\varsigma}_t^h \tilde{\sigma}$$

  - $\sigma_t^{xK^b} = (1-\bar{\kappa})\sigma - \frac{\sigma^{\vartheta} - \sigma^{\mathcal{B}}}{1-\vartheta}$

	Intermediaries	Households
Aggregate risk	$\varsigma_t^I = \sigma_t^{\eta}$	$\varsigma_t^h = -\frac{\eta_t}{1-\eta_t}\sigma_t^{\eta}$
Idiosyncratic Risk	$\tilde{\varsigma}_t^I = \frac{\chi_t}{\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}$	$\tilde{\varsigma}_t^h = \frac{1-\chi_t}{1-\eta_t}(1-\vartheta_t)\tilde{\sigma}$

$$\sigma_t^{\eta}\left((1-\bar{\kappa})\sigma - \frac{\sigma^{\vartheta} - \sigma^{\mathcal{B}}}{1-\vartheta}\right) + \left[\frac{\chi_t}{\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}\right]\varphi\tilde{\sigma} \leq$$
$$-\frac{\eta_t \sigma_t^{\eta}}{1-\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma^{\vartheta} - \sigma^{\mathcal{B}}}{1-\vartheta}\right) + \left[\frac{1-\chi_t}{1-\eta_t}(1-\vartheta_t)\tilde{\sigma}\right]\tilde{\sigma}$$

1. Money/Bond (FTPL) Evaluation + 2. η-Drift

- As before in money/nominal debt model
- Money/bond evaluation (FTPL equation)

$$\rho - \mu_t^{\vartheta/\mathcal{B}} = \eta_t \left[(\sigma_t^\eta)^2 + (\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}})^2 \right] + (1-\eta_t) \left[\left(\frac{\eta_t}{1-\eta_t} \sigma_t^\eta \right)^2 + (\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}})^2 \right]$$

- η-drift

$$\mu_t^\eta = (1-\eta_t) \left[(\sigma_t^\eta)^2 + (\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}})^2 - \left(\frac{\eta_t}{1-\eta_t} \sigma_t^\eta \right)^2 - (\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}})^2 \right] - \sigma_t^\eta \underbrace{\sigma_t^{\vartheta/\mathcal{B}}}_{\sigma_t^\vartheta - \sigma^{\mathcal{B}}}$$

η_t-Volatility and Amplification

- $\sigma_t^{\eta} = \sigma_t^{r^{\mathcal{B}}} + (1-\theta_t^I)\sigma_t^{xK^b}$, where portfolio share $1-\theta_t^I = \frac{\chi_t}{\eta_t}(1-\vartheta_t)$

$$\sigma_t^{\eta} = \sigma_t^{\vartheta} - \sigma_t^{\mathcal{B}} + (1-\vartheta_t)\frac{\chi_t}{\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma_t^{\vartheta} - \sigma_t^{\mathcal{B}}}{1-\vartheta}\right)$$

$$\Rightarrow \eta_t\sigma_t^{\eta} = \frac{(1-\vartheta_t)\chi_t(1-\bar{\kappa})\sigma + (\chi_t-\eta_t)\sigma_t^{\mathcal{B}}}{1 - \frac{\chi_t-\eta_t}{\eta_t}\left(\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\right)}$$

- Note that: $\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} = (1-\vartheta_t)\left(\underbrace{\frac{q^{K'}(\eta_t)\eta_t}{q^K(\eta_t)}}_{\text{Liquidity spiral}} + \underbrace{\frac{-q^{\mathcal{B}'}(\eta_t)\eta_t}{q^{\mathcal{B}}(\eta_t)}}_{\text{Disinflationary spiral}}\right)$

I Theory: Summary

Equations

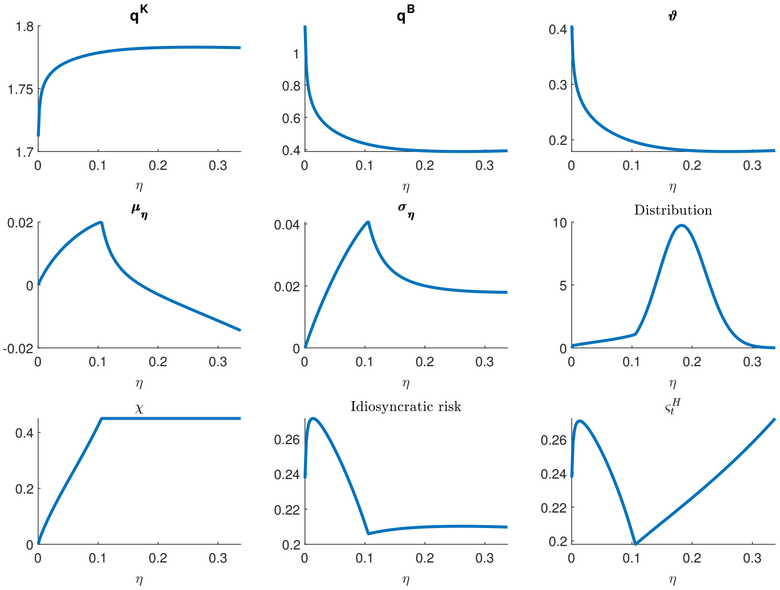
- Money evaluation equation: $\rho - \mu^{\vartheta} + \mu^{\mathcal{B}} - \sigma^{\mathcal{B}}(\sigma^{\mathcal{B}} - \sigma^{\vartheta}) = [...]$
- η-drift: $\mu^{\eta} = [...] - \sigma^{\eta}(\sigma^{\vartheta} - \sigma^{\mathcal{B}})$; η-vol: $\sigma^{\eta} =$ (ampli-equation)
- Itô's Lemma: $\vartheta\mu^{\vartheta} = \eta\mu^{\eta}\partial_{\eta}\vartheta(\eta) + \frac{1}{2}\eta^2(\sigma^{\eta})^2\partial_{\eta\eta}\vartheta(\eta)$
- Planner's condition for χ.
- Idiosyncratic risks $\tilde{\sigma}^{\tilde{\eta}^{\tilde{x}}}(\eta), x \in \{I, h\}$.

Algorithms

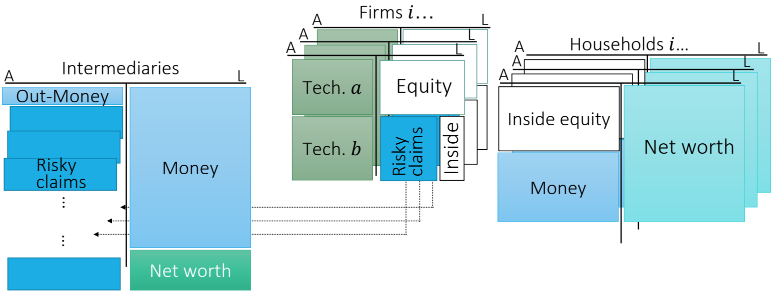
1. Construct grid for η, guess $\vartheta(\eta)$
2. Compute $\sigma^{\eta}(\eta), \chi(\eta)$ for every η
3. Compute $\mu^{\eta}(\eta)$, $\tilde{\sigma}^{\tilde{\eta}^{\tilde{x}}}(\eta), x \in \{I, h\}$ for every η
4. Update $\vartheta(\eta)$ by adding pseudo-time step.
5. Repeat 2 - 4 until it converges.

I Theory: Solutions

- $\rho = 0.05, a = .5, \delta = .03, \phi = 2, \tilde{\sigma} = 0.4, \varphi = 0.2, \mu^{\mathcal{B}} = 0, \sigma^{\mathcal{B}} = 0, \bar{\chi} = .45$



I Theory: Balance Sheets



- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits

Consequences of a Shock in 4 Steps

1. Shock: destruction of some capital
   - % loss in intermediaries net worth > % loss in assets
   - Leverage shoots up
   - Intermediaries %-loss > Household %-losses
     - η-derivative shifts losses to intermediaries
2. Response: shrink balance sheet / delever — **Paradox of Prudence**
   - For given prices no impact
3. Asset side: asset price q^K shrinks — **Liquidity Spiral**
   - Further losses, leverage ↑, further deleveraging

4a. Liability side: Banks' money supply declines
value of money $q^{\mathcal{B}}$ rises — 4a.+4b. **Disinflationary Spiral**

4b. Households' money demand rises
   - HH face more idiosyncratic risk (can't diversify)

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Policy

- **Fiscal Policy**
  - $\mu_t^{\mathcal{B}}$ affects only drift of ϑ_t
  - $\sigma_t^{\mathcal{B}}$ affects the risk of money/nominal bond value and agents portfolio choice (reaction to aggregate shock) ...
  - Alternative: policy impacts $\mathrm{d}s$ (or $\mathrm{d}\tau$)
- **"Pure" Monetary policy** without fiscal implications
  - i_t, σ_t^i, (reaction to aggregate shock) (no $\mu_t^{\mathcal{M}}$ in this lecture)
  - Definition of "Pure":
    Change in Monetary Policy has no immediate direct fiscal implications.
    - Surplus to debt ratio, $s_t/q_t^{\mathcal{B}}$, is not affected.
    - (it might alter growth rate and hence fiscal situation)
- **Macroprudential policy**

Fiscal Policy

Fiscal authority pick s_t or $\mu_t^{\mathcal{B}}$?

- If gov. can choose $\mathrm{d}\tau_t^{i,\tilde{i}}$ subject to budget constraint ($i \in \{I, h\}$) $\sum_i \int_{\tilde{i}} \mathrm{d}\tau_t^{i,\tilde{i}} = d\mathfrak{s}$(seigniorage) it can essentially complete markets
  - Recall: If transfers proportional to
    1. Output (= capital, if all a are the same)
    2. Bond holdings => no real impact
    3. Net worth $\Rightarrow$ btw 1. and 2.
- Intra-temporal Transfer Policy
  - If gov. is constrained to make only sector-specific transfers $\tau_t^{i,\tilde{i}} = \tau_t^i$ it can effectively control η_t^i (an be micro-founded by agents' hidden savings)
- Inter-temporal Transfer Policy
  - Focus on bond supply $(\mu_t^{\mathcal{B}}, \sigma_t^{\mathcal{B}})$ seigniorage is rebated to capital holders (by lowering output tax)
  - $\mu_t^{\mathcal{B}}$ affects only drift of ϑ_t
  - $\sigma_t^{\mathcal{B}}$ affects the risk of money/nominal bond value and agents portfolio choice (reaction to aggregate shock) ...

Monetary Policy: Neo-Fisherian

- Definition of "Pure MoPo":
  Change in Monetary Policy has no immediate direct fiscal implications.
- Interest rates on bond/reserves i_t is paid to bond holders.
- Fisher Equation (in setting with aggregate risk)

$$
\begin{aligned}
\mathrm{d}r_t^{\mathcal{B}} &= i_t \mathrm{d}t + \frac{\mathrm{d}(1/P_t)}{1/P_t} = i_t \mathrm{d}t + \frac{\mathrm{d}(q_t^{\mathcal{B}} K_t/B_t)}{q_t^{\mathcal{B}} K_t/B_t} \\
&= \left\{ i_t + \Phi(\iota_t) - \delta + \mu_t^{q^{\mathcal{B}}} - \left[\mu_t^{\mathcal{B}} + (\sigma_t^{q^{\mathcal{B}}} - \sigma_t^{\mathcal{B}})\sigma_t^{\mathcal{B}} \right] \right\} \mathrm{d}t + (\sigma_t^{q^{\mathcal{B}}} - \sigma_t^{\mathcal{B}}) \mathrm{d}Z_t
\end{aligned}
$$

  To study monetary policy *without* fiscal implications, then set $\sigma_t^{\mathcal{B}} = 0$:
- Unexpected permanent increase in i_t at $t = 0$,

1. **Option "Pure MoPo"**: keep $\breve{\mu}_t^{\mathcal{B}}$ constant, i.e., $\mu_t^{\mathcal{B}}$ increases

$\Rightarrow$ increases inflation (one-for-one)

- "Neo-Fisherian" – "super-neutrality of money (growth)"

Introducing Long-term Government Bonds

- Long-term bond
  - yields fixed coupon interest rate on face value $F^{(i,m)}$
  - Matures at random time with arrival rate $1/m$
  - Nominal price of the bond $P_t^{\mathcal{B}(i,m)}$
  - Nominal value of all bonds outstanding of a certain maturity:

$$\mathcal{B}_t^{(m)} = P_t^{\mathcal{B}(i,m)} F^{(i,m)}$$

  - Nominal value of all bonds $\mathcal{B}_t = \sum_m \mathcal{B}_t^{(m)}$
- Special bonds
  - Reserves: $\mathcal{B}_t^{(0)}$ and note $P_t^{\mathcal{B}(0)} = 1$ (long-term but floating interest rate)
  - Consol bond: $\mathcal{B}_t^{(\infty)}$

Debt Evolution w/o Fiscal Implications

$$d\mathcal{B}_t^{(0)} = i_t \mathcal{B}_t^{(0)} \mathrm{d}t + \sum_{i,m} \left[\left(i + \frac{1}{m} \right) F_t^{(i,m)} \mathrm{d}t - \frac{\mathcal{B}_t^{(i,m)}}{F_t^{(i,m)}} (\mathrm{d}F_t^{(i,m)} + \frac{1}{m} F_t^{(i,m)} \mathrm{d}t) \right]$$

- Reserves $\mathcal{R}_t := \mathcal{B}_t^{(0)}$ is different since it pays floating interest rate i_t
- If we have only consol bond and T-bills (=reserves if no medium of exchange friction), then

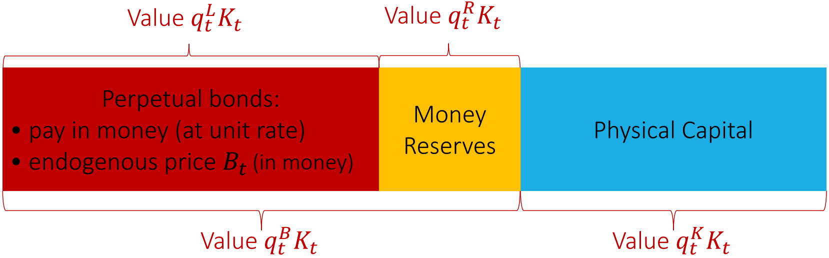
$$\mathrm{d}\mathcal{B}_t^{(0)} + \frac{\mathcal{B}_t^{(i,\infty)}}{F_t^{(i,\infty)}} \mathrm{d}F_t^{(i,\infty)} = i_t B_t^{(0)} \mathrm{d}t + i F_t^{(i,\infty)} dt$$

$$\boxed{\mathrm{d}\mathcal{R}_t + P_t^L \mathrm{d}F_t^L = i_t \mathcal{R}_t \mathrm{d}t + r^L F_t^L \mathrm{d}t}$$

New Notation: $\mathcal{B}_t^{(0)} = \mathcal{R}_t, F_t^{(i,\infty)} = F_t^L$

Introducing Long-term Gov. Bond

- Introduce long-term (perpetual) bond
  - No default ...
  - MoPo s.t. gov. bonds are held by intermediaries in equilibrium



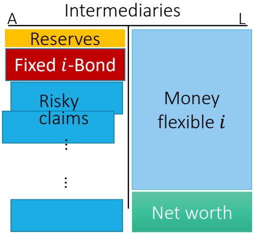
  - Value of long-term fixed *i*-bond is endogenous

$$dP_t^L / P_t^L = \mu_t^{P^L} dt + \sigma_t^{P^L} dZ_t$$

"Pure" Monetary Policy with Long-term Bonds

- Unexpected permanent cut in i_t at $t = 0$

1. **Sim's Stepping on the Rake**
   - At $t = 0$ on impact: as all $\mathcal{B}_0^{(m>0)}$ jump $\Rightarrow \mathcal{P}_0$ jumps up
   - For $t > 0$: inflation π_t is higher like in Neo-Fisherian setting
   - If long-term bonds are held proportionally to net-worth, then all citizens are affected proportionally.
2. **In I Theory**
   - Intermediaries are long long-term bonds and are short short-term money
   - Households are long short-term money paying i_t
   - Policy is Redistributive - "stealth recapitalization"
     - Long term bond price $\uparrow$
     - $\Rightarrow \eta_t \uparrow \Rightarrow$ risk premia $(\varsigma_t^I \sigma, \tilde{\varsigma}_t^I \tilde{\sigma}_t) \downarrow$



in real values

Analysis with Long-term Consol Bonds and Reserves

- Define fraction of value of bonds that are not in short-term reserves

$$\vartheta_t^L = \frac{P_t^L F_t^L}{\mathcal{B}_t},$$

- Let's postulate the price of a single long-term consol bond:

$$\frac{\mathrm{d}P_t^L}{P_t^L} = \mu_t^{P^L}\mathrm{d}t + \sigma_t^{P^L}\mathrm{d}Z_t$$

- In the total net worth numeraire the martingale pricing condition:

$$\mathbb{E}[\mathrm{d}r_t^L - \mathrm{d}r_t^{\mathcal{R}}] = \sigma_t^{P^L}\sigma_t^{\eta}$$

- for now assuming that only intermediaries find it worthwhile to hold consul bonds

$$\mathrm{d}r_t^L = \mathrm{d}r_t^{\mathcal{R}} + \sigma_t^{P^L}\sigma_t^{\eta}\mathrm{d}t + \sigma_t^{P^L}\mathrm{d}Z_t$$

0. Postulate Return Processes

- Return of total bond portfolio (in total net worth numeraire)
- $\mathrm{d}r_t^{\mathcal{B}} = \mu_t^{\vartheta}\mathrm{d}t + \sigma_t^{\vartheta}\mathrm{d}Z_t$ (since no fiscal implications)
- $\mathrm{d}r_t^{\mathcal{B}} = \mathrm{d}r_t^{\mathcal{R}} + \vartheta_t^L(\mathrm{d}r_t^L - \mathrm{d}r_t^{\mathcal{R}})$
- $\mathrm{d}r_t^{\mathcal{B}} = \mathrm{d}r_t^{\mathcal{R}} + \vartheta_t^L(\sigma_t^{P^L}\sigma_t^{\eta}\mathrm{d}t + \sigma_t^{P^L}\mathrm{d}Z_t)$

- Return of a single coin (reserve unit/short-term bond)
- $\mathrm{d}r_t^{\mathcal{R}} = (\mu_t^{\vartheta} - \vartheta_t^L\sigma_t^{P^L}\sigma_t^{\eta})\mathrm{d}t + (\sigma_t^{\vartheta} - \vartheta_t^L\sigma_t^{P^L})\mathrm{d}Z_t$
- $\vartheta_t^L\sigma_t^{P^L}$ shows importance of long-term bond price variation
  - The $\mathrm{d}Z_t$-term is a "risk-transfer"
  - The *dt*-term shows that it also affects risk premia.

η-Drift, Volatility and Amplification

Note that money is our benchmark asset

(since HH cannot go short L-bond)

- $\sigma_t^\eta = \sigma_t^{r^R} + (1 - \theta_t^{\mathcal{R},I} - \theta_t^{\mathcal{L},I})\sigma_t^{xK^b} + \theta_t^{L,I}(\sigma_t^{r^L} - \sigma_t^{r^\mathcal{R}})$
- Where portfolio share $1 - \theta_t^{\mathcal{R},I} - \theta_t^{L,I} = \frac{\chi_t}{\eta_t}(1-\vartheta_t)$ and $\theta_t^{L,I} = \vartheta_t^L \vartheta_t / \eta_t$

$$\sigma_t^\eta = \sigma_t^\vartheta - \vartheta_t^L \sigma_t^{P^L} + \frac{\chi_t(1-\vartheta_t)}{\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma_t^\vartheta}{1-\vartheta} + \vartheta_t^L \sigma_t^{P^L}\right) + \frac{\vartheta_t^L \vartheta_t}{\eta_t}\sigma_t^{P^L}$$

$$= \sigma_t^\vartheta - \vartheta_t^L \sigma_t^{P^L} + \frac{\chi_t(1-\vartheta_t)}{\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma_t^\vartheta}{1-\vartheta}\right) + \frac{\chi(1-\vartheta_t) + \vartheta_t - \eta_t}{\eta_t}\vartheta_t^L \sigma_t^{P^L}$$

- Replace: $\sigma_t^\vartheta = \frac{\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\sigma_t^\eta$ and $\sigma_t^{P^L} = \frac{P^{L'}(\eta)\eta_t}{P^L(\eta)}\sigma_t^\eta$

$$\eta_t \sigma_t^\eta = \frac{(1-\vartheta_t)\chi_t(1-\bar{\kappa})\sigma}{1 - \frac{\chi_t - \eta_t}{\eta_t}\left(-\frac{\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\right) + \vartheta_t^L \left(\frac{P^{L'}(\eta)\eta_t}{P^L(\eta)}\sigma_t^\eta\right)\frac{\chi_t(1-\vartheta_t)+\vartheta_t-\eta_t}{\eta_t}}$$

- Recall: $\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} = (1-\vartheta_t)\left(\frac{q^{K'}(\eta_t)\eta_t}{q^K(\eta_t)} + \frac{-q^{\mathcal{B}'}(\eta_t)\eta_t}{q^{\mathcal{B}}(\eta_t)}\right)$, mitigation term due to policy

Liquidity spiral   Disinflationary spiral

- μ_t^η same steps as before.

MoPo Benchmark 0: Inflation Targeting

- Pick a particular $\sigma_t^{\mathcal{B}}$, so that inflation at a constant rate.
  - $\Rightarrow$ Price level moves deterministically at a constant drift – no loading on $\mathrm{d}Z_t$-term.
  - Recall from real-vs.-nominal bond lecture:
    Inflation risk might not help to "complete markets".
- Remark:
  - $q_t^{\mathcal{B}}$ can still jump (unlike in a setting with price stickiness – see later lecture)

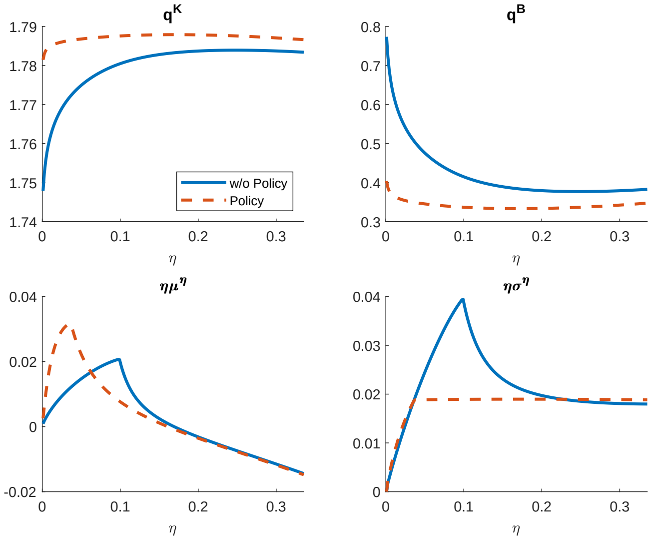
MoPo Benchmark 1: Removing Endogenous Risk

- The policy that removes endogenous risk, $\sigma_t^{\mathcal{B}} = \sigma_t^{\vartheta}$
- FOC gives:

$$\chi_t = \min\left\{\frac{\eta_t}{\eta_t + (1-\eta_t)\phi^2 + (1-\bar{\kappa})^2(\sigma^b)^2/\tilde{\sigma}^2}, \bar{\chi}\right\}$$

- η-Evolution: closed form up to ϑ_t (which is choice of planner)
  - $\sigma_t^{\eta} = (1-\vartheta_t)\frac{\chi_t}{\eta_t}(1-\bar{\kappa})\sigma^b$
  - $\eta_t\mu_t^{\eta} = \eta_t(1-\eta_t)(1-\vartheta_t)^2\left(\frac{1-2\eta_t}{(1-\eta_t)^2}\frac{\chi_t^2}{\eta_t^2}(1-\bar{\kappa})^2(\sigma^b)^2 + \frac{\chi_t^2\phi^2\tilde{\sigma}^2}{\eta_t^2} - \frac{(1-\chi_t)^2\phi^2\tilde{\sigma}^2}{(1-\eta_t)^2}\right)$
- Bond valuation equation: same as in page 41

MoPo Benchmark 1: Removing Endogenous Risk



MoPo Benchmark 2: Perfect Aggregate Risk Sharing

- Special case of Benchmark 1: Policy that ensures that $\sigma_t^\eta \to 0$
- Aggregate risk exposure of all households and intermediaries is proportional to σ^K and η_t, q_t^K, and $q_t^{\mathcal{B}}$ have no volatility.
- Remarks:
  - stochastic steady state moves closer to zero and $\sigma^\eta = 0$.
  - Boundary condition $\eta_t^I = 0$ plays no role anymore.
  - Leverage goes to infinity as $\eta_t \to 0$

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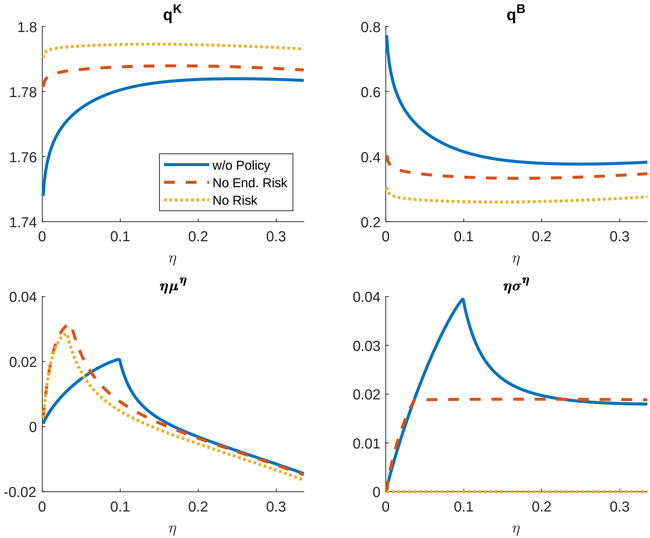

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Macroprudential Policy

- Monetary Policy cannot provide insurance and control risk taking at the same time.
  - Leverage rises endogenously the more risk sharing becomes possible.
  - Value of nominal bonds/money ϑ falls with perfect risk sharing
  - Might have adverse welfare implications

- $\Rightarrow$ Macroprudential Policy
  - Restrict intermediaries' leverage
  - Regulators simply "controls"
    intermediaries (and households) portfolio decisions $\boldsymbol{\theta}_t^i$

Optimal Policy

- Future lecture after we have covered welfare analysis

Recall

- Unified macro "Money and Banking" model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral
- Exogenous risk &
  - Sector specific
  - Idiosyncratic
- Endogenous risk **Paradox of Prudence**
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable
- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors - "Bottleneck Approach"
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking

Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes