Central Bank Balance Sheet Management

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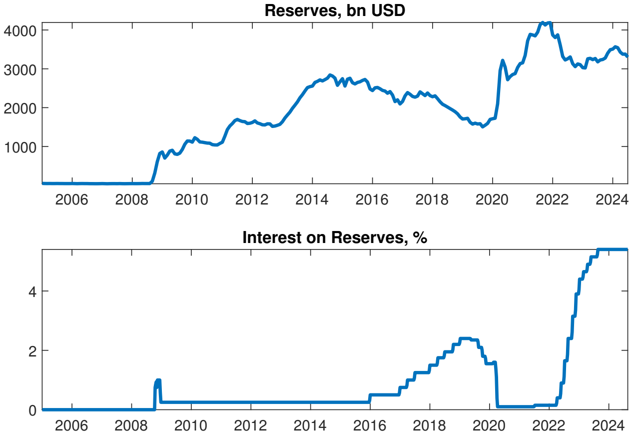
Motivation



- Interest rate payments as % of GDP: $< 0.02\%$ in 2010, $\sim 0.7\%$ in 2024

Motivation

- How to conduct interest rate policy after a long period of QE?
- How does the balance sheet policy interact with interest rate policy?
- What is the optimal size of central bank's balance sheet?
- Today: introduce a framework to study these questions
- Key takeaways:
  - QE works as a mediator of interest rate policy
    - Use QE to prepare for shocks, not to respond to them
  - More QE previously $\Longrightarrow$ more aggressive interest rate policy subsequently

Framework

- Two sectors: households and intermediaries
- Households hold capital and produce output
- Capital is subject to uninsurable idiosyncratic risk
- Intermediaries can partially diversify that risk
- Government levies taxes and conducts interest rate and balance sheet policies
- Prices are flexible (sticky-price extension if time permits)

Framework: Households

- Hold capital k_t and produce ak_t per unit of time
- Capital evolves as:
$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \tilde{\sigma}_t d\tilde{Z}_t$$
- $\tilde{Z}_t$ is uninsurable idiosyncratic risk
- Idisyncratic risk volatility $\tilde{\sigma}_t$ is time-varying:
$$d\tilde{\sigma}_t^2 = -b_s(\tilde{\sigma}_t^2 - \tilde{\sigma}_{ss}^2)dt$$
- Can offload part of their risk by issuing outside equity to *intermediaries*

Framework: Intermediaries

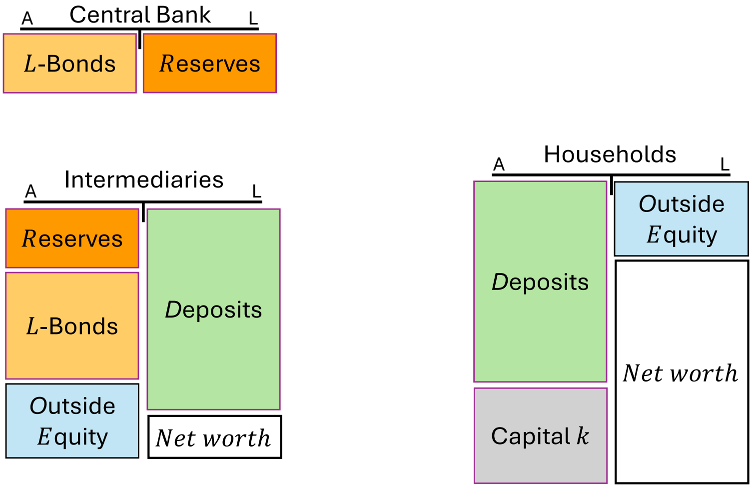
- Can not hold capital, but have a risk diversification technology
- When holding outside equity, only subject to $\varphi \in (0, 1)$ fraction of idiosyncratic risk $\tilde{\sigma}_t$
- Issue safe deposits to *households* and lever up
- Hold reserves and government long-term bonds

Framework: Government

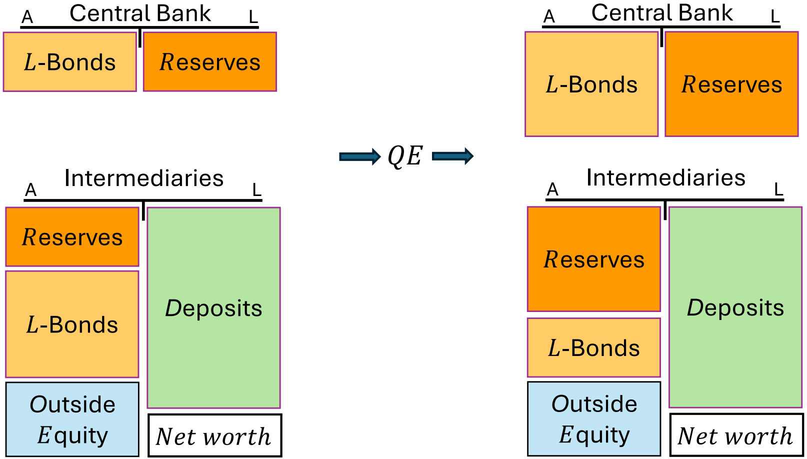
- Fiscal side:
  - Supply long-term bonds L_t, set interest rate i^L
  - Tax capital holdings τ_t^K
- Monetary side:
  - Issue reserves $\mathcal{R}_t$ and set reserve requirements $\underline{\theta}_t^{\mathcal{R}}$
  - Set interest rates on required and excess reserves $\underline{i}_t$ and i_t
  - Balance sheet policy: adjust the bonds-to-reserve ratio in the economy

$$\psi_t = \frac{L_t}{\mathcal{R}_t}$$

Model Overview



Balance Sheet Policy



Household's problem

$$\max_{\{c_t^H,\iota_t,\theta_t^K,\chi_t\}_{t=0}^{\infty}} \mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(c_t^H)dt\right] \quad \text{s.t.}$$

$$dn_t^H = -c_t^H dt + n_t^H\left[dr_t^D + \theta_t^K\left(\left(dr_t^K(\iota_t) - dr_t^D\right) - \chi_t(dr_t^{OE,H} - dr_t^D)\right)\right]$$

$$dr_t^D = r_t^D dt$$

$$dr_t^K(\iota_t) = \left(\frac{a - \iota_t - \tau_t^K}{q_t^K} + \mu_t^{q^K} + \Phi(\iota_t) - \delta\right)dt + \tilde{\sigma}_t d\tilde{Z}_t$$

$$dr_t^{OE,H} = r_t^{OE} dt + \tilde{\sigma}_t d\tilde{Z}_t$$

$$\tilde{\sigma}_t^{n,H} = \theta_t^K(1 - \chi_t)\tilde{\sigma}_t$$

Intermediary's problem

$$\max_{\{c_t^I,\theta_t^{\mathcal{R}},\theta_t^L,\theta_t^{OE,I}\}_{t=0}^{\infty}} \mathbb{E}_0\left[\int_0^{\infty} e^{-\rho t}\log(c_t^I)dt\right] \quad \text{s.t.}$$

$$dn_t^I = -c_t^I dt + n_t^I\left[dr_t^D + \theta_t^{\mathcal{R}}\left(dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) - dr_t^D\right) + \theta_t^L\left(dr_t^L - dr_t^D\right) + \theta_t^{OE,I}\left(dr_t^{OE,I} - dr_t^D\right)\right]$$

$$\theta_t^{\mathcal{R}} \geq \underline{\theta}_t^{\mathcal{R}}$$

$$dr_t^D = r_t^D dt$$

$$dr_t^{\mathcal{R}} = i(\theta_t^{\mathcal{R}})dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left(\left(1 - \frac{(\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}})^+}{\theta_t^{\mathcal{R}}}\right)\underline{i}_t + \frac{(\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}})^+}{\theta_t^{\mathcal{R}}} i_t - \pi_t\right)dt$$

$$dr_t^L = \frac{i^L}{P_t^L}dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left(\frac{i^L}{P_t^L} + \mu_t^{P^L} - \pi_t\right)dt$$

$$dr_t^{OE,I} = r_t^{OE}dt + \varphi\tilde{\sigma}_t d\tilde{Z}_t$$

Government

$$d\mathcal{R}_t + P^L_t dL_t + \mathcal{P}_t \tau^K_t K_t dt = \underline{i}_t \underline{\mathcal{R}}_t dt + i^L L_t dt + i_t(\mathcal{R}_t - \underline{\mathcal{R}}_t)dt$$

- P^L_t - nominal price of long-term bonds
- Nominal debt $\mathcal{B}_t = \mathcal{R}_t + P^L_t L_t$, real (per capital) value $q^{\mathcal{B}}_t = \frac{\mathcal{B}_t}{\mathcal{P}_t K_t}$
- Surplus-to-debt ratio:

$$\check{s}_t = \frac{\tau^K_t}{q^{\mathcal{B}}_t}$$

- QE policy:

$$\psi_t = \frac{L_t}{\mathcal{R}_t} \implies \vartheta^L_t = \frac{P^L_t L_t}{\mathcal{R}_t + P^L_t L_t} = \frac{\psi_t P^L_t}{1 + \psi_t P^L_t}$$

- Recall $\vartheta_t = \frac{\mathcal{B}_t/\mathcal{P}_t}{\mathcal{B}/\mathcal{P}_t + q^K_t K_t} = \frac{q^{\mathcal{B}}_t}{q^{\mathcal{B}}_t + q^K_t}$
- Set $\underline{\theta}^{\mathcal{R}}_t$ s.t. $\mathcal{R}_t = \underline{\mathcal{R}}_t \implies i_t$ is the marginal rate, $\underline{i}_t$ is the average rate

Equilibrium

- Key variables: $\tilde{\sigma}_t, \vartheta_t, P_t^L$ + intermediaries' wealth share $\eta_t = \frac{N_t^I}{N_t}$
- Markovian equilibrium with state variables $x \equiv \{\tilde{\sigma}, \eta\}$:
  - Laws of motion for x
  - Policy variables $\check{s}(x), \underline{\theta}^{\mathcal{R}}(x), \underline{i}(x), i(x), \vartheta^L(x)$
  - Mappings $\vartheta(x), P^L(x)$

  satisfying agents' optimality and market clearing

Solving for the Equilibrium

1. Optimal choices of households and intermediaries (SMP)
2. Combine with market clearing to derive:
   - Risk allocation χ_t as a function of state variables
   - "Money valuation equation" for ϑ_t
   - "Bond valuation equation" for P_t^L
   - Law of motion for η_t
3. Solve numerically
   - Now there are two state variables ($\tilde{\sigma}_t$ and η_t)
   - Still easy to solve since no aggregate risk

Solving for the Equilibrium: Summary

- Risk allocation:

$$\chi_t = \frac{\eta_t}{\eta_t + (1-\eta_t)\varphi^2}$$

- Money and long-term bond valuation equations:

$$\mu_t^{\vartheta} = \rho - (1-\vartheta_t)^2 \frac{\varphi^2}{\eta_t + (1-\eta_t)\varphi^2}\tilde{\sigma}_t^2 - \check{s}_t - (1-\vartheta_t)(1-\vartheta_t^L)(i_t - \underline{i}_t)$$

$$\mu_t^{P^L} = i_t - \frac{i^L}{P_t^L}$$

- Drift of η_t:

$$\mu_t^{\eta} = (1-\eta_t)\left[(1-\vartheta_t)^2 \frac{\varphi^2(1-\varphi^2)}{(\eta_t + (1-\eta_t)\varphi^2)^2}\tilde{\sigma}_t^2 - \frac{\vartheta_t}{\eta_t}(1-\vartheta_t^L)(i_t - \underline{i}_t)\right]$$

Constrained Efficiency

- To study optimal policy we need to define efficiency
- Suppose the planner can redistribute wealth between sectors but not within
- The planner freely chooses η_t but can not diversify idiosyncratic risk $\tilde{Z}_t$ away
- In addition, the planner has to respect the equilibrium mapping $\chi(\eta_t) \implies$
- The planner can not force the intermediaries to hold more (or less) risk than what realizes in equilibrium given η_t, but is free to choose η_t

Constrained Efficiency

- The constrained efficient allocation $\vartheta^*(\tilde{\sigma})$, $\eta^*(\tilde{\sigma})$, $\iota^*(\tilde{\sigma})$ solves

$$\max_{\{\iota_t,\vartheta_t,\eta_t\}_{t=0}^{\infty}} (1-\lambda)\mathbb{E}_0\left[\int_0^{\infty} e^{-\rho t}\log((1-\eta_t)\tilde{\eta}_t^H c_t K_t)dt\right]$$
$$+\lambda\mathbb{E}_0\left[\int_0^{\infty} e^{-\rho t}\log(\eta_t\tilde{\eta}_t^I c_t K_t)dt\right] \quad \text{s.t.}$$
$$c_t = a-\iota_t = \rho\frac{q_t^K}{1-\vartheta_t}, \qquad q_t^K = (1+\phi\iota_t), \qquad \chi_t = \frac{\eta_t}{\eta_t+\varphi^2(1-\eta_t)}$$
$$\frac{d\tilde{\eta}_t^I}{\tilde{\eta}_t^I} = (1-\vartheta_t)\frac{\chi_t}{\eta_t}\varphi\tilde{\sigma}_t d\tilde{Z}_t, \qquad \frac{d\tilde{\eta}_t^H}{\tilde{\eta}_t^H} = (1-\vartheta_t)\frac{1-\chi_t}{1-\eta_t}\tilde{\sigma}_t d\tilde{Z}_t$$

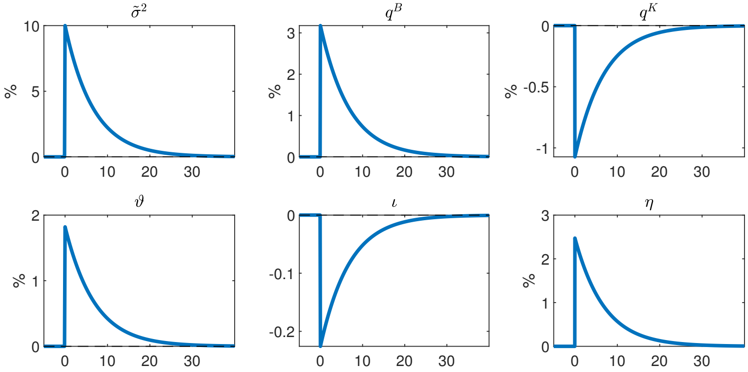
- Can be reduced to a static problem and solved state-by-state

Unanticipated shock to $\tilde{\sigma}$

- Suppose the economy is in the steady state
- At time $t = 0$ an unanticipated shock shifts $\tilde{\sigma}_0$
- Then idiosyncratic volatility converges back to the steady state deterministically:

$$d\tilde{\sigma}_t^2 = -b_s(\tilde{\sigma}_t^2 - \tilde{\sigma}_{ss}^2)dt$$

Constrained Efficient Path



Optimal Policy

- Optimal policy ensures:
  1. On impact the economy 'jumps' as in the efficient allocation:

$$\vartheta_0 = \vartheta_0^*, \;\; \eta_0 = \eta_0^*, \;\; \iota_0 = \iota_0^*$$

  2. Along the transition path the economy drifts as in the efficient allocation:

$$\mu_t^{\vartheta} = \mu_t^{\vartheta,*}, \;\; \mu_t^{\eta} = \mu_t^{\eta,*}, \;\; \mu_t^{\iota} = \mu_t^{\iota,*}$$

- Since $a - \iota_t = \rho \frac{1+\phi\iota_t}{1-\vartheta_t}$, we only need to care about ϑ_t and η_t

Optimal Policy: Transition

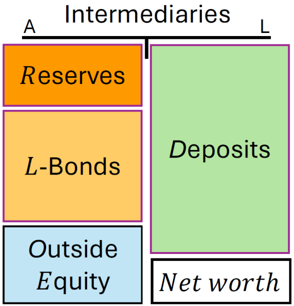
- Suppose the impact response is efficient
- For now, set $\vartheta_t^L = \vartheta^L$:

$$\mu_t^{\vartheta} = \rho - (1-\vartheta_t^*)^2 \frac{\varphi^2}{\eta_t^* + (1-\eta_t^*)\varphi^2} \tilde{\sigma}_t^2 - \check{s}_t - (1-\vartheta_t^*)(1-\vartheta^L)(i_t - \underline{i}_t) \overset{!}{=} \mu_t^{\vartheta,*}$$

$$\mu_t^{\eta} = (1-\eta_t^*)\left[(1-\vartheta_t^*)^2 \frac{\varphi^2(1-\varphi^2)}{(\eta_t^* + (1-\eta_t^*)\varphi^2)^2} \tilde{\sigma}_t^2 - \frac{\vartheta_t^*}{\eta_t^*}(1-\vartheta^L)(i_t - \underline{i}_t)\right] \overset{!}{=} \mu_t^{\eta,*}$$

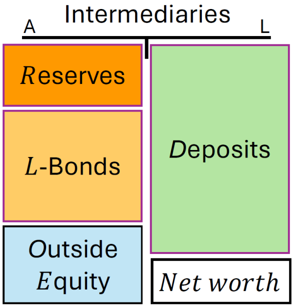
- Given balance sheet composition ϑ^L, interest rate policy $i_t - \underline{i}_t$ guides η_t
- Fiscal policy $\check{s}_t$ then guides portfolio choice ϑ_t

Optimal Policy: Impact Response



- Higher $\tilde{\sigma} \implies$ higher ϑ
- Nominal assets gain in value relative to capital
- Intermediaries are short in nominal assets and long in capital $\implies$ their net worth share decreases

Optimal Policy: Impact Response

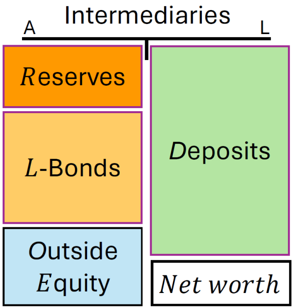


- Response of i_t changes the value of long-term bonds:

$$\mu_t^{P^L} = i_t - \frac{i^L}{P_t^L}, \qquad P_t^L = \int_t^\infty e^{-\int_t^\tau i_s ds} i^L d\tau$$

- Depending on path of i_t intermediaries gain or lose

Optimal Policy: Impact Response



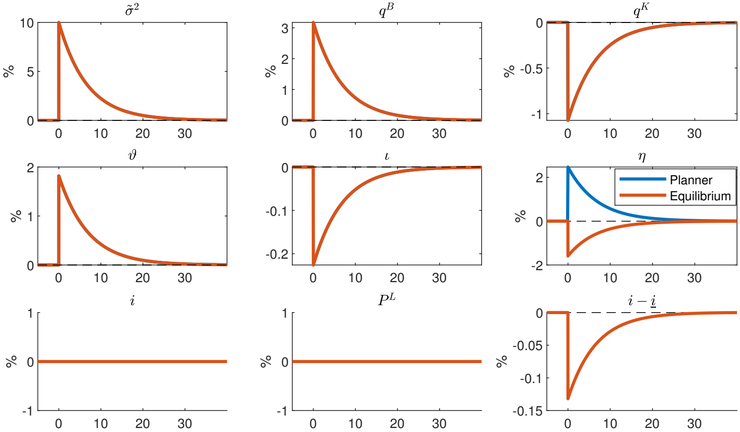
- Impact response of η_t is:

$$\eta_0 - \eta_{ss} = \frac{1}{1 + j_0^{\mathcal{B}}} \left[(\eta_{ss} - \chi_{ss}) \frac{\vartheta_0 - \vartheta_{ss}}{\vartheta_{ss}} + j_0^{\mathcal{B}} (\chi_{ss} - \eta_{ss} + \vartheta_0 (1 - \chi_{ss})) \right]$$

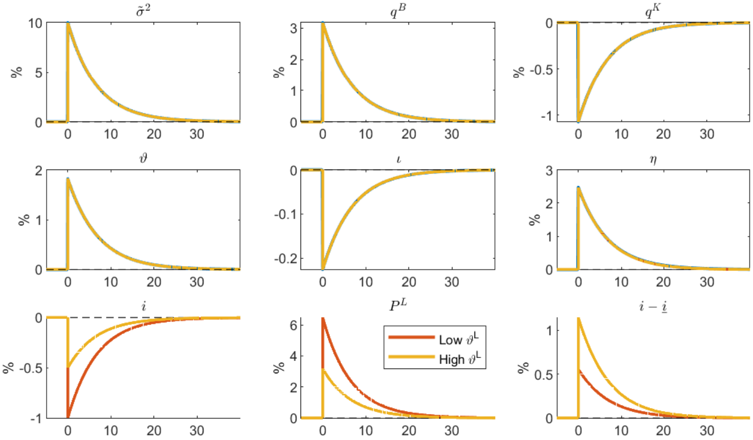
$$j_0^{\mathcal{B}} = \vartheta^L \frac{P_0^L - P_{ss}^L}{P_{ss}^L}, \qquad P_0^L = \int_0^\infty e^{-\int_0^\tau i_s ds} i^L d\tau$$

- Multiple solutions:
  - Small CB balance sheet (high ϑ^L) $\Longrightarrow$ less aggressive interest rate policy
  - Large CB balance sheet (low ϑ^L) $\Longrightarrow$ more aggressive interest rate policy

No i_t Response / No long-term bonds ($\vartheta^L = 0$)



Optimal Policy



Sticky Prices

- We can add a NK block to the model as in Li and Merkel (2023)
- This introduces another target for policymakers: capital utilization υ_t
- Same as with wealth share η_t, optimal policy needs to ensure:

$$\mu_t^{\upsilon} = \mu_t^{\upsilon,*}$$

$$\upsilon_0 - \upsilon_{ss} = \upsilon_0^* - \upsilon_{ss}^*$$

- The former one can be achieved with appropriate $\underline{i_t}$ policy
- The latter requires an adequate $j_0^{\mathcal{B}} = \vartheta^L \frac{P_0^L - P_{ss}^L}{P_{ss}^L}$
- This leads to a *trade-off* between financial and real sector efficiency

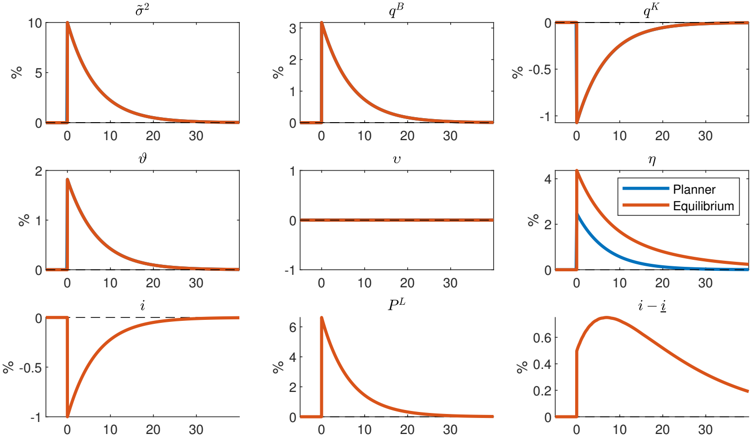
Sticky Prices

- More formally, the key variables are η_t, ϑ_t and υ_t
- Real sector efficiency: efficient output $Y_t = a\upsilon_t K_t$
- Goods market clearing + Tobin's q: $a\upsilon_t - \iota_t = \frac{1+\phi\iota_t}{1-\vartheta_t} \implies$
- Real sector efficiency depends on ϑ_t and υ_t
- Financial efficiency: efficient risk exposure in both sectors:

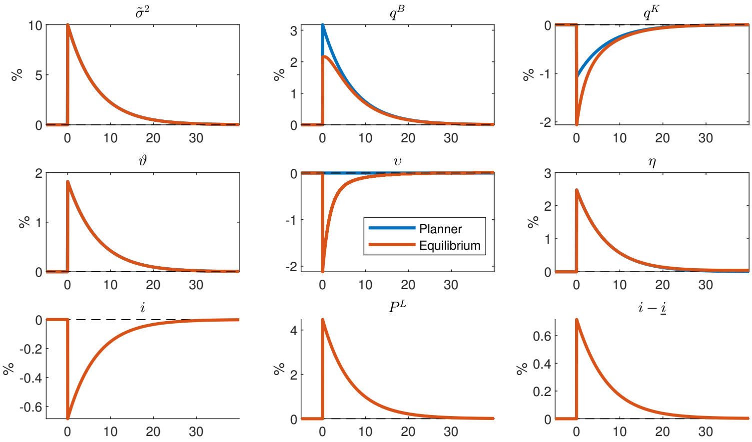
$$\tilde{\sigma}_t^{\eta} = \frac{1-\vartheta_t}{\eta_t + (1-\eta_t)\varphi^2}\varphi\tilde{\sigma}_t \qquad \tilde{\sigma}_t^{1-\eta} = \frac{(1-\vartheta_t)\varphi^2}{\eta_t + (1-\eta_t)\varphi^2}\tilde{\sigma}_t$$

- Financial efficiency depends on ϑ_t and η_t
- Policy can target either $\{\vartheta_t^*, \upsilon_t^*\}_{t=0}^{\infty}$ or $\{\vartheta_t^*, \eta_t^*\}_{t=0}^{\infty}$

Sticky Prices: Real Sector Efficiency



Sticky Prices: Financial Efficiency



Summary

- Balance sheet policy mediates the effects of conventional interest rate policy
- What matters is past QE:
  - QE is a tool to prepare for shocks, not a tool to respond to them
- More QE previously $\Longrightarrow$ more aggressive interest policy subsequently
- Under sticky prices: trade-off between real and financial efficiency