Innovations in Entrepreneurial Finance and Top Wealth Inequality

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Princeton Initiative

Question

- Entrepreneurial financing **improved** in US. Example: Growth of venture capital.
  - VC fraction of market cap less than $5\% \to 41\%$ since 70s (Gornall and Strebulaev 2021)
  - Top entrepreneur equity issuance rate $\uparrow$ (Gomez and Gouin-Bonenfant 2023)
- **This paper:** Tractable **general equilibrium** model to explore following question
- **Question:** What happens to wealth inequality when **risk sharing** improves through better financing for entrepreneurs?

Entrepreneurship and Top Wealth

- **Better equity financing** $\rightarrow$ **scale up** and **share risk**

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  - Better risk-sharing $\rightarrow$ extreme wealth trajectories less prevalent + less precautionary savings $\rightarrow$ Inequality $\downarrow$
  - **Earlier work**: Better risk-sharing $\rightarrow$ top wealth inequality $\downarrow$

    (Bonfiglioli 2012, Peter 2021, Hui 2023)

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- **This paper**: Better risk-sharing $\rightarrow$ top wealth inequality $\uparrow$
  - Entrepreneurs may choose to take more risk
  - Numerical exercise with data on **equity issuance** of top entrepreneurs to see if match facts

Second Contribution: Make Sense of Empirical Trends

- **When better risk-sharing $\rightarrow$ rising top inequality**: **Model makes sense of empirical trends:**
  - ✓ Elevated return to capital despite falling riskless rate
  - ✓ Declining aggregate labor share. Stable at firm level
  - ✓ Falling riskless rate

Literature

- **Idiosyncratic Investment Risk, Entrepreneurship and Inequality**

  Quadrini 2000, Cagetti and De Nardi 2006, Angeletos 2007, Gabaix et al. 2016, Benhabib, Bisin, and Zhu 2014, Jones and Kim 2018, Hubmer, Krusell, and A. A. Smith 2021, Peter 2021, Di Tella and Hall 2021, Atkeson and Irie 2022, Gomez and Gouin-Bonenfant 2023

- **The Venture Capital Revolution**

  Schmid 2001, Gompers and Lerner 2001, Gornall and Strebulaev 2021, Greenwood, Han, and Sanchez 2022...

- **Factor Income Distribution and Returns to Capital**

  Neiman and Karabarbounis 2014, Hartman-Glaser, Lustig, and Xiaolan 2019, Farhi and Gourio 2018 Autor et al. 2020, Barkai 2020, Moll, Rachel, and Restrepo 2019, Reis 2022

- **Measuring Wealth Inequality**

  Saez and Zucman 2016, Piketty, Saez, and Zucman 2018, Kuhn, Schularick, and Steins 2020, Bhandari et al. 2020, M. Smith, Zidar, and Zwick 2022, Gomez 2023

Returns and Risks Facing Entrepreneurs

- **Top wealth inequality depends crucially on returns and risks facing entrepreneurs**
- **How do entrepreneurs decide how much risk to take?**
- **First: Simplified Merton framework to isolate main forces**

Entrepreneurial Firms

- Entrepreneurs with logarithmic utility indexed by i
- Cobb-Douglas production

$$y_{it}dt = \bar{A}k_{it}^{\alpha}l_{it}^{1-\alpha}dt$$

- Hand-to-mouth workers supply labor at wage w_t
- Idiosyncratic risk

$$dk_{it} = (\iota_{it} - \delta)k_{it}dt + k_{it}\tilde{\sigma}dZ_{it}$$

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- **How financed?**

Capital Structure

$$\underbrace{k_{it}}_{\text{capital}} = \underbrace{v_{it}^{\text{out}}}_{\text{risky liabilities to outsiders}} + \underbrace{d_{it}}_{\text{risk-free debt}} + \underbrace{v_{it}^{\text{in}}}_{\text{entrepreneur's stake in firm}}$$

- Risky liabilities: same risk as capital, expected return determined on competitive capital market

$$dR_{it}^{out} = \underbrace{r_{it}^{\text{out}}}_{\text{expected return}} dt + \underbrace{\tilde{\sigma}}_{\text{volatility}} dZ_{it}$$

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- Risk sharing subject to skin-in-the-game constraint:

(Di Tella 2017, Brunnermeier and Sannikov 2017)

$$\underbrace{(k_{it} - v_{it}^{\text{out}})\tilde{\sigma}}_{\tilde{\sigma}_t^{\text{in}}} dZ_{it} \geq \chi k_{it} \tilde{\sigma} dZ_{it}$$

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- **Outsiders diversify** $\rightarrow r_{it}^{\text{out}} = r_t$

- risky liabilities vs risk-free debt? $\rightarrow$ Former offloads risk, latter increases.

Entrepreneurs' problem

- **Net worth:** $\underbrace{n_{it}}_{\text{net worth}} = \underbrace{k_{it}}_{\text{capital}} - \underbrace{v_{it}^{\text{out}}}_{\text{risky liabilities}} - \underbrace{d_{it}}_{\text{risk-free debt}} + \underbrace{v_{it}}_{\text{other investments}}$
- **Entrepreneurs' problem:**

$$\max_{\{c_{it},k_{it},l_{it},v_{it}^{out},d_{it},v_{it}\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(c_{it})dt\right]$$

$$dn_{it} = \left(y_{it} - w_t l_{it} - \delta k_{it} - v_{it}^{\text{out}} r_{it}^{\text{out}} - d_{it} r_t + v_{it} r_t - c_{it}\right) dt + (k_{it} - v_{it}^{\text{out}})\tilde{\sigma} dZ_{it}$$

$$\frac{k_{it} - v_{it}^{\text{out}}}{k_{it}} \geq \chi, \quad n_{it} \geq 0$$

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- **Lower χ $\Leftrightarrow$ Looser constraint $\Leftrightarrow$ Better risk-sharing**

Partial equilibrium: Better Risk Sharing $\Rightarrow$ More Risk Taking

- **Define return to capital**:

$$dR_{it}^k \equiv \underbrace{\frac{y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}}}_{\text{expected return: } r_{it}^k} dt + \tilde{\sigma} dZ_{it}$$

- Choice of $\frac{l_{it}}{k_{it}}$ independent of $i \Rightarrow r_t^k$ and r_t^{out}
- **Merton-style choice of risk exposure**:

$$\underbrace{\tilde{\sigma}_{it}^E}_{\text{risk exposure}} \equiv \frac{k_{it}}{n_{it}} \chi \tilde{\sigma} = \underbrace{\frac{r_t^k - r_t^{\text{out}}}{\chi \tilde{\sigma}}}_{\text{Sharpe ratio}}$$

- **Partial Equilibrium: Better risk-sharing $\Rightarrow$ More risk-taking**

But Excess Return Is Endogenous

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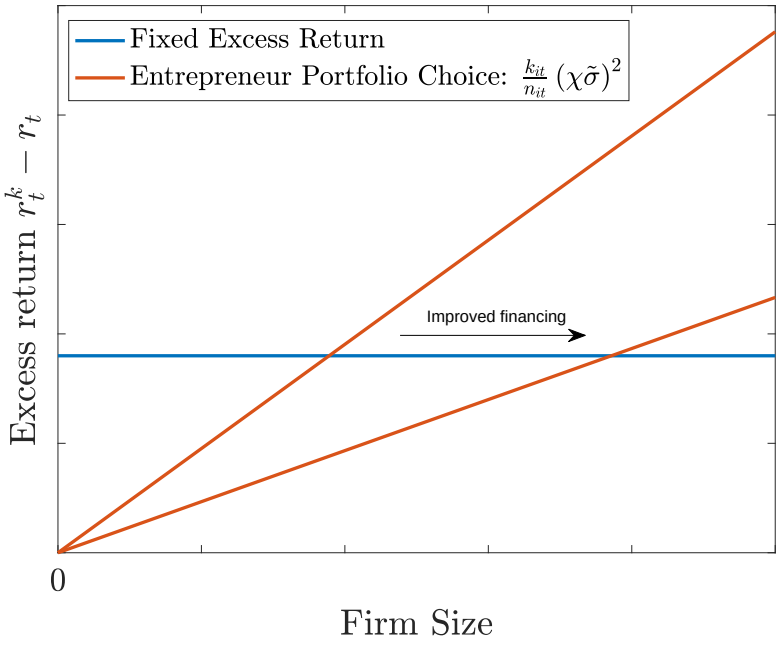
- **Partial Equilibrium: Better risk-sharing $\Rightarrow$ More risk-taking**

- **But: Endogenizing returns can reverse conclusion**
- **Simplest example:** If capital stock is limited by entrepreneurs' net worth in aggregate ($K_t = N_t^E$):

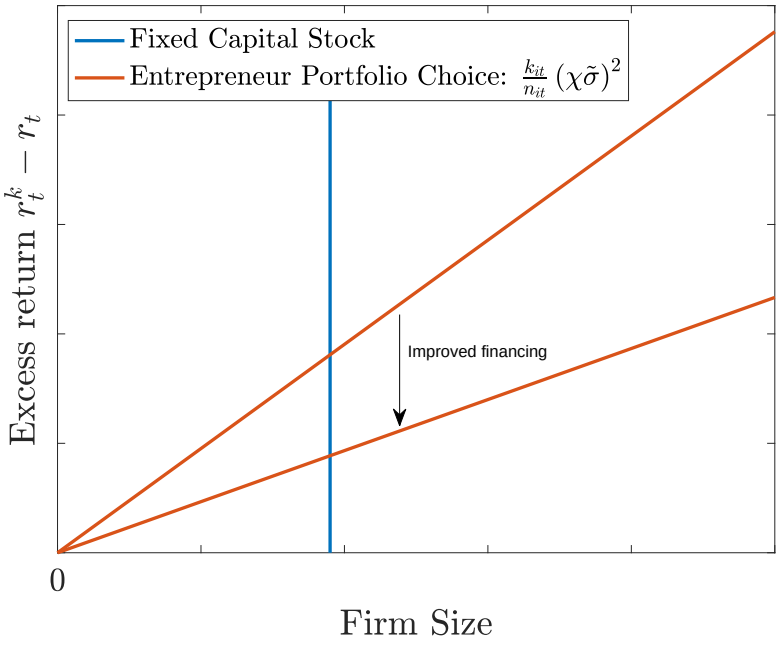
$$\tilde{\sigma}_t^E = \frac{K_t}{N_t^E} \chi \tilde{\sigma} = \chi \tilde{\sigma}$$

- Excess return $r_t^k - r_t^{\text{out}} = (\chi \tilde{\sigma})^2$ adjusts to prevent scaling up
- **Better risk-sharing $\Rightarrow$ Less risk-taking!**

Improved Risk Sharing Can Go Either Way



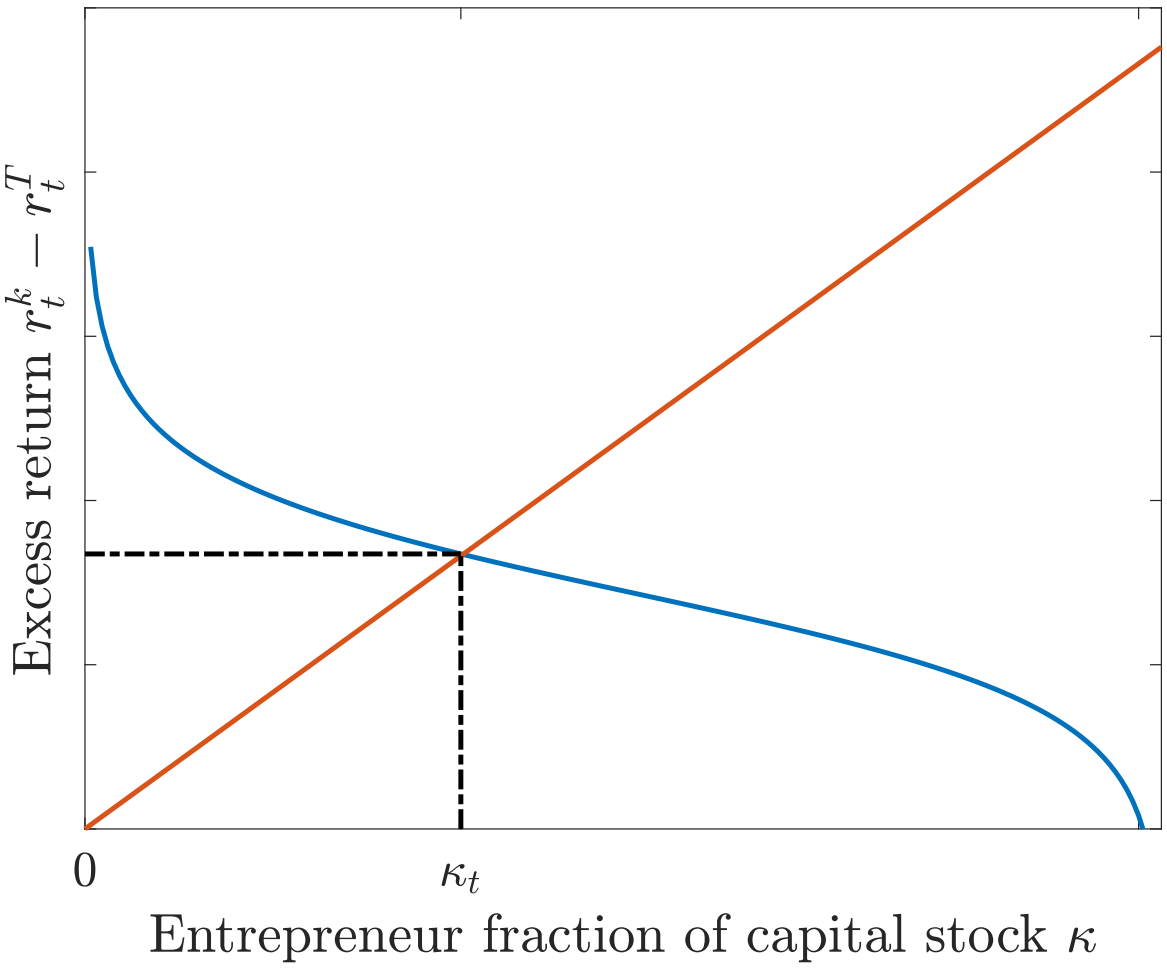
(a) Partial Equilibrium:
Scaling up (relative to net worth), returns stable.
Choose **higher** risk exposure



(b) But if $K_t = N_t^E$ in aggregate:
No Scaling up (relative to net worth), returns fall.
Choose **lower** risk exposure

This paper: Two-sector General Equilibrium

- **Entrepreneurs' portfolio choice**
- **Product market demand**



Less Constrained Equity Financing: Three Forces

- Model summarizes impact in **three economic forces:**

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- **Main theoretical result:**
  - **Strong reallocation effect** $\rightarrow$ **scaling up > risk reduction**

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Full Model

- **Three types of households:** Entrepreneur, diversified capitalist, hand-to-mouth worker.
- **Two types of firms:** Operated by entrepreneurs vs standard neoclassical (established traditional) firms (Angeletos (2007))
- **Traditional firm:** Less productive, no financing restrictions, produce substitutable good.
- **Substitutability of goods determines strength of reallocation effect**

Traditional Firm

- Also use Cobb-Douglas: $Y_t^T dt = \underline{A}(K_t^T)^{\alpha}(L_t^T)^{1-\alpha}dt$
- Less productive $\underline{A} < \bar{A}$
- No external financing constraints $\rightarrow$ fully financed externally.

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- Less productive $\underline{A} < \bar{A}$
- No external financing constraints $\rightarrow$ fully financed externally.
- Same labor market and same market for external capital:
  $\Rightarrow$ common wage w_t rate, cost of capital $r_t^T = r_t^{\text{out}}$
- Wage and cost of external capital pinned down by marginal products in traditional firm:

$$w_t = p_t^T (1-\alpha) \left(\frac{Y_t^T}{L_t^T} \right), \quad r_t^T = p_t^T \alpha \left(\frac{Y_t^T}{K_t^T} \right)$$

Final Output and Factor Markets

- Final goods producing sector. CES-production:

$$Y_t = \left[\nu \left(Y_t^E\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\nu)\left(Y_t^T\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- **Product demand**: $p_t^E = \nu\left(\frac{Y_t^E}{Y_t}\right)^{-\frac{1}{\varepsilon}}, \quad p_t^T = (1-\nu)\left(\frac{Y_t^T}{Y_t}\right)^{-\frac{1}{\varepsilon}}$

- **Clarification:** *Not* a model with imperfect competition. Imperfect substitutability is at sector level.

⏩ Some limitations and possible extensions

Modification of Entrepreneurial Risk

- **Idiosyncratic risk**

$$dk_{it} = (\iota_{it} - \delta)k_{it}dt + y_{it}\tilde{\sigma}dZ_{it}$$

  - Shocks proportional to output, as in Di Tella and Hall 2021. ⏩ Their setup
  - $\Rightarrow$ Entrepreneurs choose **same** capital-labor ratio as traditional firms.
  - $\Rightarrow$ Entrepreneur's share comes from pure capital share **and** labor share.

- $\Rightarrow$ **Tractability and Testable Implications for Factor Income Shares**

Other Households and Evolution of Aggregate State

- **Hand-to-mouth workers:** consume labor income and supply labor: $C_t^W = w_t L$
- **Diversified capitalists:** Consume fraction $C_t^D = \rho N_t^D$, save the rest at rate r_t^T
- **Aggregate state:** K_t and entrepreneurs wealth share $\eta_t = \frac{N_t^E}{N_t^E + N_t^D}$:

$$dK_t = \left(Y_t - C_t^E - C_t^D - C_t^W - \delta K_t\right) dt$$

$$d\eta_t = (1-\eta_t)\eta_t \underbrace{(r_t^E - r_t^T)}_{\text{return diff.}} dt + \underbrace{(\bar{\psi} - \eta_t)\left(\delta_d + \phi^l\right)}_{\text{demographics}} dt$$

- Steady state $\Rightarrow dK_t = d\eta_t = 0$

⏩ Equilibrium Definition

Risk Premium Determined in Equilibrium

- **Joint determination** of excess return $r_t^k - r_t^T$ and capital allocation $\kappa_t \equiv \frac{K_t^E}{K_t}$
- **Aggregate state**: K_t and entrepreneurs' share of wealth η_t
- Entrepreneurs face risk premium $r_t^k - r_t^T \Rightarrow$ **portfolio choice**
- Product demand, costs of capital $\Rightarrow$ **firms' demand for capital**

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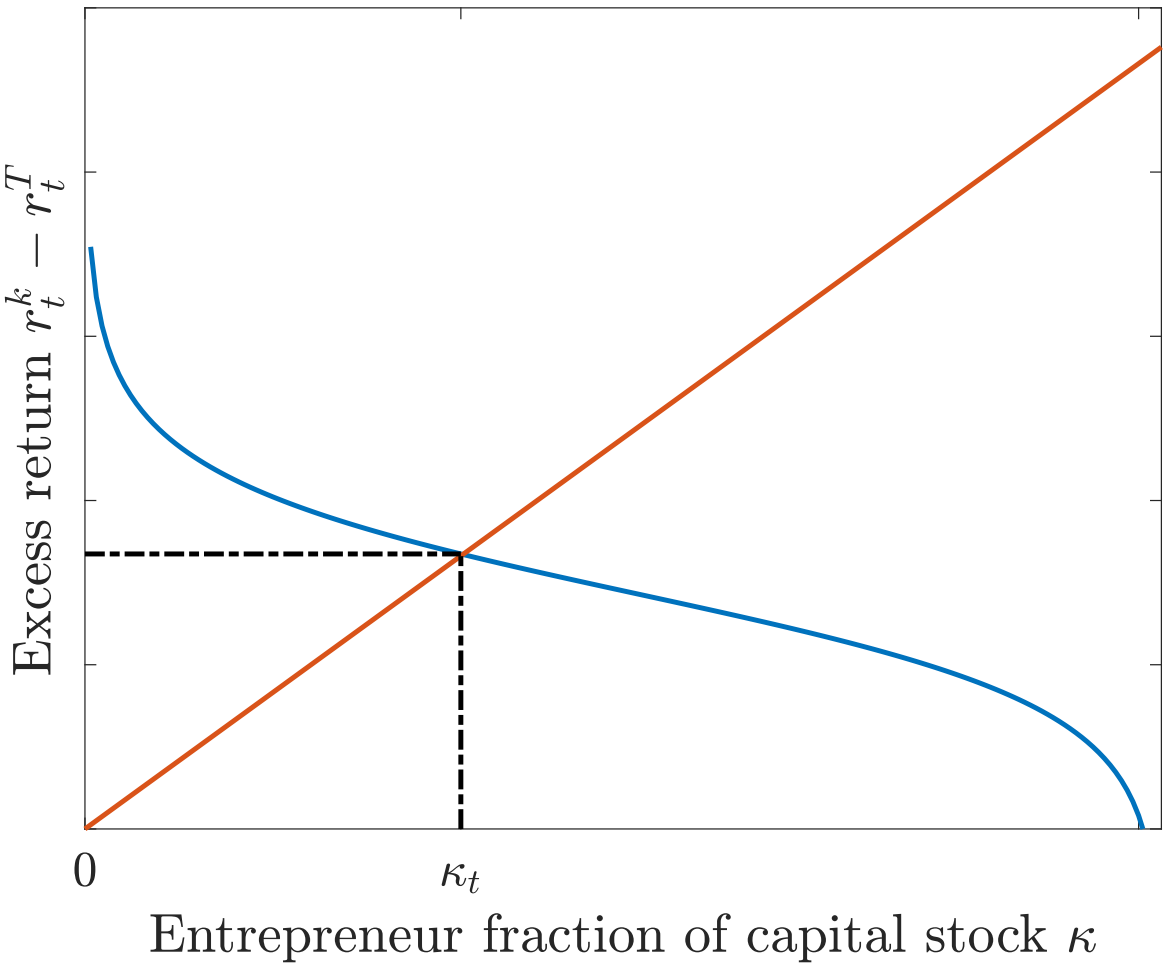

Relative Supply and Relative Demand

- Entrepreneurs' capital and wealth share: $\kappa_t = \frac{K_t^E}{K_t}$ and $\eta_t = \frac{N_t^E}{K_t}$.
- Upward sloping **relative supply schedule**:

$$r_t^k - r_t^T = \frac{\kappa_t}{\eta_t}(\chi \tilde{\sigma}_t^k)^2$$

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$$r_t^k - r_t^T = \frac{\kappa_t}{\eta_t}(\chi \tilde{\sigma}_t^k)^2$$

- Downward sloping **relative demand schedule**:

$$r_t^k - r_t^T = \underbrace{\left(\bar{A}p^E(\kappa_t) - \underline{A}p^T(\kappa_t)\right)\left(\frac{L}{K_t}\right)^{1-\alpha}}_{\text{Difference in risk-adjusted VMPK}}$$

- Product demand $p^E(\kappa_t) = \nu\left(\frac{\bar{A}\kappa_t}{A(\kappa_t)}\right)^{-\frac{1}{\varepsilon}} \rightarrow$ firms' demand for capital.

(Picture on next slide.)

Characterizing Equilibrium Excess Return

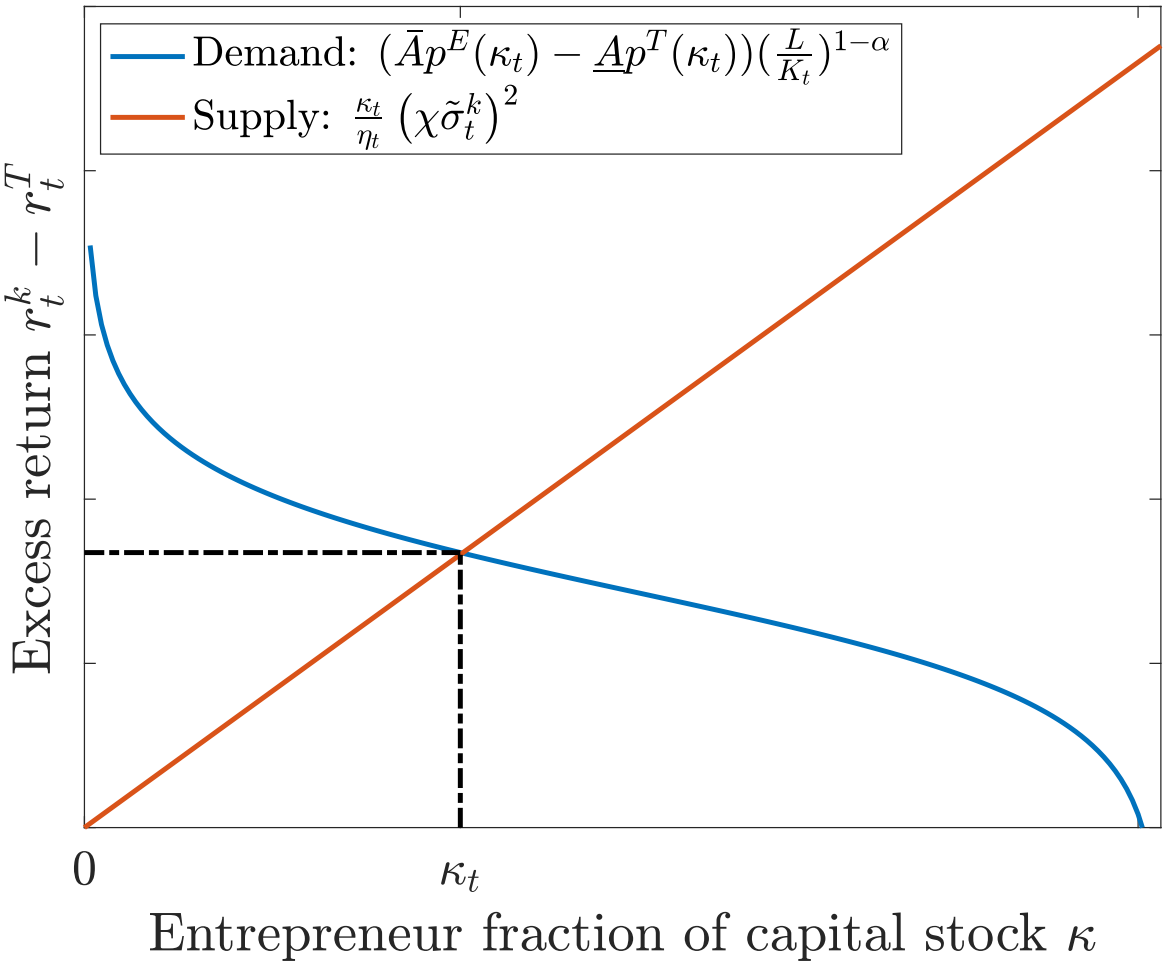
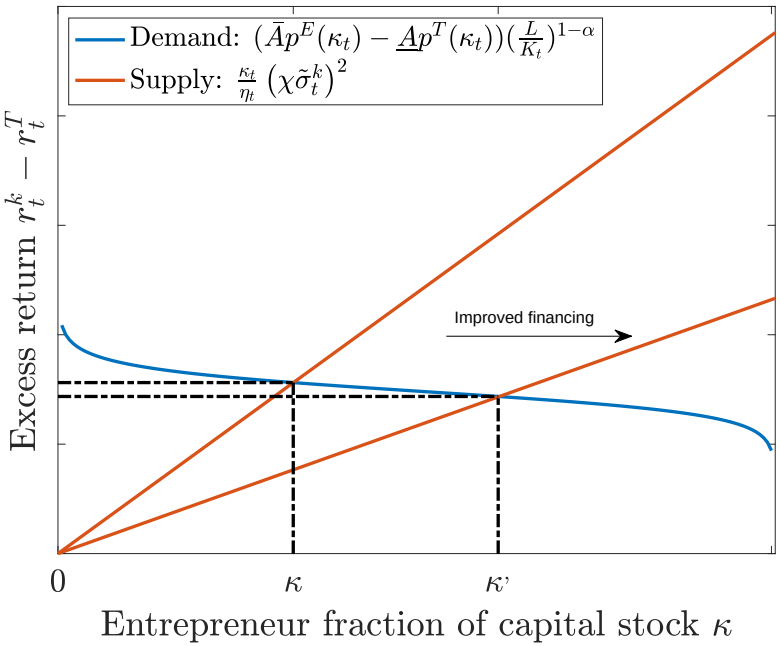
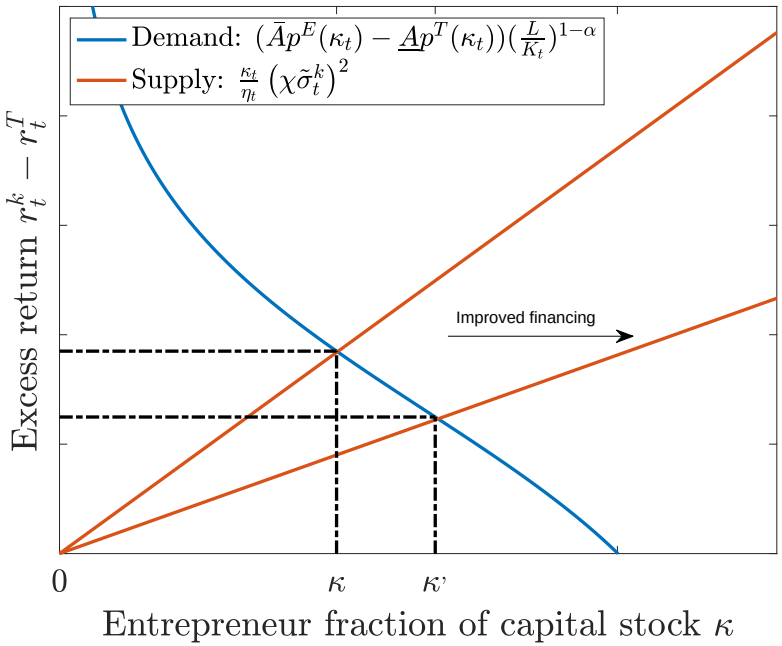


Figure: Determination of allocation of capital to entrepreneurs.

Better Risk Sharing $\Rightarrow$ Drop In $r_t^k - r_t^T$, Elasticity determines magnitude



(a) High elasticity of substitution ε



(b) Low elasticity of substitution ε

Figure: Excess return stable and large reallocation if high elasticity ε

Risk Sharing Improves $\Rightarrow$ Risk Exposure Increases if Elasticity Of Substitution Between Firms is High

- **Result:** *Starting in an interior steady state, if ε large enough, then entrepreneurs' steady state risk exposure $\tilde{\sigma}_{ss}^{E} \uparrow$ when skin-in-the-game constraint less severe $\chi \downarrow$*

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- **Macro**: Prices stable if elasticity ε high

Top Wealth Inequality in Long Run

- **Empirically:** Right-tail of wealth distribution has Pareto Shape
- **Measure of Top inequality**: Inverse of tail coefficient = $1/\zeta$
- $S(p)$ is top percentile p share of wealth. Then:

$$10^{1/\zeta - 1} = \frac{S(p)}{S(10p)}$$

⏩ Pareto Inequality In US and France

Wealth Distribution Has Pareto Shape In This Model

- **Long run**: entrepreneur's wealth geometric Brownian:

$$\frac{dn_{it}}{n_{it}} = \underbrace{\mu_{ss}^{E}}_{\text{drift}} dt + \underbrace{\tilde{\sigma}_{ss}^{E}}_{\text{volatility}} dZ_{it}$$

- $+$ demographic assumptions $\Rightarrow$ **Pareto distributed right tail**

- **In this paper** steady state implies

$$\text{Wealth growth:} \quad \mu_{ss}^{E} = r_{ss}^{T} + \left(\tilde{\sigma}_{ss}^{E}\right)^{2} - \rho$$

$$\text{Risk free rate:} \quad r_{ss}^{T} = \rho - \eta_{ss}\left(\tilde{\sigma}_{ss}^{E}\right)^{2}$$

$$\text{Wealth share:} \quad \eta_{ss} = \frac{(\eta_{ss} - \bar{\psi})(\delta_d + \phi^l)}{(\tilde{\sigma}_{ss}^{E})^2(1 - \eta_{ss})}$$

- $\Rightarrow$ **Risk exposure** $\tilde{\sigma}_{ss}^{E}+$ demographics pin **Pareto tail**!

Result 1: Risk Exposure Determines Pareto Inequality

- **Wealth process** pinned down by **risk exposure** + demography

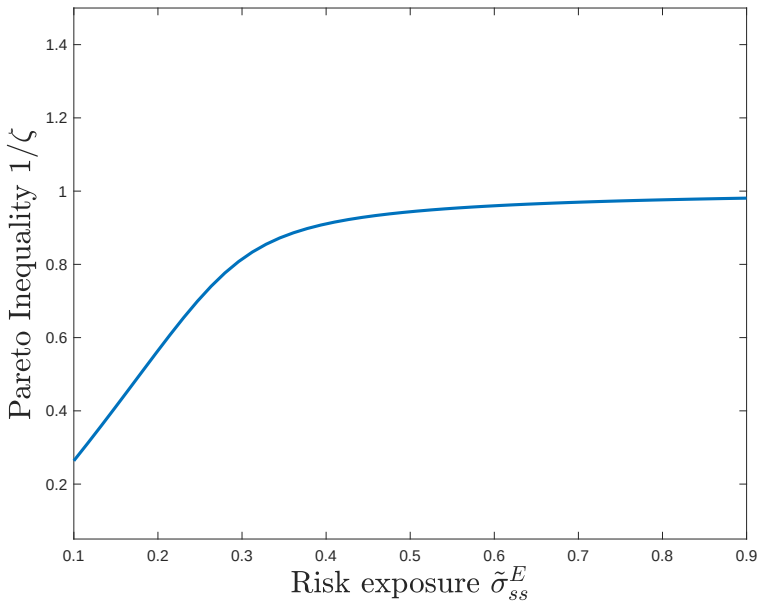

Figure: Understand risk exposure $\Leftrightarrow$ understand Pareto inequality $1/\zeta$.

⏩ Closed form expression for ζ

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⏩ Non-Monotonic

Evolution of Top Wealth Inequality in Numerical Example

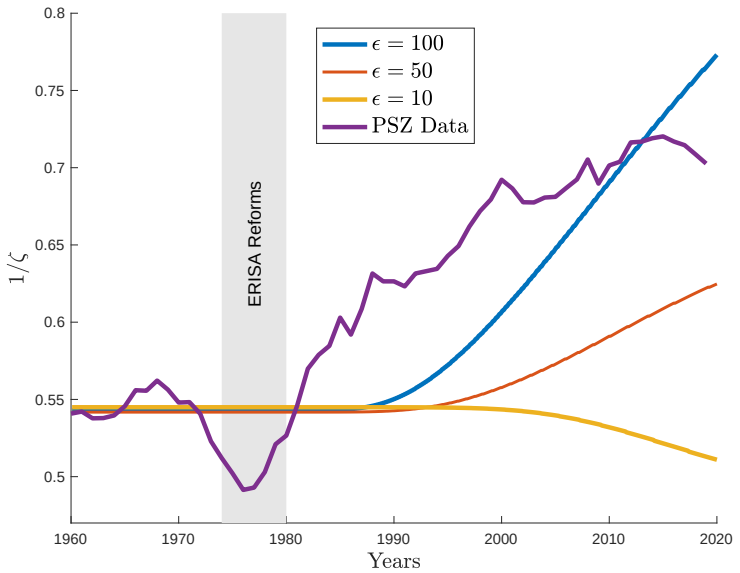

Figure: Transition of Pareto Inequality $1/\zeta$ at top 0.1%

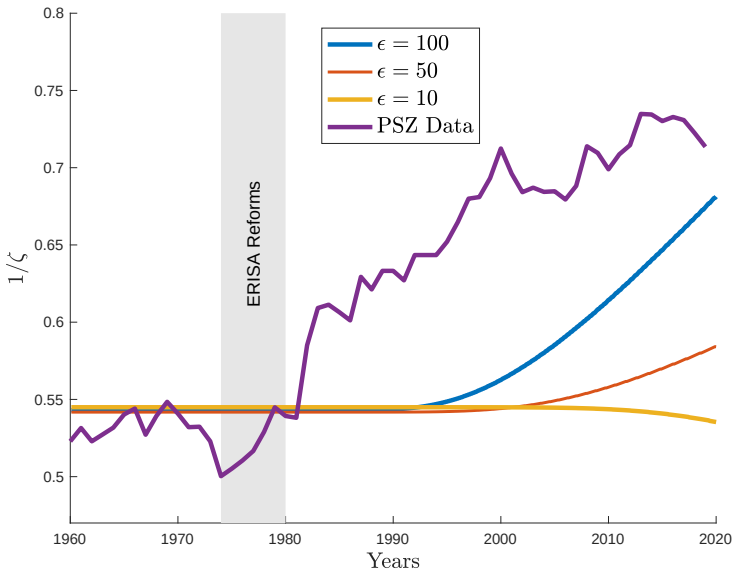

Figure: Transition of Pareto Inequality $1/\zeta$ at top 0.01%

Aggregate Return on Capital Amid Falling Risk-Free Rate

✓ Elevated return to capital despite falling riskless rate.

- How does model produce this?
- Steady state

$$r^K = \kappa r^k + (1-\kappa) r^T = r + \sigma^2 + \eta(\sigma^E)^2$$

- Even if r falls, return to capital elevated since entrepreneurs wealthier and take more idisyncratic risk.

Falling Labor Share

✓ Declining aggregate labor share. Stable at firm level.

- How does model produce this?
- Labor share lower in Entrepreneurial Firms:

$$\frac{w_t L_t^E}{p^E(\kappa_t) Y_t^E} = (1-\alpha)\frac{p^T(\kappa_t)}{p^E(\kappa_t)}\frac{A}{\bar{A}} = (1-\alpha)\left(1 - \underbrace{\frac{(r_t^k - r^T)K_t^E}{p^E(\kappa_t)Y_t^E}}_{\text{“entrepreneurial" share}}\right)$$

Falling Labor Share

✓ Declining aggregate labor share. Stable at firm level.

- How does model produce this?
- Labor share lower in Entrepreneurial Firms:

$$\frac{w_t L_t^E}{p^E(\kappa_t) Y_t^E} = (1-\alpha)\frac{p^T(\kappa_t)}{p^E(\kappa_t)}\frac{A}{\bar{A}} = (1-\alpha)\left(1 - \underbrace{\frac{(r_t^k - r^T)K_t^E}{p^E(\kappa_t)Y_t^E}}_{\text{“entrepreneurial" share}}\right)$$

- (Almost) Simpson's paradox. Aggregate labor share is weighted (by sales shares ω) average labor share:

$$LS = \omega^E LS^E + \omega^T LS^T$$

- $\omega^E \uparrow$, $LS^E \downarrow$
- Elasticity is high enough$\Rightarrow$ composition effect dominates: $LS \downarrow$.

(Modestly) Falling Riskless Rate

✓ Falling Riskless Rate.

- How does model produce this?
- Entrepreneur's Precautionary Savings Rise:

$$\text{savings out of income} = \frac{(r^E - \rho)n}{r^E n} = 1 - \frac{\rho}{r^E}$$

- Risk-free rate

$$r = \rho - \sigma^2 - \eta(\sigma^E)^2$$

(Modestly) Falling Riskless Rate

✓ Falling Riskless Rate.

- How does model produce this?
- Entrepreneur's Precautionary Savings Rise:

$$\text{savings out of income} = \frac{(r^E - \rho)n}{r^E n} = 1 - \frac{\rho}{r^E}$$

- Risk-free rate

$$r = \rho - \sigma^2 - \eta(\sigma^E)^2$$

- (Bonus: $1 - \frac{\rho}{r^E} > 1 - \frac{\rho}{r^D}$ so savings out of income positively correlated with wealth even with homothetic utility.)

Conclusion

- **This paper:** Tractable GE model to study impact of improved equity financing on top wealth inequality
- **Main take-away:** Improved risk sharing $\rightarrow$ higher top wealth inequality **if** entrepreneurs can **scale up** enough in equilibrium
  - Depends on **elasticity of substitution** with traditional firms.
  - If elasticity is high $\rightarrow$ large reallocation $\rightarrow$ model makes sense of series of trends.

Pareto Inequality In US and France

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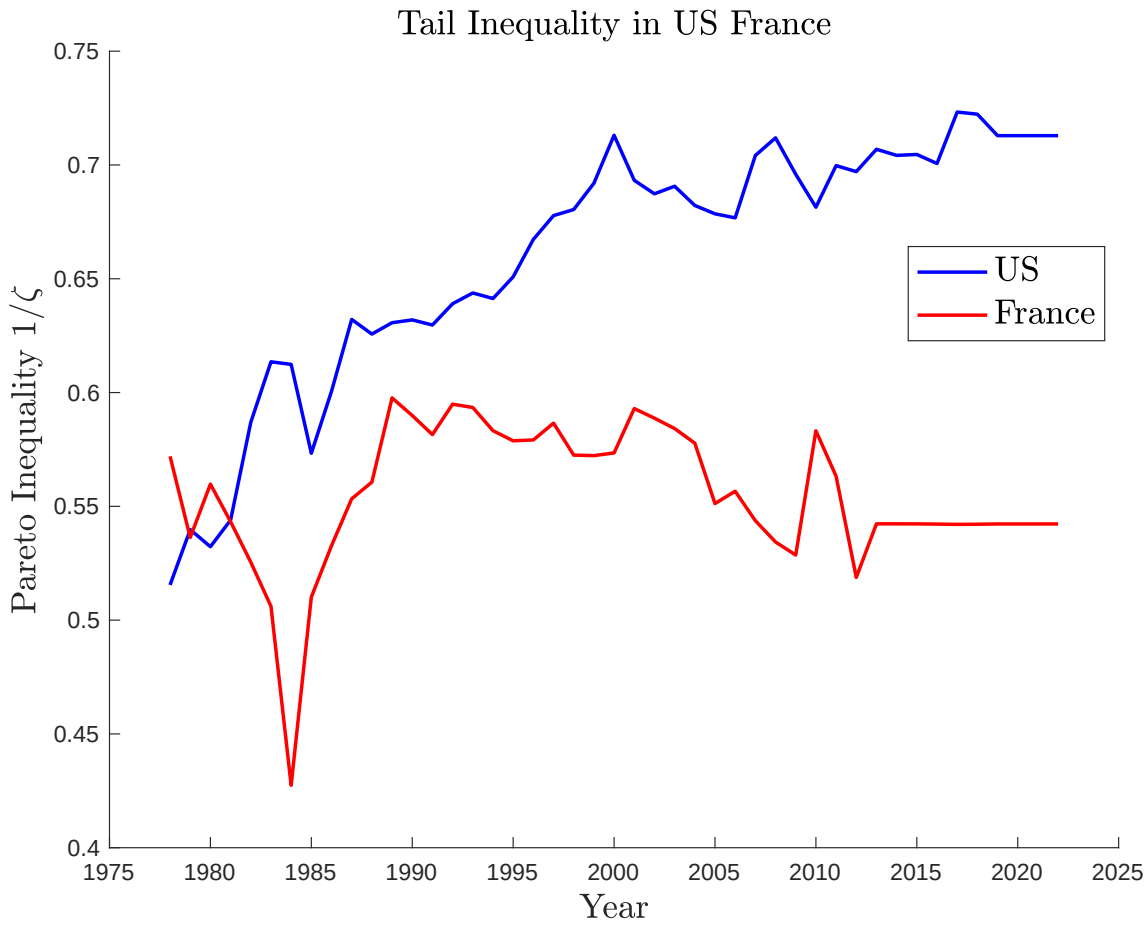

Figure: Pareto Inequality $1/\zeta$ at top 0.1% in US and France.
Data: World Inequality Database

Limitations and Extensions

- **Possible Extensions**:
  - Multiple sectors, heterogeneous substitutability ε
  - Endogenous matching between financiers and entrepreneurs
  - Heterogeneous inside risk fraction χ. Changing over firm lifetime.

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$$dY_{it} = y_{it}dt + y_{it}dZ_{it}$$

$$\Rightarrow dR^k_{it} = \frac{dY_{it} - w_t l_{it} - \delta k_{it}}{k_{it}} = \frac{y_{it} - w_t l_{it} - \delta k_{it}}{k_{it}}dt + \frac{y_{it}}{k_{it}}\tilde{\sigma}dZ_{it}$$

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Entrepreneurs' solution

$$c_{it} = \rho n_{it}, \quad y_{it} = \bar{A}\left(\frac{1-\alpha}{\alpha}\frac{r_t^T + \delta}{w_t}\right)^{1-\alpha} k_{it}, \quad k_{it} = \frac{r_{it}^k - r_t^T}{(\chi\tilde{\sigma}_t^k)^2} n_{it}$$

$$v_{it} = \frac{r_t^T - r_t}{\sigma^2} n_{it} - \chi k_{it}, \quad v_{it}^{\text{out}} = (1-\chi)k_{it}$$

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- Initial K_0, initial share of wealth held by entrepreneurs η_0
- Prices w_t, r_t^T, p_t^E, p_t^T
- Share of capital operated by entrepreneurs κ_t
- Household optimization
- Clearing labor, external capital, and product markets.
- K_t and $\eta_t = \frac{\int_{i \in E} n_{it} di}{K_t}$ evolve according to

$$dK_t = \left(Y_t - C_t^E - C_t^D - C_t^W - \delta K_t \right) dt$$

$$d\eta_t = (1 - \eta_t)\eta_t \underbrace{(r_t^E - r_t^T)}_{\text{return diff.}} dt + \underbrace{(\bar{\psi} - \eta_t) \left(\delta_d + \phi^l \right)}_{\text{demographics}} dt$$

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Risk Exposure Determines Inequality

$$\zeta = \eta_{ss} - \frac{1}{2} + \sqrt{\left(\eta_{ss} - \frac{1}{2}\right)^2 + \frac{2\eta_{ss}(1-\eta_{ss})}{\eta_{ss} - \bar{\psi}}}$$

$$\eta_{ss} = \frac{(\eta_{ss} - \bar{\psi})(\delta_d + \phi^l)}{(\tilde{\sigma}^E_{ss})^2(1-\eta_{ss})}$$

$$\frac{d\zeta}{d\eta_{ss}} < 0 \Rightarrow \frac{d\zeta}{d\tilde{\sigma}^E_{ss}} < 0 \tag{2}$$

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Entrepreneurs wealth accumulation.

- Risk and returns are constant in long run.
- Individual entrepreneurs wealth geometric Brownian:

$$\frac{dn_{it}}{n_{it}} = \left(r_{ss}^{E} - \rho\right) dt + \tilde{\sigma}_{ss}^{E} dZ_{it}$$

- Equilibrium risk and return related $\Rightarrow r_{ss}^{E} = r_{ss}^{\text{fund}} + \left(\tilde{\sigma}_{ss}^{E}\right)^2$.
- State return to agg. wealth is $\rho \Rightarrow \rho = \eta_{ss} r_{ss}^{E} + (1 - \eta_{ss}) r_{ss}^{\text{fund}}$
- Combine

$$\frac{dn_{it}}{n_{it}} = (1 - \eta_{ss}) \left(\tilde{\sigma}_{ss}^{E}\right)^2 dt + \tilde{\sigma}_{ss}^{E} dZ_{it} \tag{3}$$

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Non-Monotonicity in Sharpe Ratio

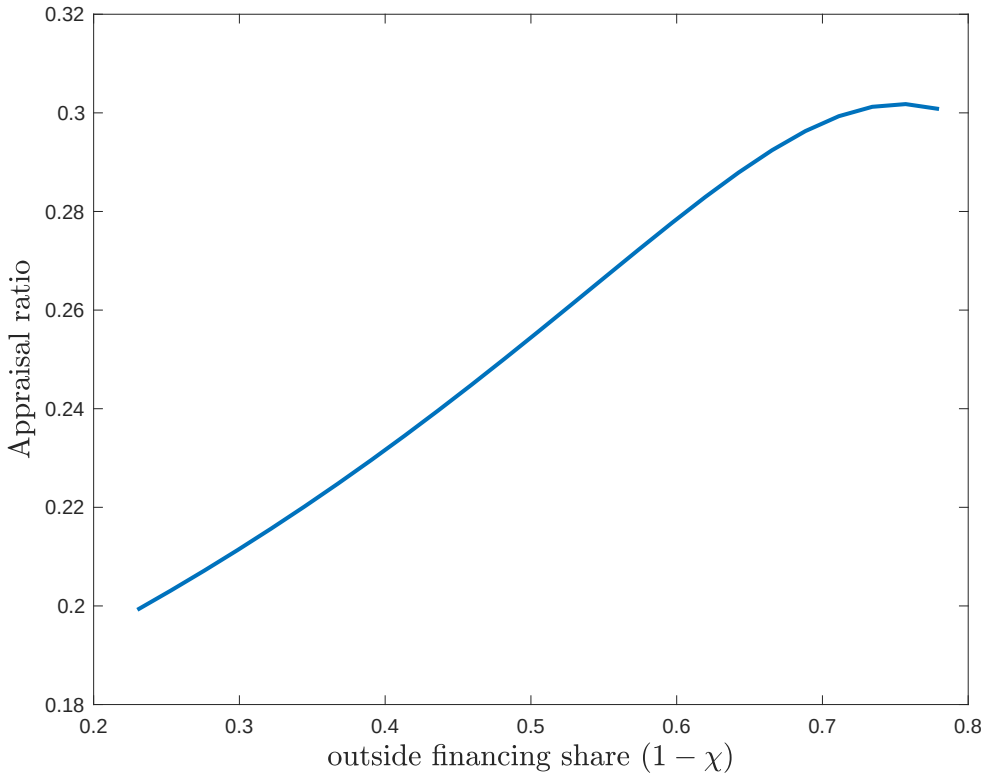

Figure: Outside Financing Fraction and Sharpe Ratio.

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