Online Summer School
Macro, Money, Finance
Problem Set 3

July 5, 2024

Please submit your solutions to the dropbox link by 7/9/2024 8:30am (EST).

1 Fire Sales

In this exercise you will solve the model from Lecture 3 numerically, under the assumption of log utility.

1. Our goal is to construct functions $q(\eta), \iota(\eta), \chi(\eta), \kappa(\eta)$ and $\sigma^q(\eta)$ on the $[0, 1]$ grid. Slide 68 provides the parameter values (ignore γ), and slide 65 provides the set of equations and the algorithm.
   (a) Solve the model at the boundaries: for $\eta = 0$ and $\eta = 1$.
   (b) Create a uniform grid for $\eta \in [0.0001, 0.9999] = \{\eta_1 = 0.0001, \eta_2, \ldots, \eta_N = 0.9999\}$.
   (c) Using the implicit method with the one-step Newton's algorithm, solve the system of equations on slide 46 (with $\chi = \alpha\kappa$) for $\eta_1, \eta_2, \ldots$ and so on.
   (d) Stop once you reach $\kappa \geq 1$. From here on, set $\kappa = 1$ and $\chi = \max\{\alpha\kappa, \eta\}$, solve for q and σ^q.
   (e) Verify your solution by plotting $q(\eta)$ and $\sigma^q(\eta)$ and comparing it with the graph on slide 50 (you won't get an exact match since on slide 50 $\gamma = 2$, but the shape will be similar). Do your functions converge to the boundary solution for $\eta = 1$ that you obtained in (a) as $\eta \to 1$?
   (f) Plot the remaining variables: $\iota(\eta), \kappa(\eta), \chi(\eta)$.
   (g) We can also look at the experts' balance sheet: derive expressions for the scaled versions of issued debt and outside equity: $\frac{D_t^e}{q_t K_t}, \frac{OE_t^e}{q_t K_t}$ and plot them against η.
2. Recall from the lecture that drift and volatility of η in the general case are given by:

$$\mu_t^\eta = (1 - \eta_t)\left[(\varsigma_t^e - \sigma - \sigma_t^q)(\sigma_t^\eta + \sigma + \sigma_t^q) - (\varsigma_t^h - \sigma - \sigma_t^q)\left(-\frac{\eta_t}{1 - \eta_t}\sigma_t^\eta + \sigma + \sigma_t^q\right) - \left(\frac{C_t^e}{N_t^e} - \frac{C_t^h}{N_t^h}\right)\right]$$

$$\sigma_t^\eta = \frac{\chi_t - \eta_t}{\eta_t}(\sigma + \sigma_t^q)$$

   (a) Which terms in the above equations can we simplify/substitute because of log utility and why? Perform these substitutions and derive the drift and volatility of η under log utility.
   (b) Verify your solution by plotting $\eta\mu^\eta(\eta)$ and $\eta\sigma^\eta(\eta)$ and comparing them with the graph on slide 50 (you can plot for $\sigma = 0.1$ only and should expect a similar shape).