Eco529: Modern Macro, Money, and International Finance

Lecture 12: One Sector Monetary Model
FTPL, Monetarism, and Sargent-Wallace

Markus Brunnermeier

Princeton University

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Course Overview

Real Macro-Finance Models with Heterogeneous Agents

1. A Simple Real Macro-finance Model
2. Endogenous (Price of) Risk Dynamics
3. A Model with Jumps due to Sudden Stops/Runs

Money Models

1. A Simple Money Model
   - FTPL, Monetarism, Sargent-Wallace
2. Cashless vs. Cash Economy and "The I Theory of Money"
3. Price Rigidities - New Keynesian Elements
4. Welfare Analysis & Optimal Policy
   - Fiscal, Monetary, and Macroprudential Policy

International Macro-Finance Models

1. International Financial Architecture

Digital Money

Overview Across Lectures 10-13

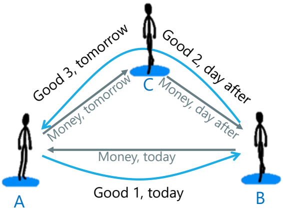
- Store of Value Monetary Model with One Sector and No Aggregate Risk
  - Safe Asset and Service Flows
  - Bubble (mining) or not
  - 2 Different Asset Pricing Perspectives/SDFs
- Store of Value Monetary Model with Time-varying Idiosyncratic Risk
  - Safe asset, Flight-to-Safety and negative CAPM-β
  - Flight-to-Safety and Equity Excess Volatility
  - Debt valuation puzzle, Debt Laffer Curve,
  - Safe Asset and Bubble Complementarity
  - Policies to Maintain Safe Asset Privilege on Gov. Bond
- Medium of Exchange Role, FTPL, Sargent-Wallace

The 3 Roles of Money

- Store of value
  - Bond is less risky than other "capital" – no idiosyncratic risk
  - Govt bond is a special safe asset
    – helps to partially overcome incomplete markets/OLG frictions
    (- helps to relax colleteral constraints)
  - Fiscal Theory of Price Level (FTPL):

$$\frac{\mathcal{B}_t+\mathcal{M}_t}{\mathcal{P}_t} = \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} (\text{primarysurpluses})_s ds + \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} \Delta i_s \frac{\mathcal{M}_s}{\mathcal{P}_s} ds + \frac{\xi_T}{\xi_t} \frac{\mathcal{B}_T+\mathcal{M}_T}{\mathcal{P}_T}$$

  - Monetary vs. fiscal dominance



- Medium of exchange
  - Overcome double-coincidence of wants problem
  - (Narrow) money is special gov. bond
    - helps to overcome double-coincidence of wants friction
    (cash-in-advance, money in utility, shopping time models)
    - lower interest rate Δi_s
  - **Monetarisms**: Quantity Equation
    $\nu_t \mathcal{M}_t = \mathcal{P}_t T_t$ (or $\mathcal{P}_t Y_t$)
- Unit of account
  - Intratemporal: Numeraire bounded rationality
  - Intertemporal: Debt contracts incomplete markets
    New Keynesian wage/price stickiness

Credit, Money, Reserves, and Government Debt

- Credit vs. Money
  - Credit — zero net supply
  - Money (Gov. bond) — positive net supply

    - Perfect credit renders money useless
- Gov. Debt vs. Money in form of Cash and Reserves
  - Gov. debt: convenience yield as it relaxes collateral constraint
  - Money $\mathcal{M}_t$ has lower interest rate Δi if it offers medium of exchange role in addition
    - Reserves: Interest bearing

      - Special form of government debt:
        - Infinite maturity — more like equity (no rollover risk)
        - Zero duration — more like overnight debt
        - Banking system can't offload it – **Financial Repression**

      - Is QE simply swapping one form of gov. debt for another one, reserves?
    - Cash: extra convenience yield and zero interest $\Rightarrow$ lower return by Δi
    - Fintech revolution erodes extra convenience yield

Price Stickiness and Phillips Curve

- Flexible prices: Prices adjust immediately
- Sticky prices:
  - Since prices adjust sluggishly, output has to adjust
    - Inflation pressure: prices too low during transition period,
      output (demand) overshoots natural (= flexible price) level
    - Deflation pressure: prices too high during transition period,
      output (demand) undershoots natural level
  - Sticky price models smooth out adjustment dynamics relative to equivalent flex price models

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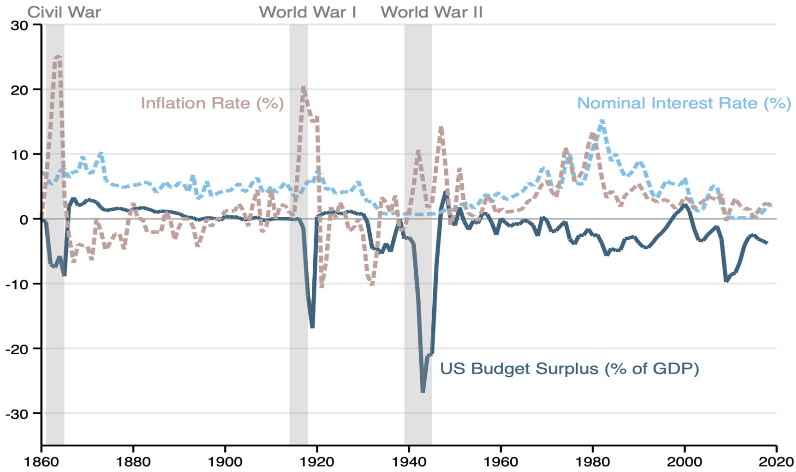
  - Temporary Anti-Fisherian: "Stepping on the Rake"
- Medium of Exchange Role of Money
  - Quantity Equation
  - Generalizing FTPL Equation (2 ways)
  - Friedman Rule
  - QE
  - Fiscal-Monetary Interaction
- Sargent-Wallace
- Price/Wage stickiness (later)

Inflation – Fiscal Link for the US

- Sims (1994): "In a fiat-money economy, inflation is a fiscal phenomenon, even more fundamentally than it is a monetary phenomenon".



Source: FRED, MeasuringWorth.com, Mitchell (1908)

Two Inflation-Fiscal Connection

- **FTPL Channel**
  Issue additional bonds to finance new economic stimulus
  + don't change future primary surpluses $s_t K_t$
  $\Rightarrow$ dilutes value of existing bonds (as # of bonds is higher)
  $\Rightarrow$ Inflation
- **Short-run Aggregate Demand Channel**
  Issue additional bonds to finance new economic stimulus
  + Commit to increase $s_t K_t$, so that bond value is not diluted
  ($\Rightarrow$ FTPL Channel is switched off)
  (extra bonds are financed by extra future $s_t K_t$)
  If economic model is:
  - Ricardian $\Rightarrow$ stimulus is neutralized by future taxes
  - Non-Ricardian $\Rightarrow$ stimulus can boost demand/output
    (if there is a negative output gap e.g. in NK models)

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Fiscal Theory of the Price Level (FTPL)

- Price level determination for a given equilibrium
  - What determines it (1/value of money)?
  - How do policy choices affect the price level/inflation
- FTPL points out the systematic link btw fiscal policy and nominal good prices
  - For a government that issues nominal debt denominated in its own currency
  - .. And is committed to not default on nominal liabilities (can be relaxed)
  - If fiscal policy is conducted in a certain way, can render the price level determinate
  - But even more generally: FTPL relationship always present in macro models
  - There are important fiscal requirements for "monetary" policy goals such a price stability
- In addition: Recall *equilibrium selection from previous lecture*
  - Bubble vs. no bubble equilibrium
  - On which asset is the bubble?

Recall Baseline Model: BruSan (AER PP 2016)

- Each heterogenous citizen $\tilde{i} \in [0,1]$:

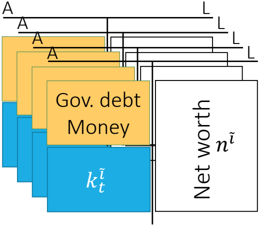
$$\mathbb{E}_t\left[\int_t^\infty e^{-\rho s}\left(\log c_s^{\tilde{i}} + f(g_s K_s)\right)\mathrm{d}s\right], \text{ where } K_s := \int k_s^{\tilde{i}}\mathrm{d}\tilde{i}$$

$$s.t. \frac{\mathrm{d}n_t^{\tilde{i}}}{n_t^{\tilde{i}}} = -\frac{c_t^{\tilde{i}}}{n_t^{\tilde{i}}}\mathrm{d}t + \mathrm{d}r_t^{\mathcal{B}} + (1-\theta_t^{\tilde{i}})(\mathrm{d}r_t^{K,\tilde{i}}(\iota_t^{\tilde{i}}) - \mathrm{d}r_t^{\mathcal{B}}) \text{ \& No Ponzi}$$

- Each citizen operates physical capital $k_t^{\tilde{i}}$
  - Output (net investment): $y_t^{\tilde{i}}\mathrm{d}t = (ak_t^{\tilde{i}} - \iota_t^{\tilde{i}}k_t^{\tilde{i}})\mathrm{d}t$
  - $\frac{\mathrm{d}k_t^{\tilde{i}}}{k_t^{\tilde{i}}} = \left(\Phi(\iota_t^{\tilde{i}}) - \delta\right)\mathrm{d}t + \tilde{\sigma}\mathrm{d}\tilde{Z}_t^{\tilde{i}} + \mathrm{d}\Delta_t^{k,\tilde{i}}$,

    ($\mathrm{d}\tilde{Z}_t^{\tilde{i}}$ idiosyncratic Brownian)
  - Output tax $\tau a k_t^{\tilde{i}}\mathrm{d}t$
- No aggregate risk $\mathrm{d}Z_t$
- Incomplete Markets Friction: no $\mathrm{d}\tilde{Z}_t^{\tilde{i}}$-claims
- Government budget constraint (fiscal/monetary)

$$\underbrace{(\mu_t^{\mathcal{B}} - i_t)}_{\check{\mu}_t^{\mathcal{B}}:=}\mathcal{B}_t + \mathcal{P}_t K_t\underbrace{(\tau a - g)}_{s:=} = 0$$



Does the fiscal authority pick s_t or $\mu_t^{\mathcal{B}}$?
- pick s_t: there are two corresponding $\check{\mu}_t^{\mathcal{B}}$. one on each side of the Laffer curve
- pick $\check{\mu}_t^{\mathcal{B}}$: doesn't have this problem

Recall Baseline Model: BruSan (AER PP 2016)

Non-Monetary	Monetary
$q_t^{\mathcal{B}} = 0$	$\frac{\mathcal{B}_0}{\mathcal{P}_0}/K_t = q^{\mathcal{B}} = \frac{\tilde{\sigma} - \sqrt{\rho + (\mu^{\mathcal{B}} - i)}[1+\phi(a-\iota\!\!\!g)]}{\sqrt{\rho+(\mu^{\mathcal{B}}-i)}+\phi\tilde{\sigma}\rho}$
$q_t^K = \frac{1+\phi(a-\iota\!\!\!g)}{1+\phi\rho}$	$q^K = \frac{\sqrt{\rho+(\mu^{\mathcal{B}}-i)}[1+\phi(a-\iota\!\!\!g)]}{\sqrt{\rho+(\mu^{\mathcal{B}}-i)}+\phi\tilde{\sigma}\rho}$
$\iota = \frac{(a-\iota\!\!\!g)-\rho}{1+\phi\rho}$	$\iota = \frac{(a-\iota\!\!\!g)\sqrt{\rho+(\mu^{\mathcal{B}}-i)}-\tilde{\sigma}\rho}{\sqrt{\rho+(\mu^{\mathcal{B}}-i)}+\phi\tilde{\sigma}\rho}$

- $g = \Phi(\iota) - \delta = \frac{1}{\phi}\log(\iota\phi + 1) - \delta = \frac{1}{\phi}\log\left(\frac{\phi(a-\iota\!\!\!g)+1}{\phi\tilde{\sigma}\rho/\sqrt{\rho+(\mu^{\mathcal{B}}-i)}+1}\right) - \delta$
- $r^f = \underbrace{\left(\Phi(\iota(\mu^{\mathcal{B}} - i)) - \delta\right)}_{=g} - (\mu^{\mathcal{B}} - i)$ ("tug-of-war" btw. $\mu^{\mathcal{B}}$ & i)
- $\pi = i - r^f = i - [g - (\mu^{\mathcal{B}} - i)] = \mu^{\mathcal{B}} - g$
- $\tilde{\varsigma} = (1-\vartheta)\tilde{\sigma} = \frac{\sqrt{\rho+(\mu^{\mathcal{B}}-i)}}{\tilde{\sigma}}\tilde{\sigma} = \sqrt{\rho + (\mu^{\mathcal{B}} - i)}$
- $\xi_t^{**} = e^{-\rho t}\frac{N_0}{N_t}$, $\frac{d\xi_t^{**}}{\xi_t^{**}} = -(\rho + g)dt$ (representative agent has no $d\tilde{Z}$-term)

Price Level Determination (via Wealth Effect)

- ξ-**FTPL equation** for $r^f > g$:
  $\frac{\mathcal{B}_0}{\mathcal{P}_0} = \int_0^\infty e^{-r^f t} s e^{gt} K_0 \mathrm{d}t = \int_0^\infty e^{(\mu^{\mathcal{B}} - i)t} s K_0 \mathrm{d}t = \frac{sK_0}{\mu^{\mathcal{B}} - i}$
- ξ^{**}-**FTPL equation**: (cash flow + service flow-term)
  $\frac{\mathcal{B}_0}{\mathcal{P}_0} = \int_0^\infty e^{-(\rho+g)t} s e^{gt} K_0 \mathrm{d}t + \int_0^\infty e^{-(\rho+g)t} (1-\vartheta)^2 \tilde{\sigma} \frac{\mathcal{B}_0}{\mathcal{P}_0} e^{gt} \mathrm{d}t$
  $= \frac{sK_0}{\rho} + \frac{\rho + \mu^{\mathcal{B}} - i}{\rho} \frac{\mathcal{B}_0}{\mathcal{P}_0}$
- Portfolio choice determines ϑ_t and with it the price level, $\mathcal{P}_t$ when there are nominal assets
- Recall goods market clearing condition

$$C_t = \rho \left(q_t^K K_t + \frac{\mathcal{B}_t}{\mathcal{P}_t} \right) = (a - \iota_t - g\!\!\!/) K_t$$

- For a given state $\mathcal{B}_0$, price level $\mathcal{P}_0$ is uniquely determined as long as fiscal policy is "active" (has its own goals)
  - $\mathcal{P}_t$ too high $\rightarrow$ total bond wealth $\mathcal{B}_t/\mathcal{P}_t$ too low $\rightarrow$ insufficient goods demand $\rightarrow$ $\mathcal{P}_t$ falls
  - $\mathcal{P}_t$ too low $\rightarrow$ total bond wealth $\mathcal{B}_t/\mathcal{P}_t$ too high $\rightarrow$ excess goods demand $\rightarrow$ $\mathcal{P}_t$ falls
  - Except if fiscal policy $s_{>t}$ is "passive" and reacts sufficiently strongly, i.e., ϑ_t reacts to $\mathcal{P}_t$

Price Level Determination: Active/Passive Fiscal Policy

- "Passive" fiscal policy $s_{>t}$ that does not pursue its own goal and hence ϑ_t, reacts sufficiently strong to $\mathcal{P}_t$ to support other equilibria [Leeper terminology]
  - If price level rises by x%, then real debt declines by x%, which fiscal reaction justifies by lowering primary surpluses by x%
  - Example: fiscal policy $s_t = \alpha_s \vartheta_t$, then
    $\vartheta_t = \int_t^\infty \rho e^{-\rho(\tau - t)} s_\tau d\tau = \int_t^\infty \rho e^{-\rho(\tau - t)} \alpha_s \vartheta_t d\tau$
    Has many solutions since $\vartheta_t = \vartheta_0 e^{(\rho - \alpha)t}$ for any ϑ_0
    (they also satisfy the transversality condition $e^{-\rho t} \vartheta_t \to 0$)
    Hence, for this fiscal policy any initial portfolio weight ϑ_0 and price level $\mathcal{P}_0$ are consistentwith "some" equilibrium
- "Active" fiscal policy $\Rightarrow$ uniqueness
  Fiscal authority pursues its own goal and does not react strongly to different $\mathcal{P}_t$
- Out-off-equilibrium fiscal policies to rule out possible non- or bubble-decaying equilibria
  - Out-off equilibrium fiscal support to secure minimum of $\underline{\vartheta}$ a la Obstfeld-Rogoff (see Lecture 10)

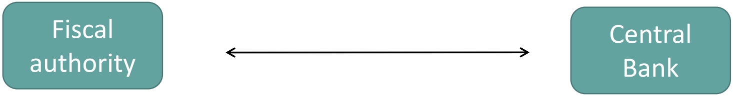
Remark: Price Level Determination

- An “active” fiscal policy is only feasible for the government if
  - Government’s nominal debt represents liability to something it can create out of FIAT
    i.e. it does not need to expend real resources to honor this liability
  - All other agents must expend real resources to service their nominal debt
  - Remark: ... but it is not required that
    - Taxes are payable in money
    - Government is a large player
- Government debt represents net worth for private sector.

Effectiveness of Monetary Policy to Impact Price Level

- Monetary Policy can be maximally effective ("Monetary Dominance") if fiscal policy generates indeterminancy (multiple possible price levels) (i.e. FTPL is switched off, e.g. via passive fiscal policy rule)
  - In representative agent setting:
    Passive fiscal policy rule (real surplus react sufficiently to real value of debt) [Leeper terminology]
    is Ricardian, i.e. it has no real impact [Woodford terminology]
- Monetary Policy has power since it can select an equilibrium e.g. via the Taylor Rule
  - $i_t = \phi_0(\tilde{\sigma}) + \phi_\pi(\pi_t - \pi^*(\tilde{\sigma}))$ (no output gap reaction with flexible prices)
  - One reasonable equilibrium
  - All others are explosive and seem implausible
    - Due to Taylor Principle: $\phi_\pi > 1$
- Remark: Monetary Dominance, i.e. passive fiscal policy + MoPo-Taylor rule, is implicitly assumed in most NK-DSGE models.

Monetary vs. Fiscal Dominance



- **Monetary dominance**
  - Monetary tightening leads fiscal authority to reduce fiscal deficit
- **Fiscal dominance**
  - Interest rate increase does not reduce primary fiscal deficit
  - . . . only lead to higher inflation

Game of chicken

See YouTube video 4, minute 4:15

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Inflation – Fiscal Link for the US

- Fisher equation: $i_t = r_t^f + \pi_t$
  - Erdogan's experiment with Turkey (until 2023)

- Unexpected permanent increase in i_t at $t = 0$

  1. **Option "Pure MoPo"**: keep $\check{\mu}_t^{\mathcal{B}}$ constant, i.e., $\mu_t^{\mathcal{B}}$ increases
  $\Rightarrow$ increases inflation (one-for-one)
     - "Neo-Fisherian" – "super-neutrality of money (growth)"
  2. **Option "Reacting Fiscal Pol"**: keep $\mu_t^{\mathcal{B}}$ constant, i.e. $\check{\mu}_t^{\mathcal{B}}$ decreases
  $\Rightarrow r^f = \underbrace{(\Phi(\iota(\check{\mu}^{\mathcal{B}}) - \delta)}_{=g} - \check{\mu}^{\mathcal{B}}$ due to the growth effect inflation decreases (slightly)

Introducing Long-term Government Bonds

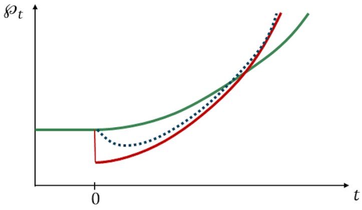
- Long-term bond
  - yields fixed coupon interest rate on face value $F^{(i,m)}$
  - Matures at random time with arrival rate $1/m$
  - Nominal price of the bond $P_t^{\mathcal{B}(i,m)}$
  - Nominal value of all bonds outstanding of a certain maturity:

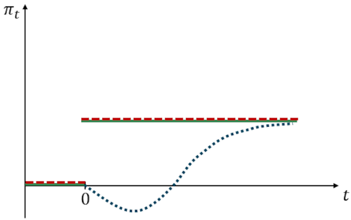
$$\mathcal{B}_t^{(m)} = P_t^{\mathcal{B}(i,m)} F^{(i,m)}$$

  - Nominal value of all bonds $\mathcal{B}_t = \sum_m \mathcal{B}_t^{(m)}$
- Special bonds
  - Reserves: $\mathcal{B}_t^{(0)}$ and note $P_t^{\mathcal{B}(0)} = 1$ (long-term but floating interest rate)
  - Consol bond: $\mathcal{B}_t^{(\infty)}$

Sims' Stepping on the Rake: "Bond Reevaluation Effect"

- Unexpected permanent increase in $i_t^{(0)}$ at $t = 0$ for all $t > 0$
  $\Rightarrow$ nominal value $\mathcal{B}_t^{(m>0)}$ of any long-term bond declines
  1. Option "Pure MoPo": keep s_t constant, i.e., "debt growth" increases, ϑ_t is constant and so is $q^{\mathcal{B}}$ (aside $s_t/q_t^{\mathcal{B}}$ also stays constant)
  - At $t = 0$ on impact: as all $\mathcal{B}_0^{(m>0)}$ decline $\Rightarrow \mathcal{P}_0$ has to jump down
  - For $t > 0$: inflation π_t is higher like in Neo-Fisherian setting
    (with price stickiness like dotted curve)





Sims' Stepping on the Rake: "Bond Reevaluation Effect"

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  $\Rightarrow$ nominal value $\mathcal{B}_t^{(m>0)}$ of any long-term bond declines
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  - At $t = 0$ on impact: as all $\mathcal{B}_0^{(m>0)}$ decline $\Rightarrow \mathcal{P}_0$ has to jump down
  - For $t > 0$: inflation π_t is higher like in Neo-Fisherian setting (with price stickiness like yellow curve)

  2. Option "Reacting Fiscal Pol": keep $\mu_t^{\mathcal{B}}$ (growth rate of nominal bond value) constant $\Rightarrow$ raise $s_t \Rightarrow \vartheta_t$ and $q_t^{\mathcal{B}}$ go up.
  - At $t = 0$ on impact: as all $\mathcal{B}_t^{(m>0)}$ decline $\Rightarrow \mathcal{P}_0$ has to jump down by more than option **1**
  - For $t > 0$: inflation π_t is higher like in Neo-Fisherian setting
- In sum, "Stepping on the Rake" only changes inflation (price drop) at $t = 0$.
  ... only with price stickiness (price drop down is smoothed out).

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ADD "Medium of Exchange" to Store of Value

- Store of Value Role (only)
  - Bond (T-Bill) = Money
  - **FTPL equation** determines price level
- *Add* Medium of Exchange Role
  - Cash-in-advance constraint, transaction cost, shopping time model,
  - $\Rightarrow r^{\mathcal{M}} < r^{\mathcal{B}}$ ("money convenience yield")
  - **Quantity equation** $\mathcal{M}_t \nu \geq \mathcal{P}_t Y_t$ determines price level (if it binds)
  - Add money as an additional asset to the model
  - Monetarists assume that velocity ν is constant (sluggish)
- Milton Friedman (1961): "inflation is always and everywhere a monetary phenomenon"
- Sims (1994): "In a fiat-money economy, inflation is a fiscal phenomenon, even more fundamentally than it is a monetary phenomenon".

Medium of Exchange: Additional Model Elements

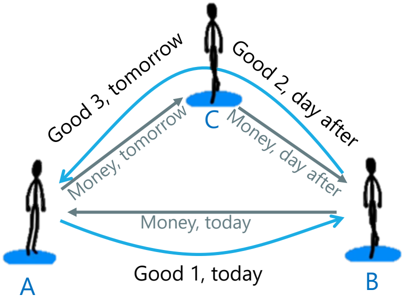
- Bond and Money
  - Money is medium of exchange as well as store of value (but worse store than bond)
  - Nominal quantity $\mathcal{M}_t$ (cash, CBDC, reserves)
  - Initial stock $\mathcal{M}_0 > 0$
  - Evolution: $d\mathcal{M}_t = \mu_t^{\mathcal{M}} dt$ controlled by monetary authority
  - Does not pay interest (or lower interest on reserves)
  - Real value (real money balances) $\frac{\mathcal{M}_t}{\mathcal{P}_t} =: q_t^{\mathcal{M}} K_t$
- Share notations: $\vartheta_t = \frac{q_t^{\mathcal{B}} + q_t^{\mathcal{M}}}{q_t^K + q_t^{\mathcal{B}} + q_t^{\mathcal{M}}}$ fraction of nominal to total wealth
  - $\vartheta_t^{\mathcal{M}} = \frac{q_t^{\mathcal{M}}}{q_t^{\mathcal{B}} + q_t^{\mathcal{M}}}$, i.e., $\vartheta_t \vartheta_t^{\mathcal{M}}$ = money as a fraction of total net worth
  - $\vartheta_t^{\mathcal{B}} = \frac{q_t^{\mathcal{B}}}{q_t^{\mathcal{B}} + q_t^{\mathcal{M}}}$, i.e., $\vartheta_t \vartheta_t^{\mathcal{B}}$ = fraction of total net worth
- Monetary authority transfers seigniorage to fiscal authority
- Gov. Budget constraint: (fiscal vs. monetary)

$$(\mu_t^{\mathcal{B}} - i_t)\mathcal{B}_t = \mathcal{P}_t(s_t + \mu_t^{\mathcal{M}} q_t^{\mathcal{M}})K_t$$

where s_t is primary surplus and $\mu_t^{\mathcal{M}} q_t^{\mathcal{M}}$ seigniorage per unit of K_t

Medium of Exchange – Transaction Role

- Overcome double-coincidence of wants



- Quantity equation: $\mathcal{P}_t T_t = \nu \mathcal{M}_t$
  - ν is velocity (Monetarism: ν exogeneous, constant)
  - T transactions $\quad C + \iota K = Y$
    - Consumption $\quad C$
    - New investment production $\quad \iota K \quad$ produce own machines
    - Transaction of physical capital $\quad d\Delta^k \quad$ infinite velocity
    - Transaction of financial claims $\quad d\boldsymbol{\theta}^{j \notin \mathcal{M}} \quad$ infinite velocity

Models of Medium of Exchange

- Reduced form models
  - Cash in advance: $T_t = \nu \frac{\mathcal{M}_t}{\mathcal{P}_t}$ Only assets $j \in \mathcal{M}$ with money-like features

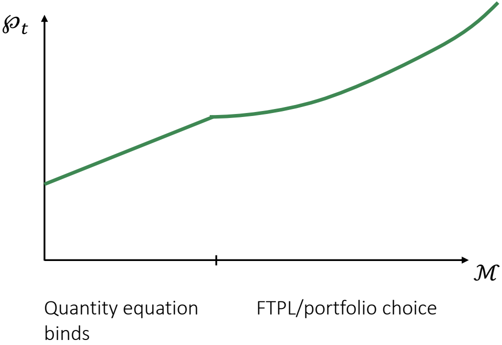
$$\boxed{c_t^i \leq \sum_{j \in \mathcal{M}} \nu^j \theta_t^{j,i} n_t^i \text{ with velocity } \nu > \rho}$$

  - Shopping time models $c = (c^c, l)$
  - Money in the utility function consume money CES
    $u(c, \mathcal{M}/\mathcal{P}) = u(c, \theta^{j \in \mathcal{M}} n)$ DiTella extension of BruSan2016
    - New Keynesian Models
    - No satiation point
  - New Monetary Economics

For generic setting encompassing all models: see Brunnermeier-Niepelt 2018

Medium of Exchange: Additional Model Elements

- 2 regimes depending on parameters



- CIA binds
  - Yes $\Rightarrow$ Quantity Equation $\mathcal{P}_t T_t = v\mathcal{M}_t$ determines $\mathcal{P}_t$
  - No & $\mu_t^{\mathcal{M}} = \mu_t^{\mathcal{B}} - i_t \Rightarrow$ price level is determined as in "nominal gov. bond model"

Stochastic Maximum Principle

- Notation: $\boldsymbol{\theta}_t = \int \theta_t^{(m)} \mathrm{d}m, \boldsymbol{\mathcal{B}} = \int \mathcal{B}^{(m)} \mathrm{d}m$, (Note: $\mathcal{M} \neq \mathcal{B}^{(0)}$)
- Agent's problem:

$$\max_{\boldsymbol{\theta}_t, c} \left[\int_0^\infty e^{-\rho t} u(c_t) \mathrm{d}t \right], s.t. \frac{\mathrm{d}n_t}{n_t} = -\frac{c_t}{n_t}\mathrm{d}t + \mathrm{d}r_t^{n^*} + (\boldsymbol{\theta}_t - \boldsymbol{\theta}_t^*)\mathrm{d}r_t^{\boldsymbol{\mathcal{B}}}, \text{and } c_t \leq \nu \vartheta_t^{\mathcal{M}} n_t$$

- Hamiltonian (in consumption numeraire):

$$H_t = e^{-\rho t} u(c_t) + \xi_t \mu_t^n n_t - \varsigma_t \xi_t \sigma_t^n n_t - \tilde{\varsigma}_t \xi_t \sigma_t^n n_t + \lambda_t^{\mathcal{M}} \xi_t n_t \left(\nu \theta_t^{\mathcal{M}} - \frac{c_t}{n_t} \right)$$

- First order conditions:

$$\begin{cases} e^{-\rho t} u'(c_t) = \xi_t (1 + \lambda_t^{\mathcal{M}}) \\ r_t^{n^*} - r_t^{\mathcal{B}^{(m)}} = \varsigma_t \left(r_t^{n^*} - r_t^{\mathcal{B}^{(m)}} \right), & \text{for bonds} \\ r_t^{n^*} - r_t^{\mathcal{M}} = \varsigma_t \left(r_t^{n^*} - r_t^{\mathcal{M}} \right) + \nu \lambda_t^{\mathcal{M}}, & \text{for money} \end{cases}$$

Understanding rs

$$\begin{aligned}
r^{f**} &= \rho + \gamma\mu_t^C - \overbrace{\frac{1}{2}\gamma(\gamma+1)[\overset{\text{agg risk}}{(\sigma_t^c)^2} + }^{\text{precautionary saving/self-insurance}} && \text{(rep. agent risk-free rate)} \\
r^f &= \qquad\qquad\qquad\qquad + \overset{\text{idio risk}}{\color{red}{(\tilde{\sigma}_t^c)^2}}] && \text{(risk-free rate)} \\
r_t^{\mathcal{M}} &= \qquad\qquad\qquad\qquad\qquad - \underbrace{\lambda_t^{\mathcal{M}}\nu}_{\Delta i_t} && \text{(return on money)}
\end{aligned}$$

Derive FTPL Equation in Setting with (Narrow) Money

- Two ways to write FTPL equation

$$\frac{\mathcal{B}_t + \mathcal{M}_t}{\mathcal{P}_t} = \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} s_s K_s \mathrm{d}s + \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} \Delta i_s \frac{\mathcal{M}_s}{\mathcal{P}_s} \mathrm{d}s + \mathbb{E}_t \frac{\xi_T}{\xi_t} \frac{\mathcal{B}_T + \mathcal{M}_T}{\mathcal{P}_T}$$

$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} s_s K_s \mathrm{d}s + \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} \mu_s^{\mathcal{M}} \frac{\mathcal{M}_s}{\mathcal{P}_s} \mathrm{d}s + \mathbb{E}_t \frac{\xi_T}{\xi_t} \frac{\mathcal{B}_T}{\mathcal{P}_T}$$

- Take difference:

$$\frac{\mathcal{M}_t}{\mathcal{P}_t} = \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} (\Delta i_s - \mu_s^{\mathcal{M}}) \frac{\mathcal{M}_s}{\mathcal{P}_s} \mathrm{d}s + \mathbb{E}_t \frac{\xi_T}{\xi_t} \frac{\mathcal{M}_T}{\mathcal{P}_T}$$

(may contain bubble term when take $T \to \infty$)

Friedman Rule & The "Optimal" Inflation Rate

- Money better medium of exchange, i.e. transaction role services.
- ... but worse as store of value, if $i_t > 0$ since money pays no/less interest $i^{\mathcal{M}} = 0$
- Distortionary, as agents economize on money holding,
  while money is socially costless to produce.
- Friedman Rule:
  Adjust the inflation rate s.t. $r_t^{\mathcal{M}} = r_t^{\mathcal{B}}$, i.e., $\pi_t^* = -r_t^{\mathcal{B}}$ $\forall t$ (which depends on $\mu_t^{\mathcal{B}}$)
- Remarks:
  - Lucas (1987): "one of the few legitimate 'free lunches' economics has discovered in 200 years of trying."
  - Friedman Rule is not optimal in our setting, as there is an optimal degree $\mu^{\mathcal{B}}$ of "bubble mining" that also determines optimal inflation (see welfare lecture).
    - inflation tax lowers real return on gov. bond and boost investment/growth rate (Tobin effect).
    - Inflation tax lowers idiosyncratic risk-sharing, which lowers citizens' utility.

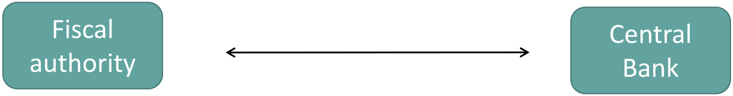
Quantitative Easing (QE)

- Assume $\mu_t^{\mathcal{M}} = \mu_t^{\mathcal{B}}$ for all t
- At $t = 0$ QE in form of an unexpected swap of $\mathcal{B}^{(0)}$-bonds (T-Bill) for money $\mathcal{M}$
- **QE Proposition:** T-Bill QE leads to positive price level jump.
  Suppose $\mathcal{P}_t$ reacts less, so that real balances $\frac{\mathcal{M}_t}{\mathcal{P}_t}$ expand
  $\Rightarrow$ Relaxes CIA constraint and
  $\Rightarrow$ permanently lowers Δi (if CIA was binding beforehand)
  $\Rightarrow$ lowers "money seigniorage"
  $\Rightarrow$ upward jump in the price level (inflation) by

$$\frac{\mathcal{B}_t + \mathcal{M}_t}{\mathcal{P}_t} = \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} s_s K_s \mathrm{d}s + \mathbb{E}_t \int_t^T \frac{\xi_s}{\xi_t} \Delta i_s \frac{\mathcal{M}_s}{\mathcal{P}_s} \mathrm{d}s + \mathbb{E}_t \frac{\xi_T}{\xi_t} \frac{\mathcal{B}_T + \mathcal{M}_T}{\mathcal{P}_T}$$

The quantity equation (with fixed velocity) $\frac{\mathcal{M}_t}{\mathcal{P}_t} = \frac{C_t}{\nu}$ would also lead to upward jump of the price level.

Fiscal and Monetary Interaction



- **Monetary dominance**
  - Monetary tightening leads fiscal authority to reduce fiscal deficit
- **Fiscal dominance**
  - Interest rate increase does not reduce primary fiscal deficit
  - . . . only lead to higher inflation

Game of chicken

See YouTube video 4, minute 4:15

Fiscal and Monetary Interaction

- Monetary authority sets $i_t, \mu_t^{\mathcal{M}}$
- Fiscal authority sets $\mu_t^{\mathcal{B}}$ ... if it undos interest rate, simply assume it sets $\check{\mu}_t^{\mathcal{B}}$
- $\mu_t^{\mathcal{M}} = \mu_t^{\mathcal{B}}$ money to bond ratio stays the same $\Rightarrow$ steady state analysis
  - i CIA binds
  - ii CIA doesn't bind
- $\mu_t^{\mathcal{M}} \neq \mu_t^{\mathcal{B}}$ not a steady state (except if the CIA constraint is slack throughout) as $\mathcal{M}_t/\mathcal{B}_t$ ratio evolves over time
  - If $\mu_t^{\mathcal{M}} > \mu_t^{\mathcal{B}}$, then convergence over time to steady-state with only money.
    The real allocation might converge there in finite time if the CIA constraint is non-binding in this long-run outcome (i.e. if idiosyncratic risk is large relative to monetary friction.)
  - If $\mu_t^{\mathcal{M}} < \mu_t^{\mathcal{B}}$ for all t (Outcome depends on CIA/money in utility specification):
    With CIA constraint on consumption, in the long run ϑ_t must converge to 1 ($\mathcal{P}_t \to 0$). If CIA holds in the extreme case: possible solution is demonetization & starvation (consumption & output converges to zero), bonds would become only store of value.
    Modification 1: Allow for (less efficient) barter trades without money, then eventually inflation is determined by the fiscal side.
    Modification 2: velocity can increase at a cost
    Modification 3: Money in Utility function (it depends whether $u(\frac{m}{P} = 0) = -\infty$ or not ... and marginal utility

Fiscal and Monetary Interaction

- Monetary authority sets $i_t, \mu_t^{\mathcal{M}}$
- Fiscal authority sets $\mu_t^{\mathcal{B}}$ ... if it undos interest rate, simply assume it sets $\breve{\mu}_t^{\mathcal{B}}$

- Prelude to Sargent and Wallace
  - Central bank can temporarily set $\mu_t^{\mathcal{M}} < \mu_t^{\mathcal{B}}$.
    Inflation will be low temporarily because the CIA determines the price level (quantity equation),
    but eventually the fiscal side takes over and raises $\mu_t^{\mathcal{M}}$ (fiscal dominancy in SW).
    Can the monetary authority contain inflation, e.g. by setting $\mu_t^{\mathcal{M}} < 0$, if fiscal authority sets a high $\breve{\mu}_t^{\mathcal{B}}$?
  - Since central bank has no taxing power, the monetary authority can only set $\mu_t^{\mathcal{M}} < 0$ until central balance sheet is used up.

Overview

- FTPL Money Delusion vs. Short-run AD Effects
- Price Level Determination
- Neo-Fisherian vs. Stepping on the Rake
  - Government Bonds with Different Maturity
  - Temporary Anti-Fisherian: "Stepping on the Rake"
- Medium of Exchange Role of Money
  - Quantity Equation
  - Generalizing FTPL Equation (2 ways)
  - Friedman Rule
  - QE
  - Fiscal-Monetary Interaction
- Sargent-Wallace
- Price/Wage Stickiness (later)

Relationship btw FTPL and Sargent and Wallace (1981)

- Sargent and Wallace (SW) point out that "*even in an economy that satisfies monetarist assumptions [...] monetary policy cannot permanent control [...] inflation*"
  - They consider an economy in which $\mathcal{P}_t$ is fully determined by money demand $\nu \mathcal{M}_t = \mathcal{P}_t Y_t$
  - but the fiscal authority is "dominant": sets *deficits* independently of monetary policy actions
- SW emphasize seigniorage from money creation
  - fiscal needs determine the total present value of *seigniorage*
  - if monetary authority provides less now, it will be forced to provide more later
- Similarity with FTPL: SW also emphasize importance of fiscal policy for inflation
- Differences to FTPL
  - Seigniorage plays important role in SW but irrelevant for FTPL
  - FTPL about tax backing (primary surplues), SW about funding deficits (negative surpluses)
  - SW about consistency of policy choices along an equilibrium path (no off-equilibrium actions)
  - price level determination in SW based on money demand, doesn't work with *i*-policy.

Recall: Model Extension with Money

- Add money as a third asset
  - nominal quantity $\mathcal{M}_t$, evolution $d\mathcal{M}_t = \mu_t^{\mathcal{M}} \mathcal{M}_t dt$
  - initial stock $\mathcal{M}_0 > 0$ given, $\mu_t^{\mathcal{M}} \geq 0$ controlled by monetary authority
  - does not pay interest
  - real value $q_t^{\mathcal{M}} := \mathcal{M}_t / \mathcal{P}_t$
- Households face a payment constraint in production $v m_t^i \geq \mathcal{P}_t y_t^i (v > \rho)$

  (as in Merkel (2020) – isomorphic to consumption cash-in-advance constraint but formally simpler)
  - if binding, $\mathcal{P} = v\mathcal{M}$ in the aggregate $\Rightarrow$ tight link between money & price level
- Monetary authority transfers seigniorage $\mathscr{s}_t := \mu_t^{\mathcal{M}} q_t^{\mathcal{M}}$ (per K_t) to fiscal authority
- Budget constraint of fiscal authority:

$$(i_t - \mu_t^{\mathcal{B}})\mathcal{B}_t = \mathcal{P}_t(s_t + \mathscr{s}_t)K_t \Rightarrow \mu_t^{\mathcal{B}} = i_t - \frac{s_t + \mathscr{s}_t}{q_t^{\mathcal{B}}}$$

New element is seigniorage income $\mathscr{s}_t$ (per K_t)

Model Solution for Binding Payment Constraint

- Let's assume that in equilibrium
  1. the payment constraint is always binding
  2. surpluses satisfy $s_t = \underline{s}, \underline{s} \leq 0$ (constant deficit-GDP ratio)
  3. $\nu > \rho$ (given log-utility)
- Then nominal wealth shares must satisfy:

$$\vartheta_t \vartheta_t^M := \frac{q_t^M}{q_t^M + q_t^{\mathcal{B}} + q_t^K} = \rho/\nu \quad \text{(from goods market clearing condition)}$$

$$\vartheta_t \vartheta_t^{\mathcal{B}} := \frac{q_t^{\mathcal{B}}}{q_t^M + q_t^{\mathcal{B}} + q_t^K}$$

$$= \int_t^\infty \rho e^{-\rho(t'-t)}(s_{t'} + \mathfrak{s}_{t'})\mathrm{d}t' = \underbrace{\underline{s}}_{<0} + \int_t^\infty \rho e^{-\rho(t'-t)}\mathfrak{s}_{t'}\mathrm{d}t'$$

A Fiscally Dominant Regime after T

- Suppose after time $T < \infty$ the fiscal authority can take control of $\mu_t^{\mathcal{M}}$.
- Fiscal authority chooses seigniorage to keep debt-GPD ratio constant, i.e.

$$\breve{s}_t = \hat{\breve{s}}(\vartheta_T^{\mathcal{B}}) := -\underline{s} + \vartheta_T \vartheta_T^{\mathcal{B}}, \quad t \geq T$$

(there are limites on feasible seigniorage but let's ignore this for simplicity)

- For $t \leq T$, the monetary authority chooses (constant) $\mu^{\mathcal{M}}$ independently
  - then also $\breve{s}_t = \mu^{\mathcal{M}} q_t^{\mathcal{M}} = \mu^{\mathcal{M}}(a - \breve{g})/\nu =: \breve{s}$ is controlled by the monetary authority
- **"Unpleasant Arithmetic" Proposition**:
Tight money now means higher inflation eventually.
  - specifically: the (constant) inflation rate over $[T, \infty)$ is strictly decreasing in $\mu^{\mathcal{M}}$ over $[0, T]$

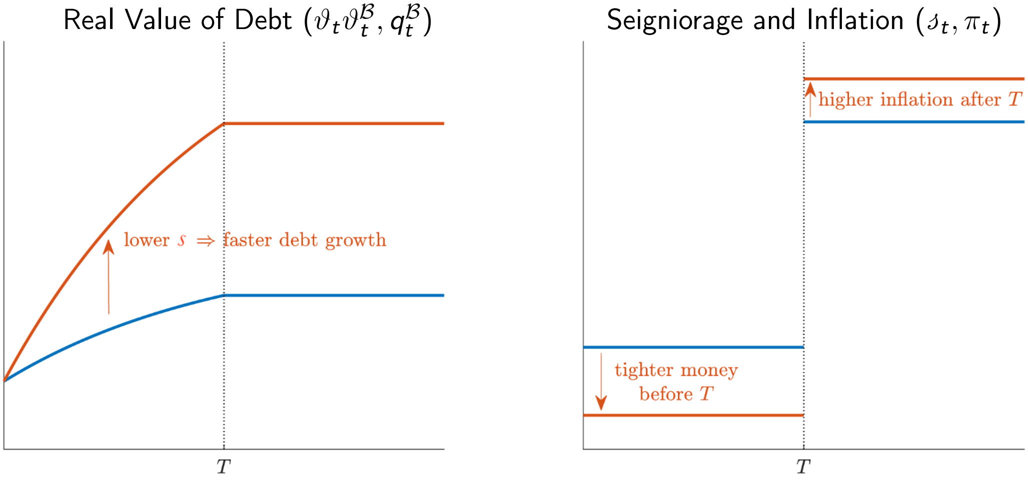
Why Does the Sargent-Wallace Proposition Hold?

- Iterating government budget constraint forward in time and dividing by total wealth yields:

$$\vartheta_T \vartheta_T^{\mathcal{B}} = \vartheta_0 \vartheta_0^{\mathcal{B}} - \int_0^T \rho e^{-\rho t}(\underline{s} + \mathcal{s})\mathrm{d}t$$

- Lower money $\mu_t^{\mathcal{M}}$ over $[0, T] \Rightarrow$ lower seigniorage transfers $\mathcal{s} = \mu^{\mathcal{M}}(a - \mathcal{g})/\nu$ $\Rightarrow$ debt grows faster
- Higher debt at T: need larger seigniorage thereafter to cover interest payments:
  - recall $\hat{\mathcal{s}}(\vartheta_T^{\mathcal{B}}) = -\underline{s} + \vartheta_T \vartheta_T^{\mathcal{B}}$ is increasing in $\vartheta_T^{\mathcal{B}}$

Illustration of Unpleasant Arithmetic



Monetary Dominance

- Suppose $T = \infty$: monetary authority is always in control of the money supply
- Is there an equilibrium? (suppose also $\mathfrak{s} \neq \vartheta_0 \vartheta_0^{\mathcal{B}} - \underline{s}$)
  - not with constant deficit/K_t-ratio $s_t = \underline{s}$
  - but: a constant deficit is not necessarily feasible policy
- Two cases
  1. if $\mathfrak{s} > \vartheta_t \vartheta_t^{\mathcal{B}} - \underline{s}$, $s_t = \underline{s} < 0$ remains feasible
     - but fiscal authority will absorb money over time, effective money suppply is smaller than $\mathcal{M}_t$
     - fiscal authority controls inflation
       (e.g. if real debt to K_t ratio is kept constant, outcomes as if $\mathfrak{s} = \vartheta_0 \vartheta_0^{\mathcal{B}} - \underline{s}$)
  2. if $\mathfrak{s} < \vartheta_t \vartheta_t^{\mathcal{B}} - \underline{s}$, s_t has to rise to avoid default on nominal bonds
     - fiscal authority effectively faces an "intertemporal budget constraint"
     - e.g. smallest constant primary surpluse (per K_t is $s = \vartheta_0 \vartheta_0^{\mathcal{B}} - \mathfrak{s}$
- *Remark*:
  Here, gov. debt is like real/foreign currency debt — very different from FTPL

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- Sargent-Wallace
- Price/Wage Stickiness (later)
  - Li-Merkel (2023)
    $q_t^{\mathcal{B}}$ is sticky and q_t^K more volatile
  - Alexandrov-Brunnermeier (2023) (Price vs. Financial Stability)