MacroFinance

Lecture 01: Introduction to Macrofinance

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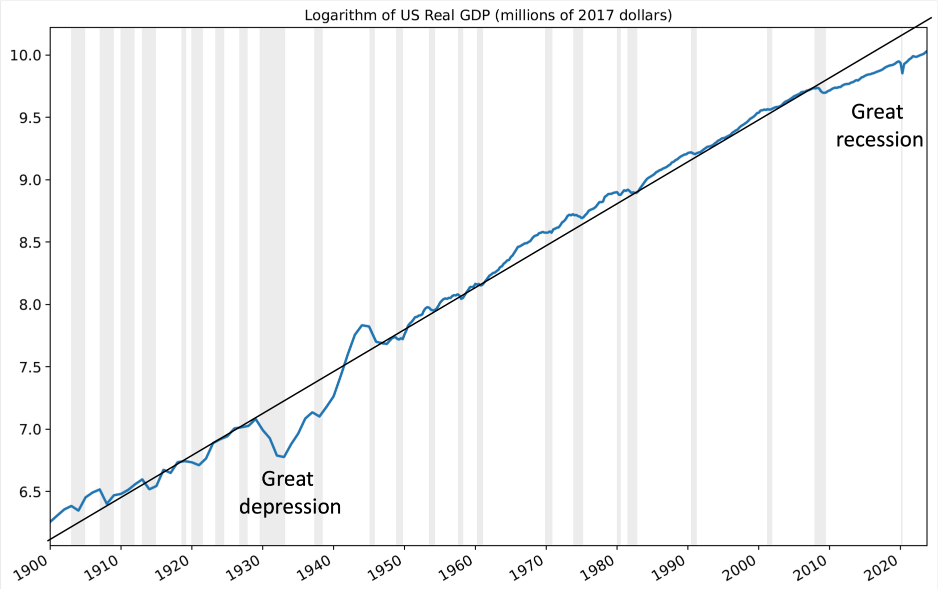
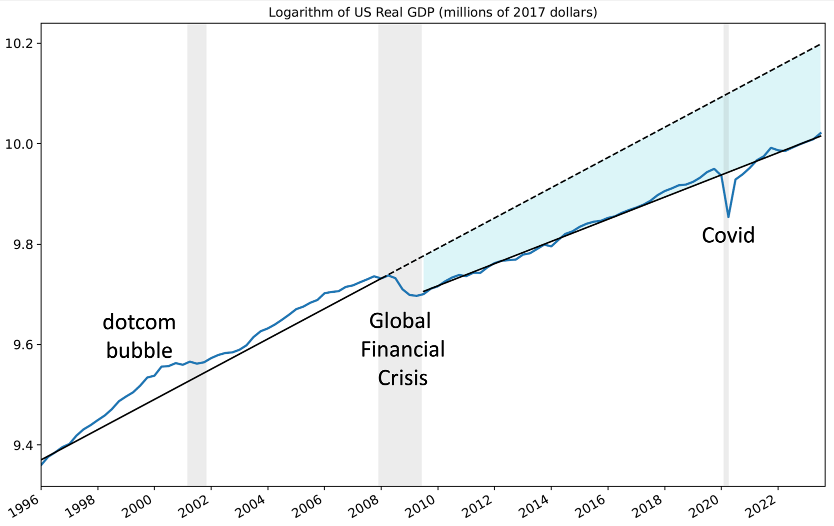
Introduction to Modern Macro, Money, and Finance

- What is Macrofinance?
- Type of Frictions
- Portfolio/investment/risk- vs. consumption-focused macro
- Amplification, Persistence, Resilience
  in 1^{st} Generation Models
  with aggregate MIT-Shock and reversion to steady state

Real US GDP in log: Financial Crises as Resilience Killers



Real US GDP in log: Financial Crises as Resilience Killers



Gap in 2023 alone $\approx 3 - 4$ trillion; Gap over the years (shaded area)

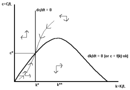
History of Macro and Finance

- *Verbal Reasoning* *(qualitative)*

Fisher, Keynes, ...

Macro

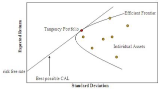
- Growth theory
  - *Dynamic (cts. time)*
  - *Deterministic*



- Introduce stochastic
  - *Discrete time*
    - Brock-Mirman, Stokey-Lucas
    - DSGE models

Finance

- Portfolio theory
  - *Static*
  - *Stochastic*



- Introduce dynamics
  - *Continuous time*
    - Options — Black Scholes
    - Term structure — CIR
    - Agency theory — Sannikov

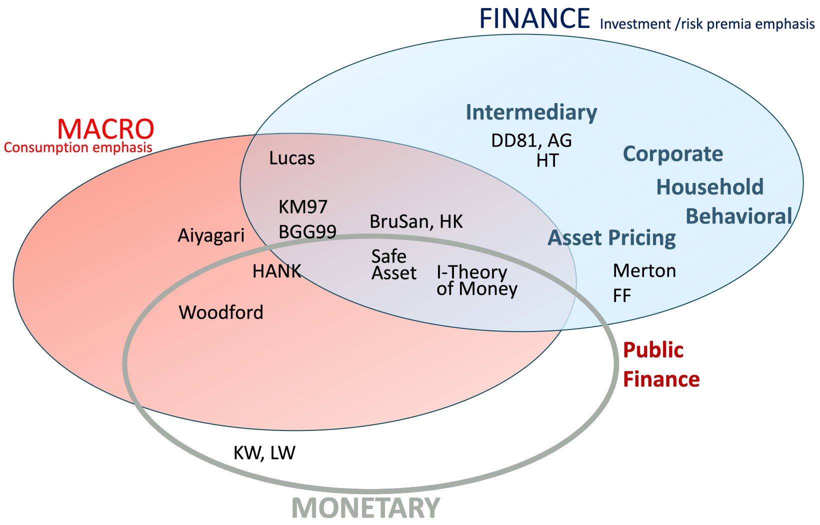
- Cts. time macro with financial frictions

What is Macro-Finance?

- Macro: aggregate impact (resource allocation and constraint)
- Finance: risk allocation
  financial/contracting frictions, heterogeneous agents
  $\Rightarrow$ institutions, liquidity
- Monetary: inside money creation

- How to design Financial Sector, Gov. bonds, etc.
  to achieve optimal resource and risk allocation
- Topics include:
  - Amplification, percolation of shocks, resilience, financial cycle
  - Financial stability, spillovers, systemic risk measures
  - (Un)conventional central bank policy and balance sheet, maturity structure, CBDC
  - Capital flows

MacroFinance: More than Intersection of Macro & Finance



Heterogeneous Agents

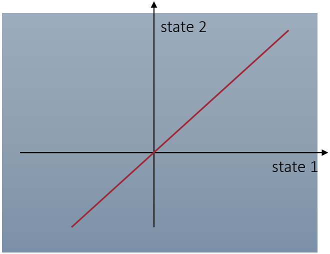
- Lending-borrowing/insuring since agents are different

▪ Poor-rich ▪ Productive ▪ Less patient ▪ Less risk averse ▪ More optimistic	Limited direct lending ⟷ due to frictions	▪ Rich-poor ▪ Less productive ▪ More patient ▪ More risk averse ▪ More pessimistic

- Friction state prices/SDF_s/MRS_s differ after transactions
- Wealth distribution matters (net worths of subgroups) matters!
- Financial sector is not a veil

Financial Frictions and Distortions

- Incomplete markets
  - "natural" leverage constraint (*BruSan*)
  - Costly state verification (*BGG*)
- + Leverage constraints
  (no "liquidity creation")
  - Exogenous limit (*Bewley/Ayagari*)
  - Collateral constraint
    - Current price $D_t \leqslant q_t k_t$
    - Next period's price $D_t \leqslant q_{t+1} k_t$ (*KM*)
    - Next period's VaR $D_t \leqslant VaR_t(q_{t+1}) k_t$ (*BruPed*)



- Search Friction (*Duffie et al.*)
- Belief distortions

Financial Sector

- Financial sector helps to
  - overcome financing frictions and
  - channels resources
  - creates money
- ... but
  - Credit crunch due to adverse feedback loops & liquidity spirals
  - Non-linear dynamics
- New insights to monetary and international economics

Macro: Finance vs. Consumer Focused

- **Portfolio and Investment decision** - Macro-finance
  - Risk-free rate and risk premia [term-risk, credit risk premia]
  - Risk-premia = price of risk * (exogenous risk + endogenous risk)

    amplification/spirals, runs/sudden

    - Δprice $= f(\Delta\mathbb{E}[\text{future cash flows}, \Delta\text{risk premia}])$
  - Non-linearities are prominent
    - around $\neq$ away from steady state
  - Heterogeneity: wealth distribution across investors (+ consumers)

- **Consumption decision**
  - Demand management [interest rate drives c_t]
    - ZLB (liquidity trap)
  - Expectation hypothesis, UIP, ... (limited role for time-varying risk premia)
  - Heterogeneity: wealth distribution across consumers (with different MPCs)

Cts.-time Macro: Macro-Finance vs HANK

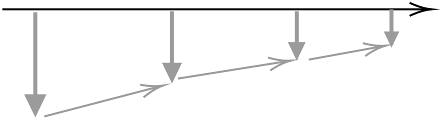
Agents	**Heterogenous investor focus** - Net worth distribution (often discrete)	**Heterogenous consumer focus** - Net worth distribution (often cts.)
Tradition:	Finance (Merton) *Portfolio and consumption choice* ■ Full/global dynamical system ■ Focused on non-linearities away from steady state (crisis ...) ■ Length of recession is stochastic	DSGE (Woodford) *Consumption choice* ■ Zero probability shock ■ Deterministic transition dynamics back to steady state ■ Length of recession deterministic
Risk	Risk and Financial Frictions	No aggregate risk (in HANK paper)
Price of risk:	Idiosyncratic and aggregate risk	N/A
Assets:	Capital, money, bonds with different risk profile ■ Risk-return trade-off ■ Liquidity-return trade-off ■ Flight-to-safety	All assets are risk free ■ No risk-return trade-off ■ Liquidity-return trade-off
Money:	Risk and Financial Frictions	Price stickiness

Overview

- Defining Macrofinance
- Type of Frictions
- Portfolio/investment/risk- vs. consumption focused macro
- Amplification, Persistence, Resilience
  in 1^{st} Generation Models with Aggregate MIT-shocks
- Kiyotaki-Moore in continuous time
- Bernanke-Gertler-Gilchrist

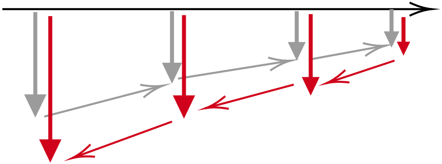
Persistence and Resilience

- Even in standard real business cycle models,
  temporary adverse shocks can have long-lasting effects
- Due to feedback effects, persistence is much stronger in models with *financial frictions*
  - Bernanke & Gertler (1989)
  - Carlstrom & Fuerst (1997)
- Negative shocks to net worth exacerbate frictions and lead to lower capital, investment and net worth in future periods



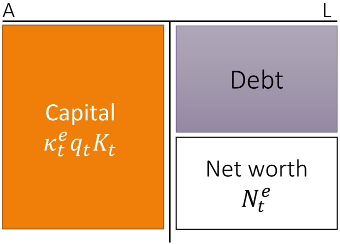
Persistence Leads to Dynamic Amplification

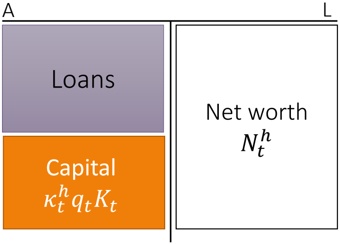
- *Static* amplification occurs because fire-sales of capital from productive sector to less productive sector depress asset prices
  - Importance of *market liquidity* of physical capital
- Dynamic amplification occurs because a temporary shock translates into a persistent decline in output and asset prices
  - Forward grow net worth via retained earnings
  - Backward asset pricing → tightens constraints



Two Sector Model: Kiyotaki Moore (1997) in Cts. Time

- Expert sector (Farmers) — Household sector (Gatherers)





- Capital shares: κ_t^e (experts), κ_t^h (households), $\kappa_t^e + \kappa_t^h = 1, \kappa_t^e, \kappa_t^h \geqslant 0$
- Experts produce with capital with linear production function $a^e k_t^e (= a^e \kappa_t^e K_t)$.
- Households' production function $a^h(\kappa_t^h) k_t^h$ is concave in (aggregate) κ_t^h.
  - Productivity $a^h(\kappa^h) \leqslant a^e$ with equality for $\kappa^h = 0$ and strictly decreasing in κ^h
- Experts can only issue debt with leverage constraint: $D_t^e \leqslant \ell \kappa_t^e q_t K_t$
- All experts' net worth $N_t^e = \int_0^1 n_t^{e,i} di = n_t^e$; all households' net worth $N_t^h = n_t^h$
- Assumption: aggregate physical capitals are in fixed supply $K_t = \bar{K}$

Kiyotaki Moore (1997) in Cts. Time

Expert Sector (Farmers)

- Output: $y_t^e = a^e k_t^e = a^e \kappa_t^e \bar{K}$
- Consumption rate: c_t^e

Household Sector (Gatherers)

- Output: $y_t^h = a^h(\kappa_t^h) k_t^h = a^h(\cdot)\kappa_t^h \bar{K}$
- Consumption rate: c_t^e

Kiyotaki Moore (1997) in Cts. Time

Expert Sector (Farmers)

- Output: $y_t^e = a^e k_t^e = a^e \kappa_t^e \bar{K}$
- Consumption rate: c_t^e
- Objective: $\int_0^\infty e^{-\rho^e t} \log(c_t^e) \mathrm{d}t$

Household Sector (Gatherers)

- Output: $y_t^h = a^h(\kappa_t^h) k_t^h = a^h(\cdot) \kappa_t^h \bar{K}$
- Consumption rate: c_t^e
- Objective: $\int_0^\infty e^{-\rho^h t} \log(c_t^h) \mathrm{d}t$

Assumptions:

- Experts are more impatient $\rho^e > \rho^h$
- Productivity $a^h(\kappa^h) \leqslant a^e$ with equality for $\kappa^h = 0$ and strictly decreasing in κ^h
- No equity issuance
- Debt issuance only w/ leverage constraint: $D_t^e \leqslant \ell \kappa_t^e q_t K_t$

  $\Leftrightarrow \frac{D_t^e}{N_t^e} \leqslant \ell \frac{\kappa_t^e q_t K_t}{N_t^e} \Leftrightarrow -(1 - \theta_t^{K,e}) \leqslant \ell \theta_t^{K,e}$

  Leverage constraint in KM97: $D_t^e(1 + r_{t+dt} dt) \leqslant \ell \kappa_t^e q_{t+dt} K_t$

Portfolio choices: Hamiltonian Approach

- Experts' problem: $\max_{c_t^e,\theta_t^{K,e}} \int_s^\infty e^{-\rho^e t} u(c_t^e)\mathrm{d}t$ s.t. $(1-\ell)\theta_t^{K,e} \leqslant 1$, and

$$\frac{\mathrm{d}n_t^e}{\mathrm{d}t} = \left[-c_t^e + n_t^e\left(r_t + \theta_t^{K,e}(r_t^{K,e} - r_t)\right)\right]$$

- Households' problem: $\max_{c_t^h,\theta_t^h} \int_s^\infty e^{-\rho^h t} u(c_t^h)\mathrm{d}t$, s.t.

$$\frac{\mathrm{d}n_t^h}{\mathrm{d}t} = \left[-c_t^h + n_t^h\left(r_t + \theta_t^{K,h}(r_t^{K,h} - r_t)\right)\right],$$

- The Hamiltonians can be constructed as

$$\begin{aligned}\mathcal{H}_t^e =& e^{-\rho^e t} u(c_t^e) + \xi_t^e \overbrace{\left[-c_t^e + n_t^e\left(r_t + \theta_t^{K,e}(r_t^{K,e} - r_t)\right)\right]}^{\mu_t^{n^e} n_t^e} + \xi_t^e n_t^e \lambda_t^\ell \left(1 - (1-\ell)\theta_t^{K,e}\right) \\ \mathcal{H}_t^h =& e^{-\rho^h t} u(c_t^h) + \xi_t^h \left[-c_t^h + n_t^h\left(r_t + \theta_t^{K,h}(r_t^{K,h} - r_t)\right)\right]\end{aligned}$$

- ξ_t^i multiplier on the budget constraint, $\xi_t^e n_t^e \lambda_t^\ell$ multiplier on leverage constraint
  - We proceed to show that ξ_t^i is SDF later.
- Fisher Separation Theorem btw. consumption and portfolio choice

Hamiltonian Approach: First order conditions

- FOC w.r.t c_t^i:

$$\begin{cases} e^{-\rho^e t} u'(c_t^e) = \xi_t^e \\ e^{-\rho^h t} u'(c_t^h) = \xi_t^h \end{cases} \Rightarrow c_t^i = \rho^i n_t^i, \text{ log utility}$$

Hamiltonian Approach: First order conditions

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$$\begin{cases} e^{-\rho^e t} u'(c_t^e) = \xi_t^e \\ e^{-\rho^h t} u'(c_t^h) = \xi_t^h \end{cases} \Rightarrow c_t^i = \rho^i n_t^i, \text{ log utility}$$

- FOC w.r.t $\theta_t^{K,i}$:

$$\begin{cases} r_t^{K,e} - r_t = (1-\ell)\lambda_t^\ell \\ r_t^{K,h} - r_t = 0 \end{cases}$$

- Where capital returns are: (dividend + price drift)

$$\begin{cases} r_t^{K,e} = \dfrac{a^e}{q_t} + \dfrac{1}{q_t}\dfrac{\mathrm{d}q_t}{\mathrm{d}t} \\ r_t^{K,h} = \dfrac{a^h(\kappa_t^h)}{q_t} + \dfrac{1}{q_t}\dfrac{\mathrm{d}q_t}{\mathrm{d}t} \end{cases}$$

Aside: Understanding Asset Prices

- Price dynamics (with some proper initial conditions):

$$\frac{1}{q_t}\frac{\mathrm{d}q_t}{\mathrm{d}t} + \frac{a^h(\kappa_t^h)}{q_t} = r_t,$$

$$q_t = \int_t^{\infty} e^{-\int_t^s r_u du} a^h(\kappa_s^h) ds$$

- Discrete time analogy:

$$\frac{q_{t+1} - q_t}{q_t} + \frac{a^h(\kappa_t^h)}{q_t} = r_t$$

$$q_t = \sum_{s=0}^{\infty} \left[\prod_{u=0}^{s} \frac{1}{(1 + r_{t+u})} \right] a^h(\kappa_{t+s}^h)$$

- Asset price = sum of discounted dividend flows.
- Asset prices are **solved backward**

Dynamics

- Equilibrium objects are functions of state, net worth share, $\eta_t = \frac{N^e_t}{N_t} = \frac{N^e_t}{q_t \bar{K}}$
- Price dynamics: (No arbitrage for households)

$$\frac{a^h(\kappa^h_t)}{q_t} + \frac{1}{q_t}\frac{\mathrm{d}q_t}{\mathrm{d}t} = r_t,$$

- State dynamics:

$$\mu^N_t dt = \frac{dN_t}{N_t} = \underbrace{\frac{N^e_t}{N_t}}_{\eta_t} \mu^{N^e}_t dt + \underbrace{\frac{N^h_t}{N_t}}_{(1-\eta_t)} \mu^{N^h}_t dt$$

$$\begin{aligned}
\mu^\eta_t &= \mu^{N^e}_t - \mu^N_t = (1-\eta_t)(\mu^{N^e}_t - \mu^{N^h}_t) \\
&= (1-\eta_t)[-(\rho^e - \rho^h) + \theta^{K,e}_t(\frac{a^e}{q_t} + \frac{1}{q_t}\frac{\mathrm{d}q_t}{\mathrm{d}t} - r_t) - \theta^{K,h}_t(\frac{a^h(\kappa^h_t)}{q_t} + \frac{1}{q_t}\frac{\mathrm{d}q_t}{\mathrm{d}t} - r_t)] \\
&= (1-\eta_t)[-(\rho^e - \rho^h) + \theta^{K,e}_t \underbrace{(\frac{a^e}{q_t} - \frac{a^h(\kappa^h_t)}{q_t}}_{=r^{K,e}_t - r^{K,h}_t})]
\end{aligned}$$

Equilibrium Conditions

- Equilibrium objects $(\kappa^e, \kappa^h, q, r)$ are functions of state, net worth share, $\eta_t = \frac{N_t^e}{N_t} = \frac{N_t^e}{q_t \bar{K}}$
- pinned down by:

$$q_t \bar{K}[\rho^e \eta_t + \rho^h(1-\eta_t)] = [a^e \kappa_t^e + a^h(\kappa_t^h)\kappa_t^h]\bar{K} \qquad \text{(Goods market)}$$

$$\underbrace{\theta_t^{Ke}\eta_t}_{=\kappa_t^e} q_t \bar{K} + \underbrace{\theta_t^{Kh}(1-\eta_t)}_{=\kappa_t^h} q_t \bar{K} = q_t \bar{K} \qquad \text{(Capital market)}$$

$$\kappa_t^e \leqslant \frac{\eta_t}{1-\ell} \qquad \text{(Collateral Constraint)}$$

$$\mu_t^\eta = (1-\eta_t)\left[-(\rho^e - \rho^h) + \theta_t^{K,e} \frac{a^e - a^h(\kappa_t^h)}{q_t}\right]$$

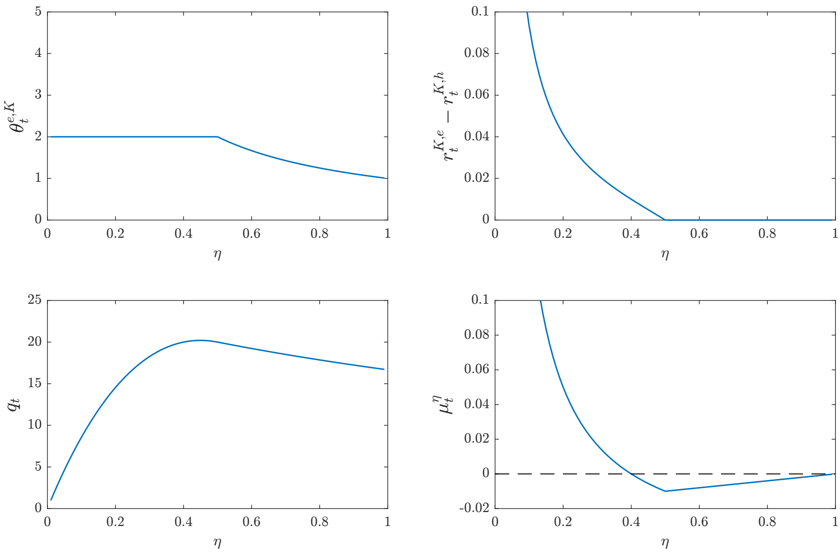
- simplified to (and define $\kappa_t := \kappa_t^e = 1 - \kappa_t^h$)

$$q_t[(\rho^e - \rho^h)\eta_t + \rho^h] = \kappa_t a^e + (1-\kappa_t)a^h(1-\kappa_t)$$

$$\kappa_t \leqslant \frac{\eta_t}{1-\ell}$$

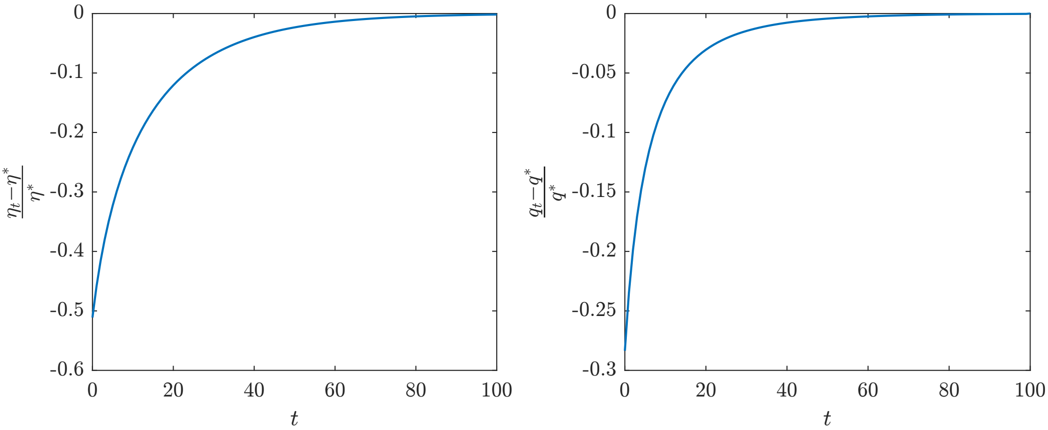
$$\mu_t^\eta = (1-\eta_t)\left[-(\rho^e - \rho^h) + \frac{\kappa_t}{\eta_t}\frac{a^e - a^h(1-\kappa_t)}{q_t}\right]$$

Global Non-linear Solution



Parameters: $\rho^e = 0.06, \rho^h = 0.04, \ell = 0.05, a^e = 1.0, a^h(1-\kappa) = \kappa$

Impluse Responses



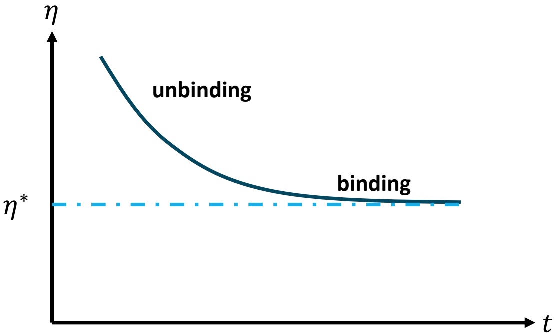
Impulse response function with 30% (of η) negative redistribution shock.
Parameters: $\rho^e = 0.06, \rho^h = 0.04, \ell = 0.5, a^e = 1.0, a^h(1-\kappa) = \kappa$

Log-linearization around Steady State

1. Derive steady state with $\mu^\eta = 0$
   with its properties
2. Log-linearize around steady state
   characterize dynamical system locally around the steady state

The Steady State: Binding Collateral Constraint

- The collateral constraint always binds in the steady state
  - If collateral constraint does not bind $\lambda_t^\ell = 0$ and hence $r^{K,e} = r^{K,h}$, i.e. $a^e = a^h(\cdot)$
- Note, the constraint does not need to bind only if $\kappa_t = 1$.
  - Then $\mu_t^\eta = (1 - \eta_t)(\rho^h - \rho^e)$
  - as $\rho^e > \rho^h \Rightarrow \mu_t^\eta < 0$, i.e. η declines
- Characterization of Steady State (Next Page)



Steady State

- Since Collateral constrained binds, steady state capital share

$$\kappa^* = \frac{\eta^*}{1-\ell}$$

- Expert sector's net worth share is $\eta_t := \frac{N_t^e}{q_t \bar{K}}$, is constant, i.e. $\mu_t^\eta := \frac{\mathrm{d}\eta_t}{\mathrm{d}t} = 0$

$$q^*[(\rho^e - \rho^h)\eta^* + \rho^h] = \kappa^* a^e + (1-\kappa^*)a^h(1-\kappa^*)$$

$$(\rho^e - \rho^h) = \frac{\kappa^*}{\eta^*} \frac{a^e - a^h(1-\kappa^*)}{q^*} \quad \text{for } \mu^\eta = 0$$

- Combine

$$\kappa^* a^e - \kappa^* a^h(1-\kappa^*) + q^*\rho^h = \kappa^* a^e + (1-\kappa^*)a^h(1-\kappa^*)$$

$$\Rightarrow \quad q^* = a^h(1-\kappa^*)/\rho^h,$$

where the steady state κ^* is implicitly given by:

$$\frac{\rho^e - \rho^h}{\rho^h} = \frac{1}{1-\ell} \frac{a^e - a^h(1-\kappa^*)}{a^h(1-\kappa^*)}.$$

- For specific functional form $a^h(1-\kappa_t) = a^e \kappa_t$:

$$\kappa^* = \frac{1}{(1-\ell)(\rho^e - \rho^h)/\rho^h + 1} \quad \Rightarrow \eta^* = \frac{1-\ell}{(1-\ell)(\rho^e - \rho^h)/\rho^h + 1}$$

Steady State: Comparative Static

- For the specific example $a^h(\cdot) = a^e\kappa$:

- For higher leverage, ℓ, (i.e. less tight collateral constraint)
  - κ^*, SS-capital share, is higher.
  - η^*, SS-net worth share, is lower.
  - $q^* = \frac{a^h}{\rho^h}$, price of capital, is higher.
    $q^*\bar{K}$, total wealth in the economy, is higher too.
  - $N^{e,*}$ SS-experts' net worth, is higher (Check?)
  - Comparative Static = permanent (long-run) shift to new steady state
  - Next: Dynamics of how to return to the old steady state
    (after an unanticipated shock)

Log-linearized Dynamics Around Steady State

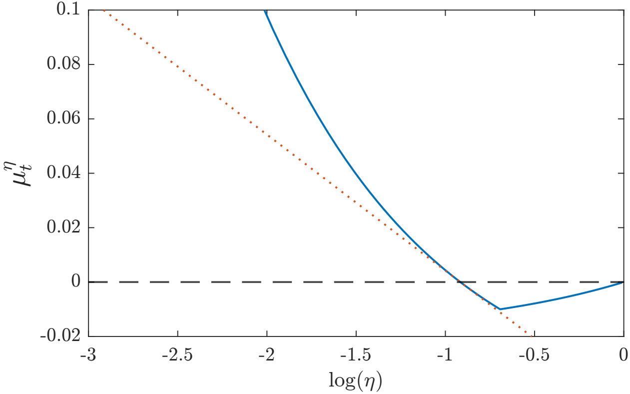
- Analytical solutions to η_t, q_t dynamics are hard to obtain. Expansion around the steady state:

$$\log(\eta_t/\eta^*) = \hat{\eta}_t$$
$$\log(q_t/q^*) = \hat{q}_t$$
$$\log(r_t/r^*) = \hat{r}_t$$
$$\log(a_t^h/a^{h,*}) = \hat{a}_t^h$$

- Expression for $\hat{a}_t^h, \hat{q}_t^h$ as a function of $\hat{\eta}_t$
- State dynamics and price dynamics become:

$$\frac{\mathrm{d}\hat{\eta}_t}{\mathrm{d}t} = \frac{1-\eta^*}{1-\ell}\left(-\frac{a^{h,*}}{q^*}\hat{a}_t^h - \frac{a^e - a^{h,*}}{q^*}\hat{q}_t\right)$$
$$\frac{\mathrm{d}\hat{q}_t}{\mathrm{d}t} = r^*(\hat{r}_t + \hat{q}_t - \hat{a}_t^h)$$

Global vs. Log-linearized Solution for η-drift

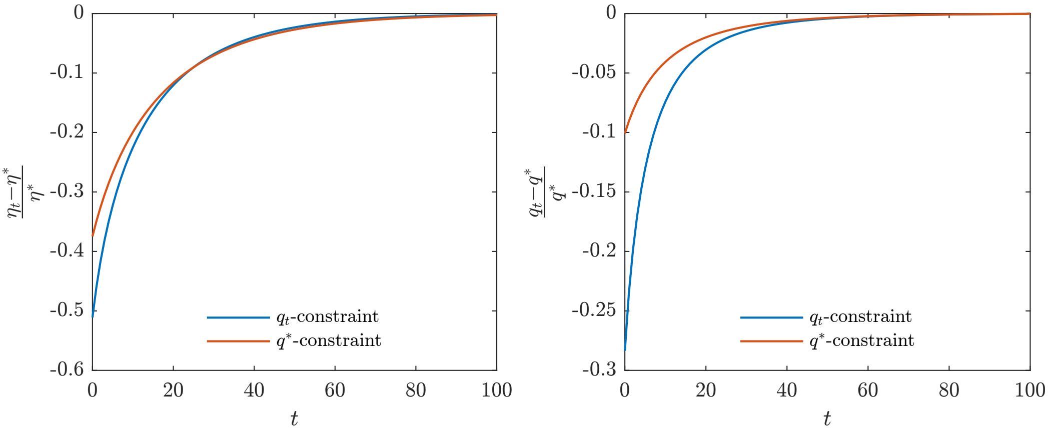


- Note: x-axis is $\log(\eta)$, since log-linearization

Decomposing Amplification Effects

- Start at steady state $\{q^*, \eta^*, \kappa^*\}$
- Shock: redistribution of a fraction of experts' net worth share to households
  - In KM productivity shock lasts for one period (not for an instant), causes initial redistribution
- **Impulse response function** (with deterministic recovery)
- Immediate impact at $t = 0$
  - direct redistributive effect/shock
  - **price-net worth effect**
    decline in q_t reduces experts' net worth share as they are levered $\Rightarrow$ feedback
  - **price-collateral effect**
    decline in q_t tightens collateral constraints $\Rightarrow$ feeds back on price-net worth effect
- Subsequent impact $t > 0$ (which feeds back to immediate impact)
- **Decomposition:**
  Switch off price-collateral effect by assuming that
  collateral constraint is determined by SS-price q^* instead of equilibrium price q_t.

Decomposition of Amplification: Impulse Response Fcn



Impulse response function with 30% (of η) negative redistribution shock.
Parameters: $\rho^e = 0.06, \rho^h = 0.04, \ell = 0.5, a^e = 1.0, a^h(1-\kappa) = \kappa$

Decomposing Amplification at $t = 0$

- At time t, the economy is at steady state $\{q^*, \eta^*, \kappa^*\}$.
- Negative initial/direct redistributive shock $\eta' = (1-\epsilon)\eta^*$, new price q', and capital holding κ' solves:

$$q' = \frac{\kappa' a^e + (1-\kappa')a^h(1-\kappa')}{(\rho^e - \rho^h)\eta' + \rho^h} \qquad \text{(Goods market)}$$

$$\kappa' = \frac{\eta^*(1-\epsilon)}{1-\ell} \qquad (q_t\text{-constraint})$$

$$\kappa' = \frac{\eta^*(1-\epsilon)}{1-\ell q^*/q'} \qquad (q^*\text{-constraint})$$

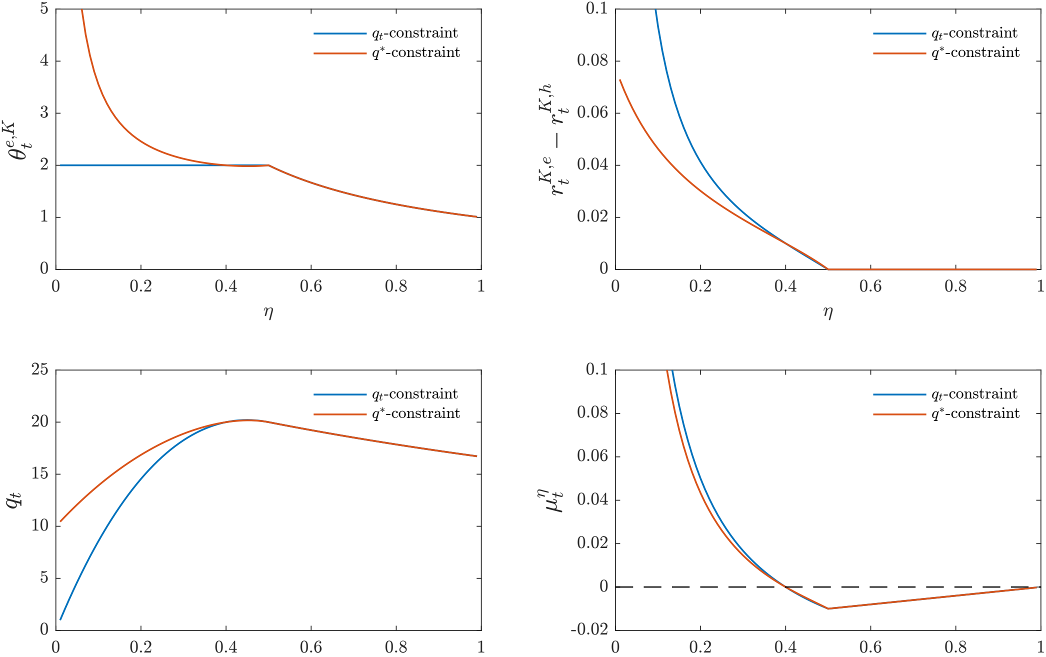
- However, debt contract was signed by old price $q^* \Rightarrow \eta$ drops further
- Consider the balance sheet (first round effect):

$$\frac{\eta'}{1-\ell}q' = \frac{\ell}{1-\ell}\eta' q^* + \eta'' q'$$

To get the convergence result, we need to do this procedure iteratively.

Decomposing Amplification for $t > 0$ (global solution)

$\rho^e = 0.06, \rho^h = 0.04, \ell = 0.05, a^e = 1.0, a^h(1-\kappa) = \kappa$



Decomposing Amplification for $t > 0$ (log-linearized sol.)

- Price dynamics:

$$\frac{\mathrm{d}\hat{q}_t}{\mathrm{d}t} = r^*\hat{r}_t - r^*\hat{a}_t^h + r^*\hat{q}_t$$

- State dynamics with q_t-collateral constraint:

$$\frac{\mathrm{d}\hat{\eta}_t}{\mathrm{d}t} = \frac{1-\eta^*}{1-\ell}\left(-\frac{a^{h,*}}{q^*}\hat{a}_t^h - \frac{a^e - a^{h,*}}{q^*}\hat{q}_t\right)$$

- State dynamics with q^*-collateral constraint:

$$\frac{\mathrm{d}\hat{\eta}_t}{\mathrm{d}t} = \frac{1-\eta^*}{1-\ell}\left(-\frac{a^{h,*}}{q^*}\hat{a}_t^h - \frac{1}{1-\ell}\frac{a^e - a^{h,*}}{q^*}\hat{q}_t\right)$$

$\hat{q}_t, \hat{a}_t^h, \hat{r}_t$ are different with different constraints.

Adding Investments/Physical Capital Formation

- Instead of fixed aggregate capital stock $\bar{K}$, convert goods into physical capital
- Capital conversion function $\Phi(\iota)$ (increasing and concave)

$$dk_t = \Phi(\iota_t)k_t - \delta k_t$$

  - ι_t is the investment **rate** (real investment is $\iota_t k_t$)
  - occurs within the period (no "time-to-build") $\Rightarrow$ static problem
  - δ is the depreciation rate of capital
- Optimal investment rate depends on price of physical capital q_t.
  - Tobin's Q:

$$q_t = 1/\Phi'(\iota_t)$$

  - attractive functional form with adjustment cost ϕ: $\Phi(\iota) = \frac{1}{\phi}\log(\phi\iota + 1)$

- Homework: Redo continuous time KM analysis with ι-investment.

Bernanke, Gertler, Gilchrist 1999

- Fully fledged DSGE Model with price stickiness, idiosyncratic firm risk, ...
- Aggregate shocks are unanticipated zero-probability shocks (MIT shocks)
- No fire-sale to less productive household sector (unlike in KM97)
- Divestment: Convert physical capital back to consumption good at a cost (captured by $\Phi(\cdot)$-adjustment cost function)
- Financial Frictions:
  - No equity issuance
  - Debt issues with costly state verification (instead of collateral constraint)
    - If firm defaults (after negative idiosyncratic shock), creditor has to pay cost to verify true (remaining) cash flow
    - Optimal contract is a debt contract (debt payoff is hockey stick function of cash flow)
    - De-facto borrowing firms pay verification costs in expectations (in form of higher interest rate/funding costs)
  - A negative aggregate shock, lowers firms' net worth $\Rightarrow$ firm's default prob. rises $\Rightarrow$ expected verification cost rise $\Rightarrow$ Firms funding costs rise

"Single Shock Critique"

- Critique: After the shock all agents in the economy know that the economy will deterministically return to the steady state.
  - Length of slump is deterministic (and commonly known)
    - No safety cushion needed
- In reality an adverse shock may be followed by additional adverse shocks
  - Build-up extra safety cushion for an additional shock in a crisis
- Impulse response vs. volatility dynamics

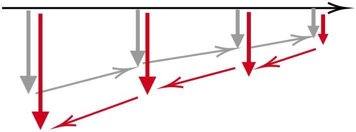
Conclusion & Takeaways

- Defining Macrofinance
- Contrasting Different Financial Frictions
- First-Generation Macrofinance Models
  - Zero Probability Aggregate Shocks
  - Log-linearization Around Steady State
  - Agents believe deterministic return to Steady State
- Without (anticipated) risk, collateral constraint binds in equilibrium
  i.e. no difference between normal times and crisis times
- Log-linearlization is a good approximation

- NEXT: Stochastic Modeling
  2*nd* Generation Macrofinance Models

Endogenous Volatility & Volatility Paradox

- Endogenous Risk/Volatility Dynamics in BruSan
  - Beyond Impulse responses



  - Input: constant volatility
  - Output: endogenous risk, time varying volatility

⇒ Precautionary savings

  - Role for money/safe asset

⇒ Nonlinearities in crisis

⇒ endogenous fait tails, skewness

- Volatility Paradox
  - Low exogenous (measured) volatility leads to high build-up of (hidden) endogenous volatility (Minksy' financial instability hypothesis)