The Market for Attention

Daniel Chen

Princeton University

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Motivation

- This talk is about platforms that profit primarily from targeted advertising.

- This is a large and important market.

Stylized Features

1. Platforms often provide their services for <u>free</u> to consumers.
2. They compete for attention by <u>investing in the quality</u> of their services.
3. They earn revenue by <u>selling ads</u> to firms in the product market.
4. They sell ads via <u>dynamic individualized auctions</u> that arise in real time.
5. The ads are <u>targeted</u> using individual level <u>consumer data</u>.
6. But, firms generally do <u>not personalize prices</u>.

Contribution

- I build a search-theoretic model of platform competition that endogenizes outcomes on the different market sides including the product market.
- The model shows how ad revenues, platform quality, and the allocation in the product market are determined and how they depend on data and platform interoperability.
- Analyze the welfare impacts of platforms from their two productive roles: 1. provision of quality services, 2. mitigation of product market frictions by advertising.
- Study effects of policies on data and interoperability on "whole" economic system.
- Methodological contribution: show how this can be done tractably.

Main Takeaway

A broad multi-sided perspective is essential:

1. Effects of policies may flip when all sides are accounted for.

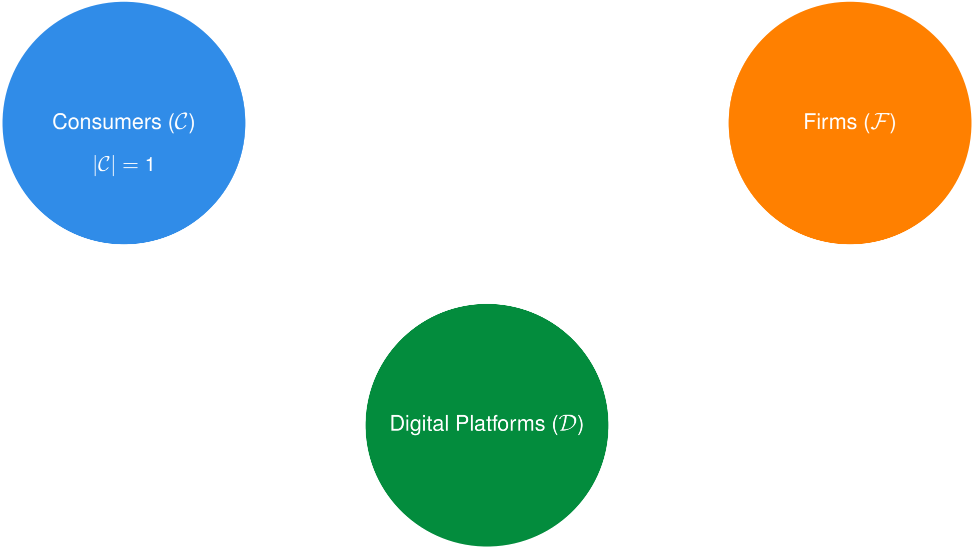
2. Short run effects may differ substantially from the long run.

3. Nontrivial tradeoffs among the different market sides.

Related Literature

1. Platforms and two-sided markets
   - Surplus generated by interactions of agents on the two sides often exogenous
   - So far, no model of competing heterogeneous data services with market power
2. Advertising
   - Typically do not model endogenous content provision for attention
   - Traditional ad models have no role for consumer data, ad targeting
3. Ad auctions
   - Mostly squarely auction focused; no product market
   - Usually, little distinguishes objects sold as ads
4. Competing auctions
   - Solve for steady state but not full dynamics
   - Not tailored to microstructure of platforms

Baseline Model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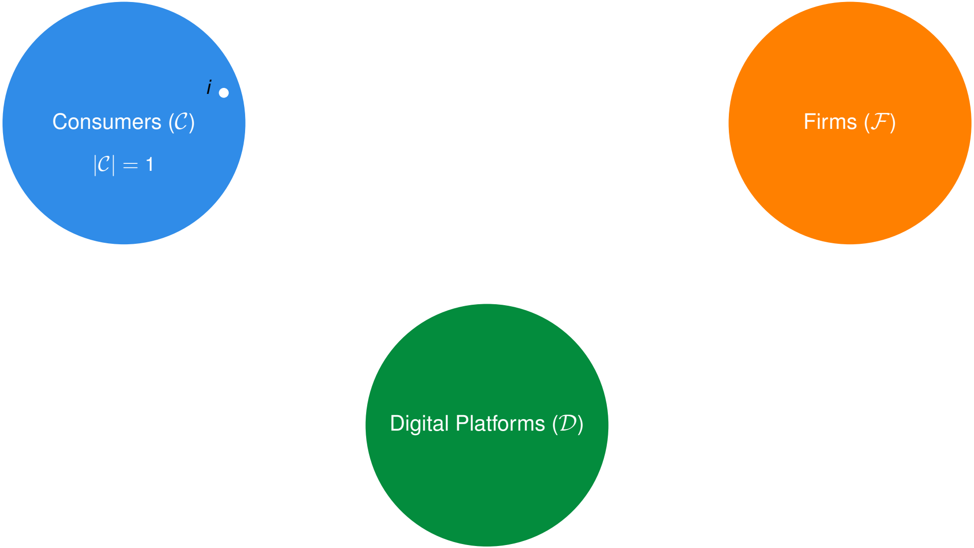

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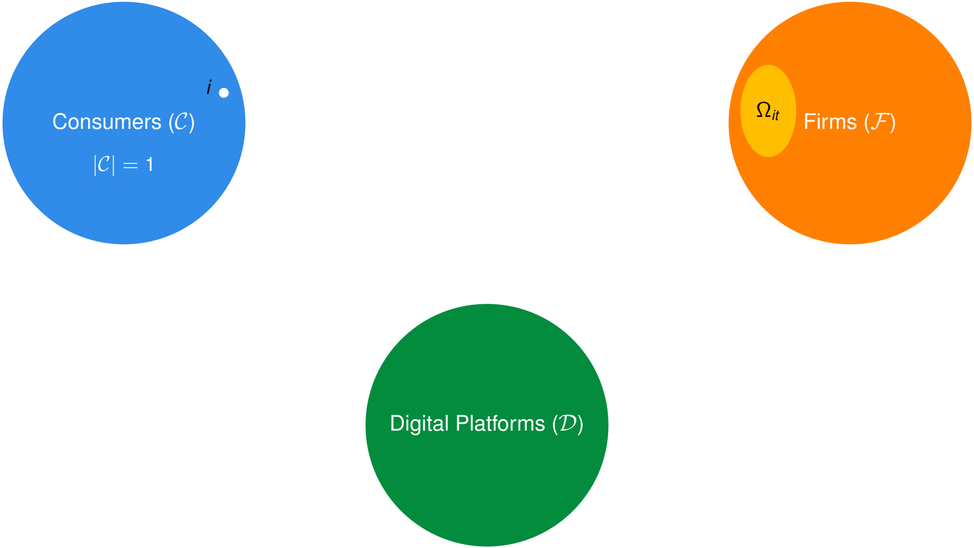

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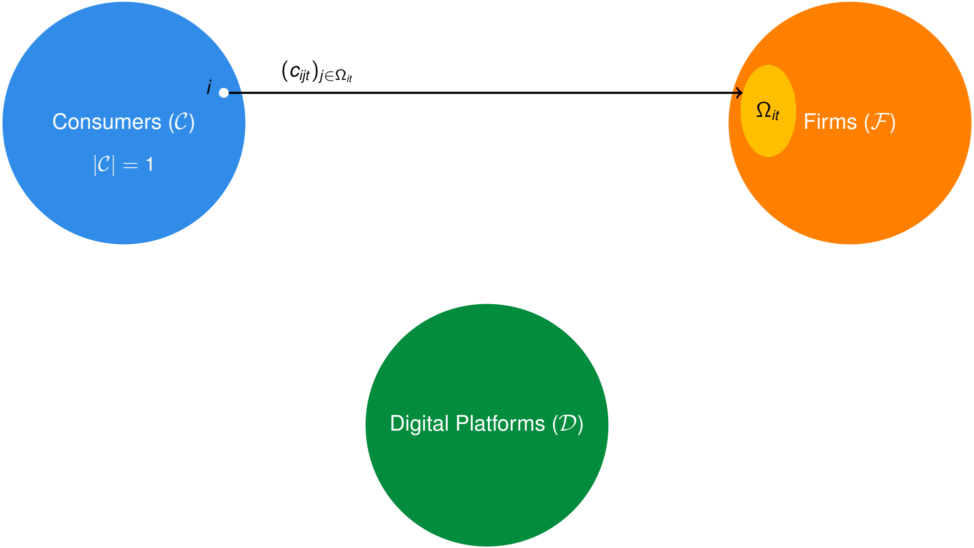

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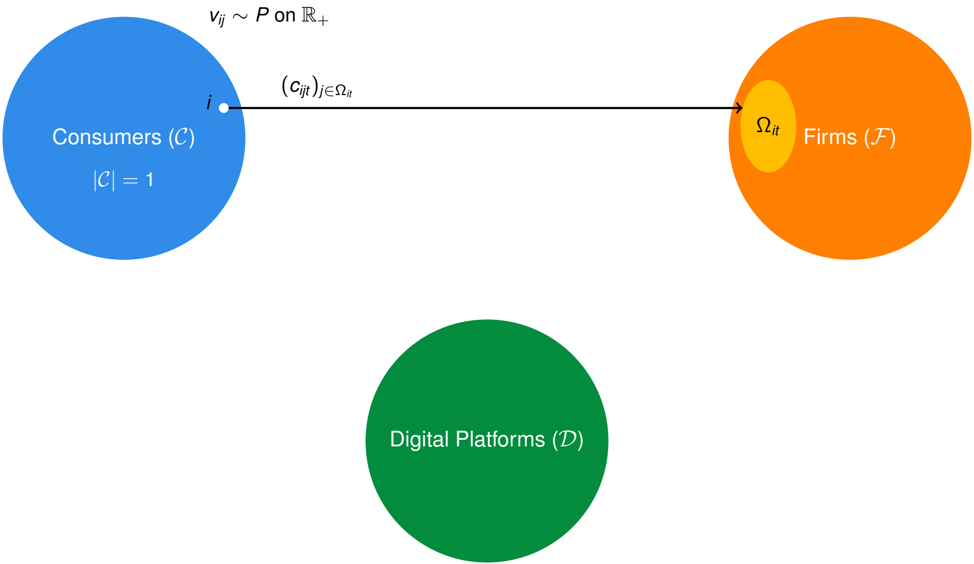
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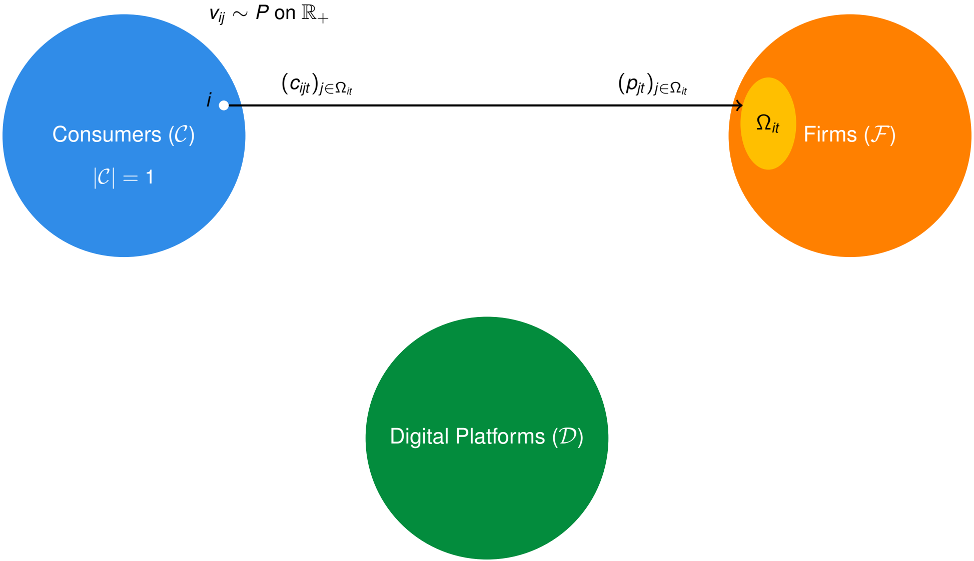
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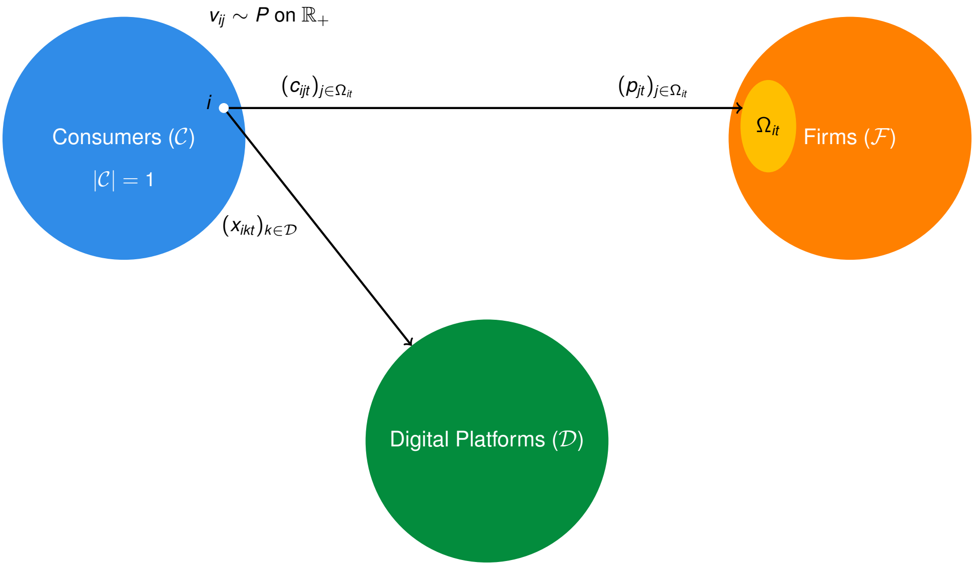
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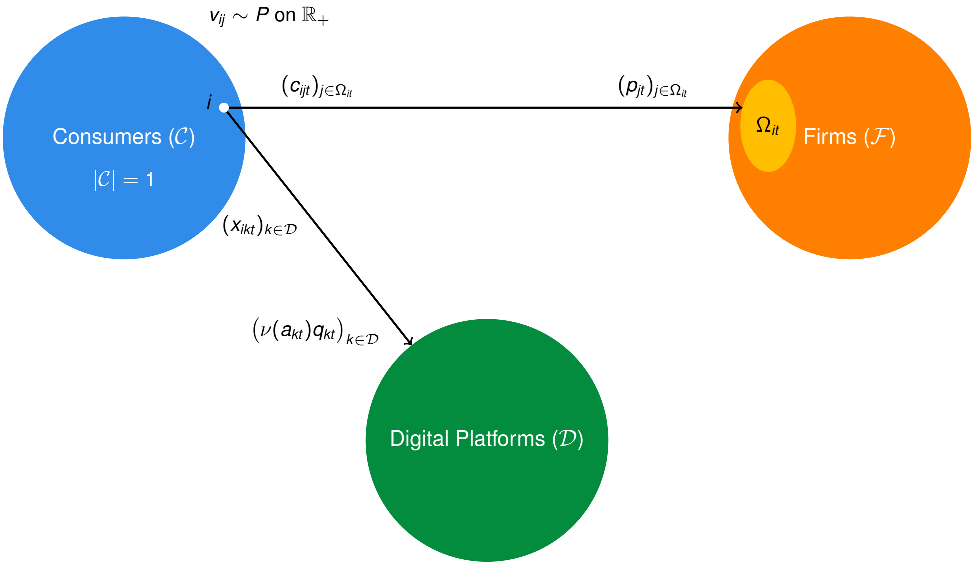
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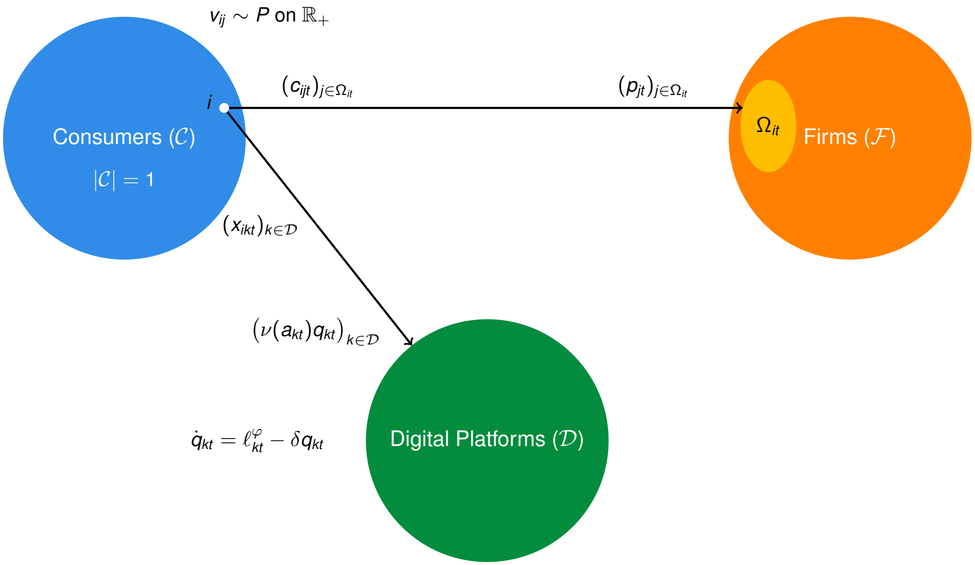
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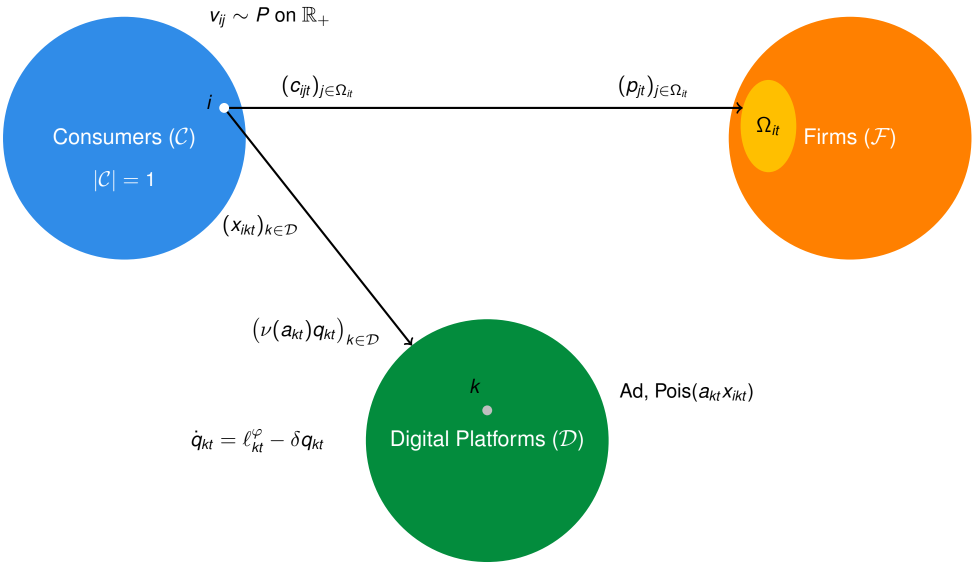

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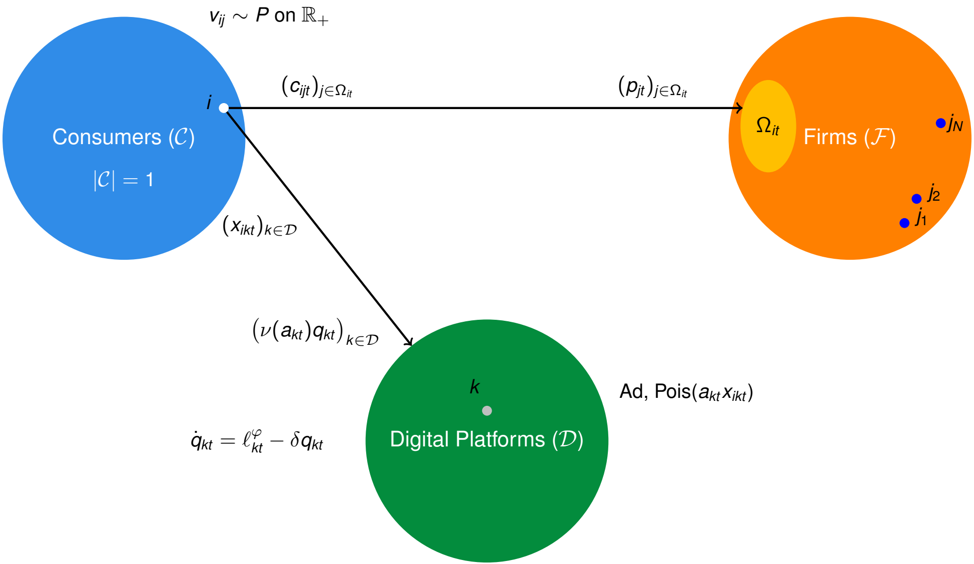

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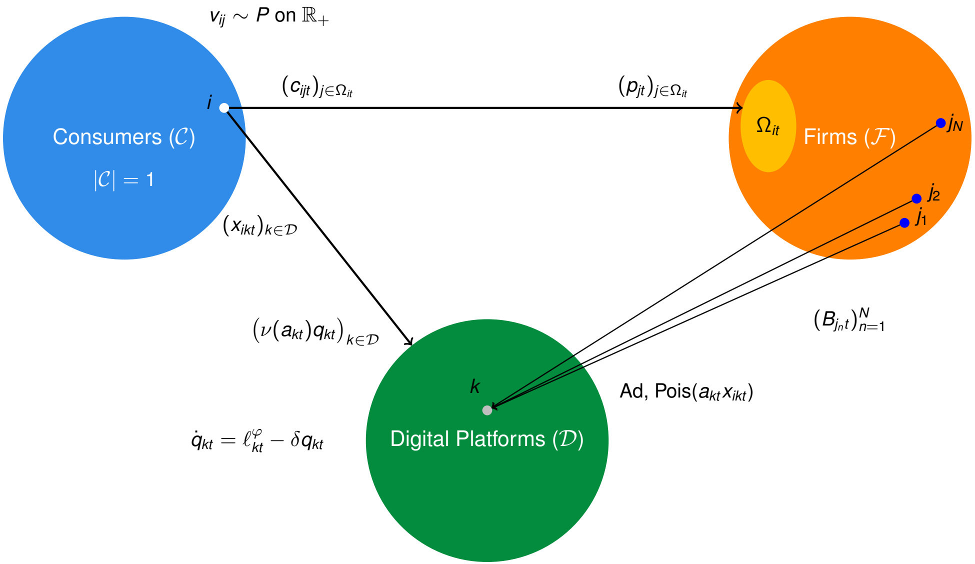

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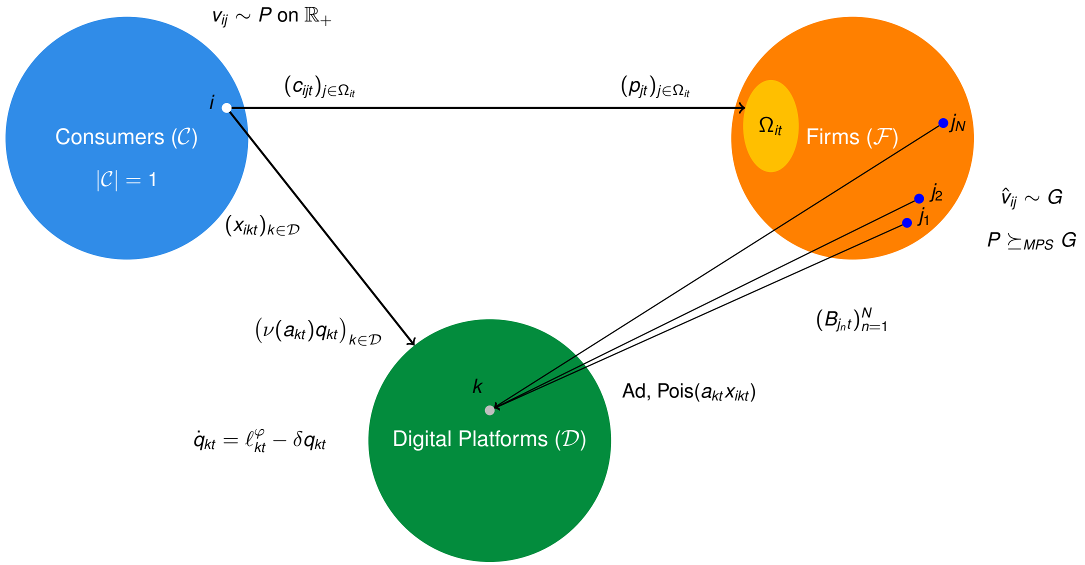
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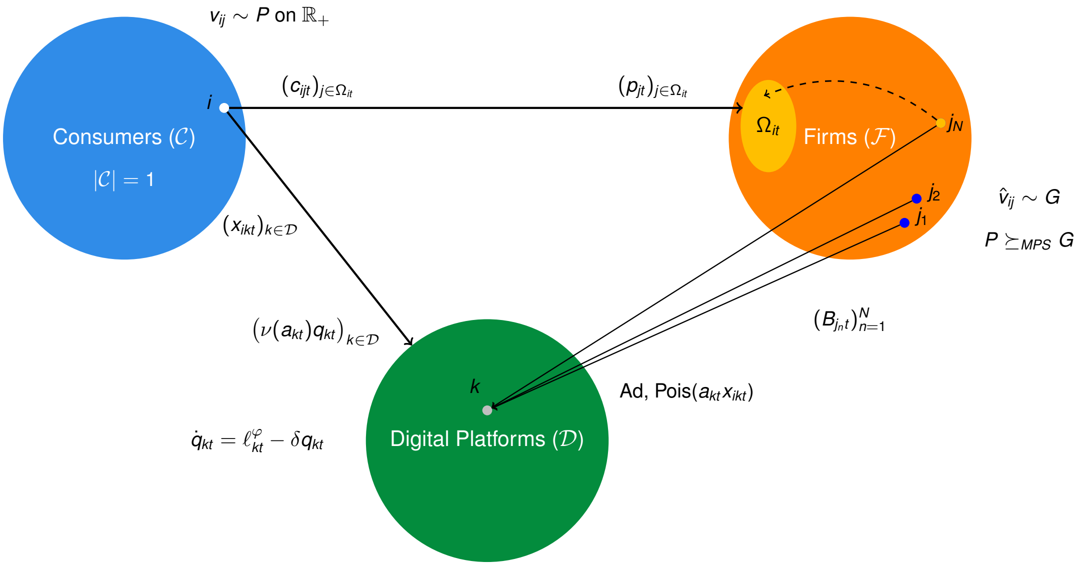

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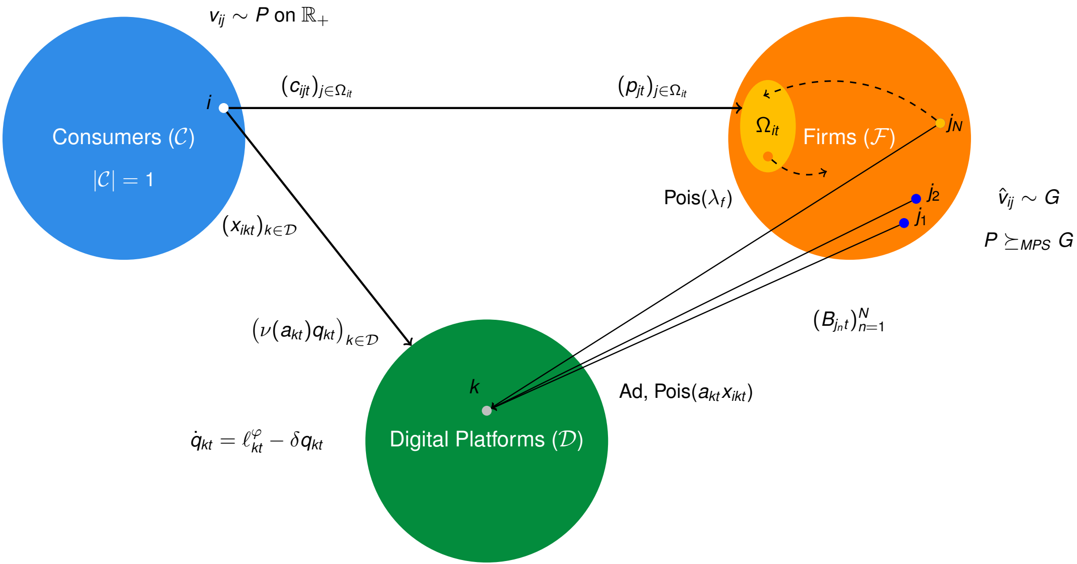
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Preferences

- Consumer i's flow utility is $u(C_{it}, X_{it})$ where:

$$C_{it} = \left[\int_{\Omega_{it}} v_{ij}^{\frac{1}{\sigma}} c_{ijt}^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}j \right]^{\frac{\sigma}{\sigma-1}}$$

$\sigma > 1$: product substitutability

$$X_{it} = \left[\int_{\mathcal{D}} (\nu(a_{kt}) q_{kt} x_{ikt})^{\frac{\rho-1}{\rho}} \, \mathrm{d}k \right]^{\frac{\rho}{\rho-1}}$$

$\rho > 1$: platform substitutability

- Firms and platforms are profit maximizers.

- Everyone discounts time at rate $r > 0$.

Equilibrium Definition

An *equilibrium* is a collection of processes for

1. product demands $\{c_{ijt}\}$,
2. platform demands $\{x_{ikt}\}$,
3. the measure of varieties in consideration sets $\{M_t\}$,
4. the cdf of the expected values of consumers for those varieties $\{H_t\}$,
5. prices $\{p_{jt}\}$,
6. bidding functions $\{B_{jt}\}$,
7. ad frequencies $\{a_{kt}\}$,
8. investment rates $\{\ell_{kt}\}$,
9. and quality levels $\{q_{kt}\}$

such that firms, consumers, and platforms are optimizing*
and $\{M_t\}$, $\{H_t\}$, and $\{q_{kt}\}$ satisfy their respective laws of motion.

Equilibrium

Equilibrium Characterization

Thm. Under technical parameter conditions, there exists a unique equilibrium. Equilibrium converges to a steady state and has the following properties:

1.
2.
3.
4.
5.
6.
7.
8.

1. Consumers' Demands

1. Consumer i's demand for product $j \in \Omega_{it}$ is

$$c_{ijt} = \frac{I v_{ij}}{\int_{\Omega_{it}} v_{iz} p_{zt}^{1-\sigma} \mathrm{d}z} p_{jt}^{-\sigma}.$$

2. Consumer i's demand for platform $k \in \mathcal{D}$ is

$$x_{ikt} = \frac{[\nu(a_{kt}) q_{kt}]^{\rho-1}}{\int_{\mathcal{D}} [\nu(a_{zt}) q_{zt}]^{\rho-1} \mathrm{d}z}.$$

2. Firms' Flow Profits and Prices

1. Firm j's flow profit from selling to consumer i is

$$\underbrace{\frac{I v_{ij}}{\int_{\Omega_{it}} v_{iz} p_{zt}^{1-\sigma} \mathrm{d}z} p_{jt}^{-\sigma}}_{\text{demand}} \underbrace{(p_{jt} - 1)}_{\text{markup}}.$$

2. Firm j's profit-maximizing price is

$$p_{jt} = \frac{\sigma}{\sigma - 1}.$$

It is optimal for firm j *not* to personalize prices.

3. Ad Frequency

Platform k sets ad frequency

$$A = \underset{a_{kt} \in [0,\bar{a}]}{\arg\max} \underbrace{\pi_{\mathcal{D}t} a_{kt} \frac{[\nu(a_{kt}) q_{kt}]^{\rho-1}}{\int_{\mathcal{D}} [\nu(a_{zt}) q_{zt}]^{\rho-1} \mathrm{d}z}}_{\text{expected ad price} \times \text{ad frequency} \times \text{attention}} = \boxed{\underset{a_{kt} \in [0,\bar{a}]}{\arg\max}\, a_{kt} \nu(a_{kt})^{\rho-1}}.$$

Thus, ad frequency is constant over time and depends only on ν and ρ.

In fact, we can show that $A\downarrow$ as $\rho\uparrow$.

4. Size and Composition of Consideration Sets

- The measure of varieties in Ω_{it} is

$$M_t = \frac{A}{\lambda_f} - \left(\frac{A}{\lambda_f} - M_0\right) e^{-\lambda_f t}$$

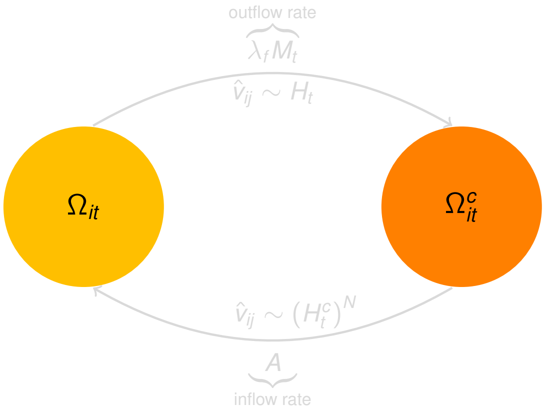
and $M_t \to M = A/\lambda_f$.

- H_t^c is characterized by

$$\int_{H_0^c(\cdot)}^{H_t^c(\cdot)} \frac{1}{|\mathcal{F}|G(\cdot) - Mu^N - (|\mathcal{F}| - M)u}\,\mathrm{d}u$$

$$= \frac{\ln\left[M - M_0 + (|\mathcal{F}| - M)e^{\lambda_f t}\right]}{|\mathcal{F}| - M}.$$

- Given M_t, H_t^c get H_t from accounting.



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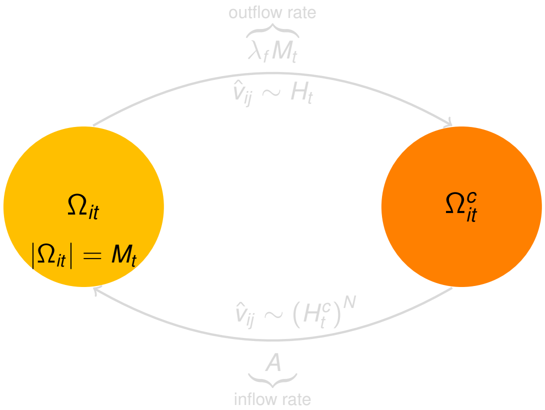

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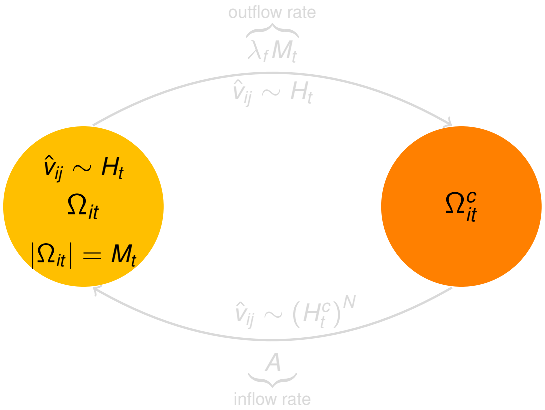

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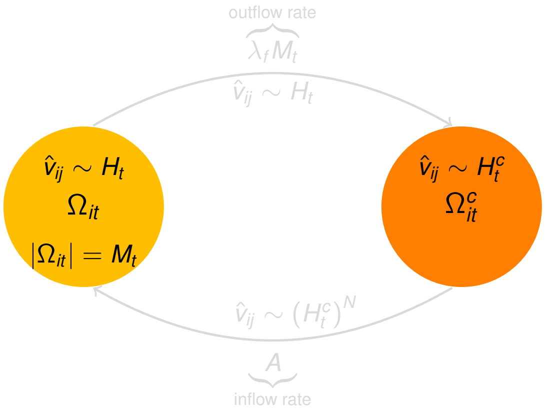

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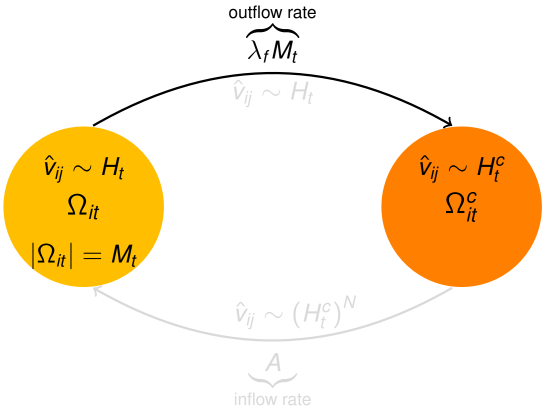

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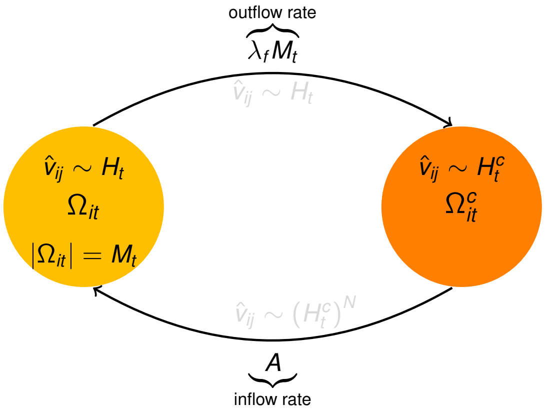

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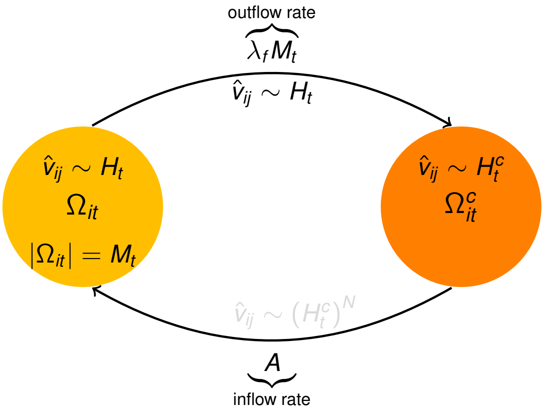

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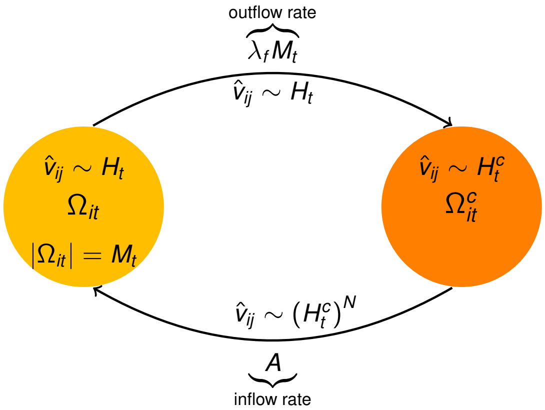

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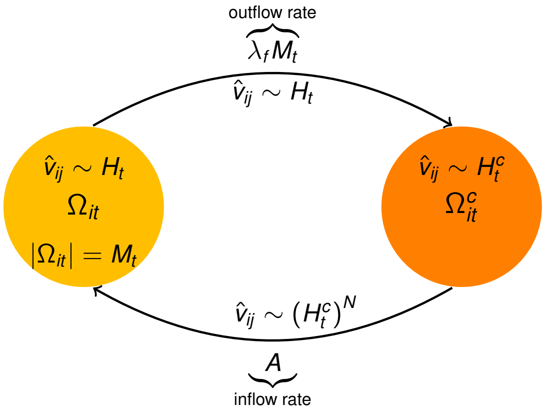

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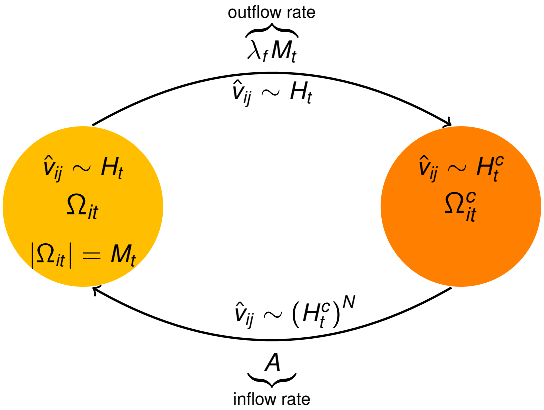
$$= \frac{\ln\left[M - M_0 + (|\mathcal{F}| - M)e^{\lambda_f t}\right]}{|\mathcal{F}| - M}.$$

- Given M_t, H_t^c get H_t from accounting.



4. Size and Composition of Consideration Sets

- The measure of varieties in Ω_{it} is

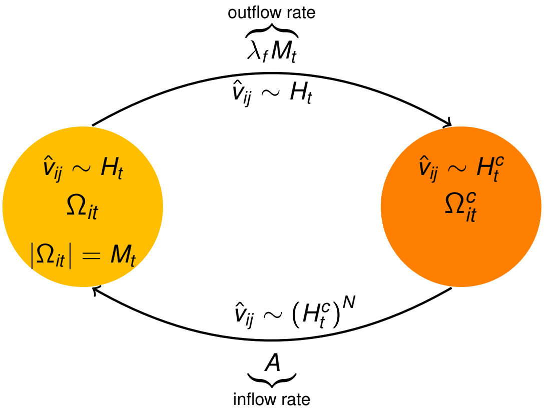
$$M_t = \frac{A}{\lambda_f} - \left(\frac{A}{\lambda_f} - M_0\right) e^{-\lambda_f t}$$

and $M_t \to M = A/\lambda_f$.

- H_t^c is characterized by

$$\int_{H_0^c(\cdot)}^{H_t^c(\cdot)} \frac{1}{|\mathcal{F}|G(\cdot) - Mu^N - (|\mathcal{F}| - M)u} \mathrm{d}u$$

$$= \frac{\ln\left[M - M_0 + (|\mathcal{F}| - M)e^{\lambda_f t}\right]}{|\mathcal{F}| - M}.$$



- Given M_t, H_t^c get H_t from accounting.

5. Firms' Expected Flow Profits and Match Delay

1. Firm j's expected flow profit from selling to consumer i is

$$\pi_{\mathcal{F}t}\hat{v}_{ij}$$

where

$$\pi_{\mathcal{F}t} := \frac{I}{\sigma M_t \mu_{H_t}}.$$

2. Firm j's Poisson rate of entry into consumer i's consideration set Ω_{it} is

$$\lambda_{et}(\hat{v}_{ij}) := \underbrace{\frac{NA}{|\mathcal{F}| - M_t}}_{\text{auction entry rate}} \underbrace{H_t^c(\hat{v}_{ij})^{N-1}}_{\text{win probability}}.$$

6. Bidding

Each firm bids according to

$$B_t(\hat{v}) = \int_0^{\hat{v}} \int_t^{\infty} \pi_{\mathcal{F}s} e^{-\int_t^s [r+\lambda_f+\lambda_{ez}(y)]\,\mathrm{d}z}\,\mathrm{d}s\,\mathrm{d}y, \quad \hat{v} \in \mathbb{R}_+.$$

Given B_t, the expected ad price is $\pi_{\mathcal{D}t} = \mathbb{E}\left[B_t\left(\hat{v}_{(2)}\right)\right]$.

In the limit as $\lambda_{et} \to \infty$ pointwise for all t, $\pi_{\mathcal{D}t} \to 0$ and we recover a classical economy.

7. Platform Investment and Quality

Platform k's investment ℓ_{kt} and quality q_{kt} solve the BV problem:

$$\dot{\ell}_{kt} = \frac{r+\delta}{1-\varphi}\ell_{kt} - \frac{\varphi}{1-\varphi}\frac{\pi_{\mathcal{D}t}A(\rho-1)}{|\mathcal{D}|q_{kt}}\ell_{kt}^{\varphi}$$

$$\dot{q}_{kt} = \ell_{kt}^{\varphi} - \delta q_{kt}$$

with boundary $\lim_{t\to\infty}\ell_{kt} = \ell^*$ where

$$\ell^* = \lim_{t\to\infty}\frac{\varphi\delta\pi_{\mathcal{D}t}A(\rho-1)}{|\mathcal{D}|(\delta+r)}$$

and initial condition $q_{k0} = q_0$.

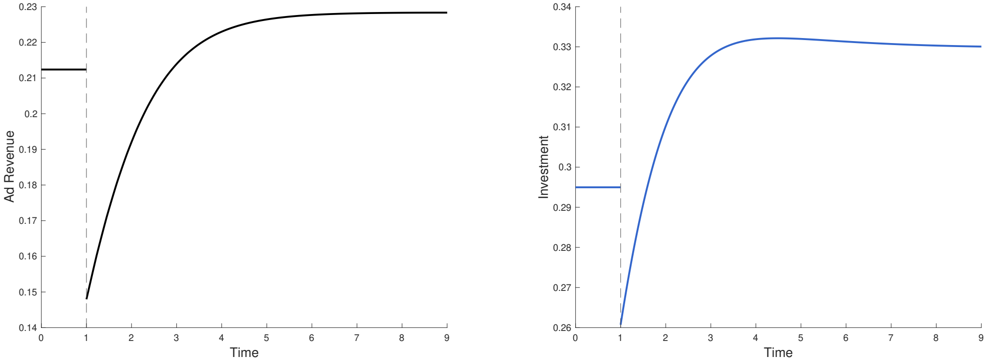
8. Surpluses

- CS: $U_i = \int_0^\infty e^{-rt} u(C_{it}, X_{it})\,\mathrm{d}t$ where
  1. $C_{it} = I(M_t \mu_{H_t})^{\frac{1}{\sigma - 1}}$
  2. $X_{it} = |\mathcal{D}|^{\frac{1}{\rho - 1}} \nu(A) q_t$
- FS: $\int_0^\infty e^{-rt} \left(I - \pi_{\mathcal{D}t} A\right) \mathrm{d}t$
- PS: $\int_0^\infty e^{-rt} \left(\pi_{\mathcal{D}t} A - |\mathcal{D}| \ell_t\right) \mathrm{d}t$

Comparative Statics

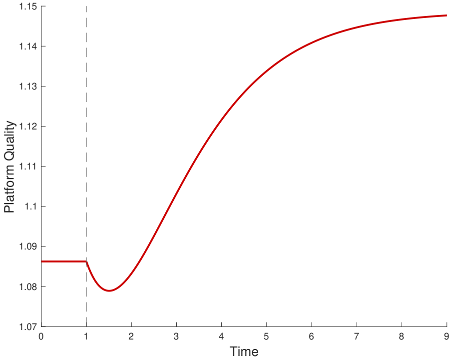
1. A Shock to Platform Substitutability

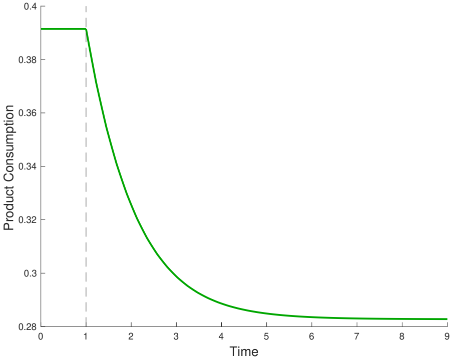
At $t = 1$, $\rho = 4/3 \rightsquigarrow \rho' = (1.01)(4/3)$...



Parameters: $G = \text{U}[0,1]$; $r = .1$; $\lambda_f = 1$; $D = .1$; $F = 1$; $\sigma = 3$; $N = 5$; $I = 1$; $\varphi = .5$; $\delta = .5$; $\nu(a) = (1 - .8395a^{.01})^{63.1472}$

1. A Shock to Platform Substitutability

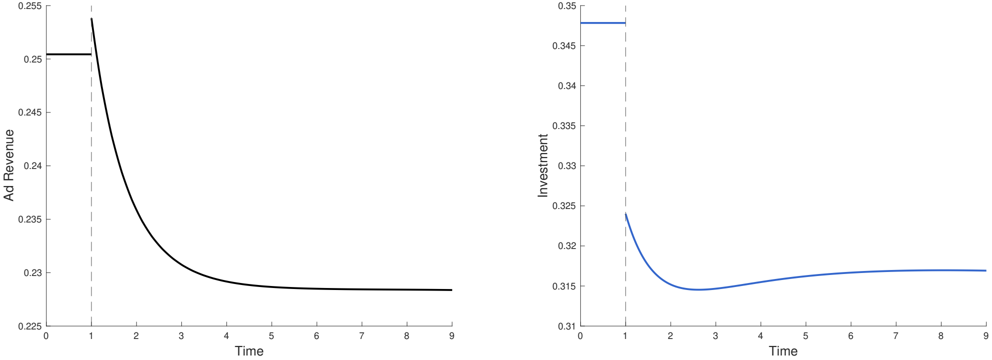




Parameters: $G = \text{U}[0, 1]$; $r = .1$; $\lambda_f = 1$; $D = .1$; $F = 1$; $\sigma = 3$; $N = 5$; $I = 1$; $\varphi = .5$; $\delta = .5$; $\nu(a) = (1 - .8395a^{.01})^{63.1472}$.

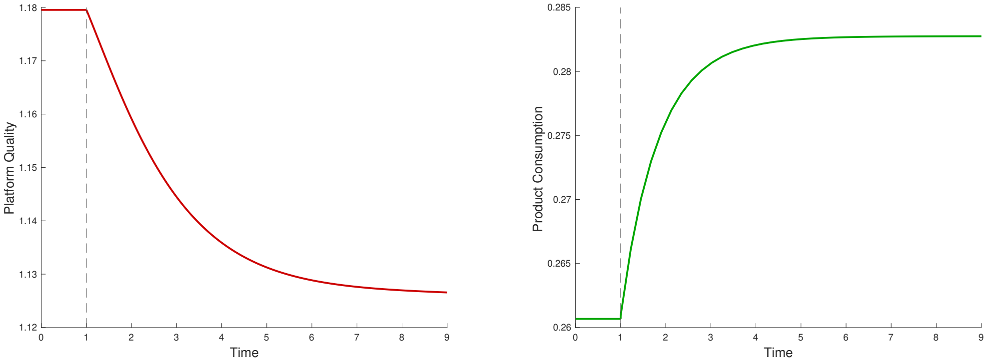
2. A Shock to Data

At $t = 1$, $G = U[.2, .8] \rightsquigarrow G' = U[0, 1]$...



Parameters: $r = .1$; $\rho = 4/3$; $\lambda_f = 1$; $D = .1$; $F = 1$; $\sigma = 3$; $N = 5$; $I = 1$; $\varphi = .5$; $\delta = .5$; $\nu(a) = 1 - 7.5a$.

2. A Shock to Data



Parameters: $r = .1$; $\rho = 4/3$; $\lambda_f = 1$; $D = .1$; $F = 1$; $\sigma = 3$; $N = 5$; $I = 1$; $\varphi = .5$; $\delta = .5$; $\nu(a) = 1 - 7.5a$.

Extended Model Matches Two Empirical Trends

Silk et. al (2021) document that

1. the market share of digital advertising has grown dramatically in the past decade (2010-2019) at the expense of traditional advertising.

2. advertising revenue as a fraction of GDP has historically been stable but has declined some in the past decade (2010-2019).

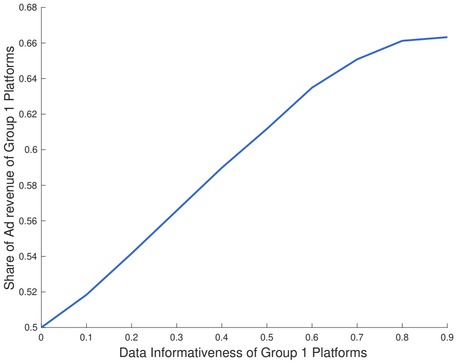
> *"Perhaps the most puzzling feature...is that the rapid growth of digital advertising has occurred over a period during which the share of U.S. economic activity (as measured by GDP) represented by total advertising expenditures has been in decline."*
>
> — Silk et. al (2021).

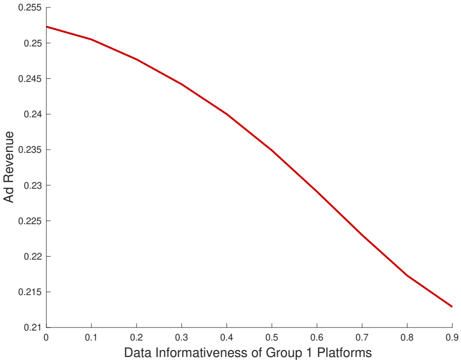
Extended Model Matches Two Empirical Trends

In the paper, I extend the analysis of steady state to a setup where:

- Consumers' values are log-normal: $v_{ij} = e^{Z_{ij}}$ where $Z_{ij} \sim N(0, .5)$.
- Two groups of platforms of equal size.
- Platforms in group 1 have signals $\zeta_{1ij} = Z_{ij} + \Delta\epsilon_{ij}$ where $\epsilon_{ij} \sim N(0, 2)$.
- Platforms in group 2 have $\zeta_{2ij} = Z_{ij} + \epsilon_{ij}$.
- Group 1 platforms are data rich ($\Delta < 1$) and represent digital platforms.
- Group 2 platforms represent traditional media.

Extended Model Matches Two Empirical Trends





Parameters: $I = 1$; $A = .01$; $\lambda_f = 1$; $F = .1$; $N = 20$; $\rho = 1.6$; $\sigma = 3$; $\varphi = .75$; $\nu(a) = 1 - 62.5a$.

Welfare Analysis

Extensions

Conclusion

Conclusion

- A new model of platform competition that emphasizes interactions among the different sides of the market including the product market.
- Takeaway: It is essential to have a broad market perspective
  1. seemingly intuitive comparative statics may flip
  2. the long run may look different from the short run
  3. there are nontrivial tradeoffs among the different market sides.
- Model's tractability and flexibility suggests potential for future work to build on it.

Thank you for your attention!

More on Bidding

Let $V_t^{\mathrm{In}}(\hat{v})$ denote firm j's continuation value from selling to a consumer i with $\hat{v}_{ij} = \hat{v}$ if it is in Ω_{it} and let $V_t^{\mathrm{Out}}(\hat{v})$ be defined analogously.

V_t^{In} and V_t^{Out} must solve the HJB equations:

$$\dot{V}_t^{\mathrm{In}}(\hat{v}) = rV_t^{\mathrm{In}}(\hat{v}) - \lambda_f \left[V_t^{\mathrm{Out}}(\hat{v}) - V_t^{\mathrm{In}}(\hat{v})\right] - \pi_{\mathcal{F}t}\hat{v}$$

$$\dot{V}_t^{\mathrm{Out}}(\hat{v}) = rV_t^{\mathrm{Out}}(\hat{v}) - \lambda_{et}(\hat{v})\left(V_t^{\mathrm{In}}(\hat{v}) - V_t^{\mathrm{Out}}(\hat{v}) - \mathbb{E}\left[B_t^{(1)} \middle| B_t(\hat{v}) > B_t^{(1)}\right]\right)$$

$$B_t(\hat{v}) = V_t^{\mathrm{In}}(\hat{v}) - V_t^{\mathrm{Out}}(\hat{v}).$$

These three equations can be solved explicitly for B_t.

More on a Platform's Problem

Platform k solves

$$\Pi_{\mathcal{P}} = \max_{\{a_{kt} \leq \bar{a}, \ell_{kt}\}} \int_0^{\infty} e^{-rt} \left(\pi_{\mathcal{P}t} a_{kt} x_{kt}(a_{kt}, q_{kt}) - \ell_{kt} \right) \mathrm{d}t$$

subject to $\dot{q}_{kt} = \ell_{kt}^{\varphi} - \delta q_{kt}$ given $q_{k0} = q_0$.

More on Welfare Analysis

- If $u(C, X) = C^{1-\tau} X^{\tau}$ where τ is the weight on platform consumption, then the deviation of equilibrium investment from first best is increasing in

$$\frac{\sigma}{\sigma - 1} \frac{\pi_{\mathcal{P}} A}{I} (\rho - 1) - \frac{\tau}{1 - \tau}.$$

- The only terms in this condition are: product markups, ad revenue, income, platform substitutability, and τ.
- Ad revenue and income are endogenous, but suggests a sufficient statistics approach to gauge market efficiency.

More on Network Effects

- The effective quality of a platform k is now $\eta(x_{kt})\nu(a_{kt})q_{kt}$ where η is increasing.
- In equilibrium, the attention that a consumer pays to platform k is

$$x_{kt} = \frac{[\eta(x_{kt})\nu(a_{kt})q_{kt}]^{\rho-1}}{\int_{\mathcal{D}}[\eta(x_{zt})\nu(a_{zt})q_{zt}]^{\rho-1}\mathrm{d}z}.$$

- To solve for x_{kt} explicitly, assume that $\eta(x) = x^{\zeta}$ where $\zeta > 0$.
- For each subset $\mathcal{E}_t \subset \mathcal{D}$ of positive measure, there is a solution that sets

$$x_{kt} = \frac{[\nu(a_{kt})q_{kt}]^{\frac{\rho-1}{1-\zeta(\rho-1)}}}{\int_{\mathcal{E}_t}[\nu(a_{zt})q_{zt}]^{\frac{\rho-1}{1-\zeta(\rho-1)}}\mathrm{d}z}$$

if $k \in \mathcal{E}_t$ and otherwise sets $x_{kt} = 0$. Unique equilibrium in which $\mathcal{E}_t = \mathcal{D}$ at all t.

More on Related Literature

1. Platforms and two-sided markets
   - Jullien et al. 2021; Bergemann, Bonatti, and Gan 2019; Bergemann and Bonatti 2023; Prat and Valletti 2021; Anderson and Coate 2005...
2. Advertising
   - Anderson and Coate 2005; survey by Bagwell 2007...
3. Ad auctions
   - Edelman et al. 2007; Athey and Ellison 2011; Varian 2007; Board 2009; Bergemann, Heumann, et al. 2021; Hummel and McAfee 2016...
4. Competing auctions
   - Wolinsky 1988; McAfee 1993...