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Fiscal Theory of the Price Level (FTPL) without and with Bubbles

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Key Issue in Monetary Eocnomics: Price Level Determination

- Fundamental questions in monetary economics:
  - what determines the price level (the value of money)?
  - how do policy choices affect the price level/inflation?
- Classic monetarist answer: money supply and demand

  $\rightarrow$ monetary authority can determine price level/inflation by setting money supply
- Issues with this answer:
  1. Sargent-Wallace 1981: fiscal considerations may matter
     - monetary authority may be forced to create money to supply seigniorage revenues
  2. in many modern models: money is endogenous due to interest rate policy
     - leads to *nominal indeterminacy*: no nominal anchor that determines the price level

Fiscal Theory of the Price Level (FTPL)

- FTPL points out systematic link between fiscal policy and nominal goods prices
  - for a government that issues nominal debt denominated in its own currency
  - and is committed to not default on nominal liabilities (this can be relaxed)
- If fiscal policy is conducted in a certain way, can render the price level determinate
- But even more generally: FTPL relationship always present in macro models

  $\rightarrow$ there a important fiscal requirements for "monetary" policy goals such as price stability
- Sims (1994): *"In a fiat-money economy, inflation is a fiscal phenomenon, even more fundamentally than it is a monetary phenomenon."*
- This Lecture
  - Introduction to basic FTPL
  - Connection with Sargent, Wallace (1981)
  - FTPL with a Bubble (Brunnermeier, Merkel, Sannikov 2023)

Outline

Outline

Simple AK Economy Model: Preferences, Technology, Market Clearing

- Household preferences ($i \in [0, 1]$)

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right]$$

- Each agent i manages capital k_t^i
  - production flow $y_t^i dt = k_t^i dt$
  - capital tax levied by government $\tau_t k_t^i dt$ (equivalent to lump-sum tax)
  - no investment, no depreciation, no idiosyncratic risk, $dk_t^i = k_t^i d\Delta_t^{k,i}$
  - traded on capital markets at (real) price q_t^K
- Aggregates and market clearing
  - normalize $K_t := \int k_t^i di = 1$
  - goods market clearing $C_t := \int c_t^i di = \int y_t^i di =: Y_t = 1$

Simple AK Economy Model: Government

- Government issues nominal bonds
  - nominal face value $\mathcal{B}_t$, evolution $d\mathcal{B}_t = \mu_t^{\mathcal{B}} \mathcal{B}_t dt$
  - initial debt $\mathcal{B}_0 > 0$ given (state variable of the model)
  - pays (floating) interest i_t
  - real value $q_t^B := \mathcal{B}_t / \mathcal{P}_t$

- Interest paid with new bonds or taxes τ_t

- Nominal budget constraint

$$i_t \mathcal{B}_t = \mu_t^{\mathcal{B}} \mathcal{B}_t + \mathcal{P}_t \tau_t \qquad \Rightarrow \qquad \mu_t^{\mathcal{B}} = i_t - \frac{\tau_t}{q_t^B}$$

- A *feasible government policy* specifies i_t, $\mu_t^{\mathcal{B}}$, τ_t – possibly contingent on histories $\{q_s^B, q_s^K\}_{s=0}^t$ – such that this constraint always holds

  example: fix $i_t, \tau_t \geq 0$, adjust $\mu_t^{\mathcal{B}}$ to satisfy the constraint

Model Solution: Household Problem and Optimal Choice Conditions

- Household i chooses $\{c_t^i\}$, $\{\theta_t^i\}$ to maximize utility subject to net worth evolution

$$dn_t^i = -c_t^i dt + n_t^i \left(\theta_t^i dr_t^{\mathcal{B}} + (1-\theta_t^i) dr_t^K\right)$$

  with returns

$$dr_t^{\mathcal{B}} = \left(\frac{\tau_t}{q_t^B} + \mu_t^{q,B}\right) dt \qquad dr_t^K = \left(\frac{1-\tau_t}{q_t^K} + \mu_t^{q,K}\right) dt$$

- Optimal consumption choice:
  - consume constant fraction of wealth, $c_t^i = \rho n_t^i$
- Optimal portfolio choice (no arbitrage): for interior θ_t^i

$$dr_t^{\mathcal{B}} = dr_t^K \Leftrightarrow \mu_t^{q,B} - \mu_t^{q,K} = \frac{1-\tau_t}{q_t^K} - \frac{\tau_t}{q_t^B}$$

  - relates expected appreciation of relative price q_t^B / q_t^K to difference in payout yields

Optimal Portfolio Choice in Equilibrium

- Rewriting portfolio choice in terms of $\vartheta_t := q_t^B/(q_t^B + q_t^K)$:

$$\mu_t^\vartheta = \rho \frac{\vartheta_t - \tau_t}{\vartheta_t}$$

- Integrating forward in time and imposing asset market clearing ($\vartheta = \theta$)

$$\theta_t = \vartheta_t = \int_t^\infty \rho e^{-\rho(s-t)} \tau_s ds$$

- *In words*: equilibrium portfolio weight on bonds is a weighted average of future taxes
  - *Remark*: in more general model, need to replace τ_t with ratio of primary surpluses to output
- Example: $\tau_t = \tau$ constant; then $\vartheta_t = \tau$ for all t

Remark: Portfolio Choice and Debt Valuation

- The portfolio choice condition

$$\vartheta_t = \int_t^\infty \rho e^{-\rho(s-t)} \tau_s ds$$

is equivalent to a debt valuation equation

$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = q_t^B = \int_t^\infty \exp\left(-\int_t^s r_u du\right) \tau_s ds$$

- To derive latter: multiply by $q_t^B + q_t^K$ plus some algebra
- Interpretation: households willing to absorb any amount of bonds as long as they expect sufficient future taxes to back them

Price Level Determination

- Have just seen: fiscal policy (τ_t) affects portfolio choice ($\theta_t = \vartheta_t$) and thus *relative* asset valuations (q_t^B / q_t^K)
- What determines *level* of asset prices q_t^K, q_t^B?

  $\rightarrow$ consumption-savings choice and wealth effects (& goods market clearing)

  - goods market clearing:

$$1 = C_t = \rho(q_t^B + q_t^K) = \rho \frac{q_t^B}{\vartheta_t}$$

  - solving for q_t^B

$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = q_t^B = \frac{\vartheta_t}{\rho}$$

- This is a condition for the equilibrium price level $\mathcal{P}_t$

  (because $\mathcal{B}_t$ is a pre-determined state variable)

Interpretation: Portfolio Choice can Determine the Price Level

- Previous result suggests: portfolio choice can determine the price level when there are nominal assets
- Economic logic, for given ϑ_t
  - $\mathcal{P}_t$ too high $\rightarrow$ total wealth q_t^B/ϑ_t too low $\rightarrow$ insufficient goods demand $\rightarrow$ price level falls
  - $\mathcal{P}_t$ too low $\rightarrow$ total wealth q_t^B/ϑ_t too high $\rightarrow$ excess goods demand $\rightarrow$ price level rises
- Key to this logic: some asset value is fixed in nominal terms (here bonds)
- Also: logic may break down if ϑ_t reacts to $\mathcal{P}_t$ (because future τ_ts do) (will consider this issue later)

Fiscal Theory of the Price Level (FTPL)

- Nominal government bonds and tax policy play a key role in previous argument
  - tax policy determines portfolio choice and relative asset prices (ϑ_t)
  - q_t^B and total wealth are inversely linked to $\mathcal{P}_t$ because bonds are nominal
- The idea is therefore called the *Fiscal Theory of the Price Level* (FTPL)

Equilibrium Selection

- Discussion so far: focus on a given equilibrium
  - equilibrium equations that pin down the price level
    (portfolio choice and goods market clearing)
  - economic story about the equilibrating mechanism
    (wealth effects from nominal government bonds)
- Focus of most of the FTPL literature:
  - is the equilibrium unique?
  - or, if not: is there at least a unique price level prediction across equilibria?
- Whether FTPL can be used as a selection device depends on fiscal policy rule
  - "active" fiscal policy: taxes do not react (strongly) to stabilize debt
    $\rightarrow$ FTPL selects a unique price level
  - "passive" fiscal policy: taxes rise in response to rising debt levels
    $\rightarrow$ FTPL cannot select a unique price level

(active/passive terminology is due to Leeper 1991)

Illustration: Uniqueness with Active Fiscal Policy

- Example of active fiscal policy: constant $\tau_t = \tau$
  (and some specification for i_t and $\mu_t^{\mathcal{B}}$ consistent with gov. budget constraint)

- Have seen before: portfolio choice and asset market clearing imply $\vartheta_t = \tau$

- Goods market clearing implies

$$\mathcal{P}_t = \frac{\rho}{\vartheta_t}\mathcal{B}_t = \frac{\rho}{\tau}\mathcal{B}_t,$$

  $\rightarrow \mathcal{P}_t$ is uniquely determined as function of $\mathcal{B}_t$

- $\mathcal{B}_0$ is a given state, so this determines $\mathcal{P}_0$ uniquely

- Can show further: also evolution of $\mathcal{B}_t$ and thus $\mathcal{P}_t$ uniquely determined

Illustration: Indeterminacy with Passive Fiscal Policy

- Example of a passive fiscal policy: $\tau_t = \alpha/\rho \cdot \vartheta_t$, $\alpha > 0$
  (similar to $\tau_t = \alpha q_t^B$, which is more natural but requires more algebra)

- Equation for ϑ_t then becomes

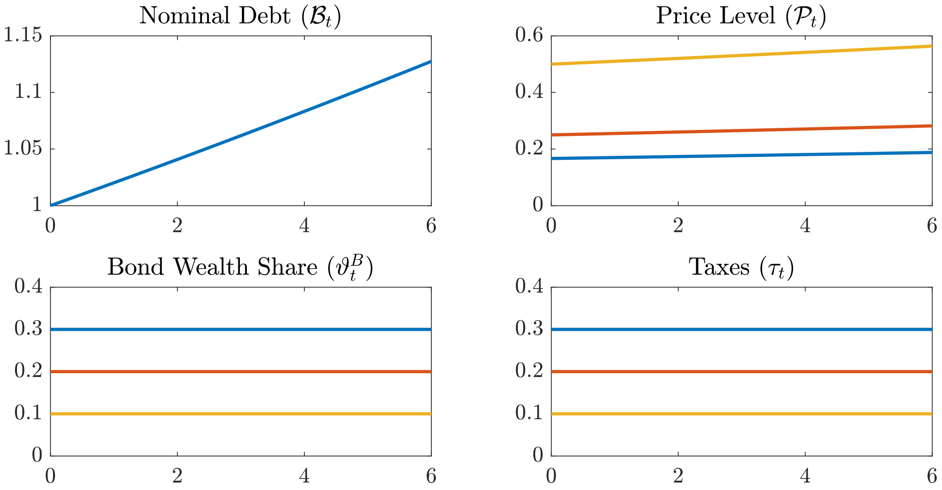
$$\vartheta_t = \int_t^\infty \rho e^{-\rho(s-t)} \tau_s ds = \int_t^\infty \alpha e^{-\rho(s-t)} \vartheta_s ds$$

  - this has many solutions: $\vartheta_t = \vartheta_0 e^{(\rho-\alpha)t}$ for any ϑ_0
  - all these solutions are valid because $e^{-\rho t}\vartheta_t \to 0$ ("transversality condition")

- Corollary: for this fiscal policy *any* initial portfolio weight ϑ_0 and price level $\mathcal{P}_0$ are consistent with some equilibrium

- *But note*: different equilibria are associated with different predictions for fiscal variables

Passive Policy with $\alpha = \rho$: Three Equilibria



What Is Special about Government Bonds?

- How is government debt special? What about other nominal assets in the economy?
  (e.g. private bonds, bank deposits, etc.)

  1. aggregate wealth effects require that these assets represent *net wealth* for private sector
  2. for uniqueness: an "active" policy is only feasible for the government
     - government's nominal debt represents liability to something it can create
     - it does not need to expend real resources to honor this liability
     - all other agents must expend real resources to service their nominal debt

- *Remark*: however, the following are *not* relevant for observation 2
  - that the government is a large player
  - that the government can tax other agents
  - that taxes are payable in money

Outline

Relationship to Sargent and Wallace (1981)

- Sargent and Wallace (SW) point out that *"even in an economy that satisfies monetarist assumptions [...] monetary policy cannot permanently control [...] inflation"*
  - they consider an economy in which $\mathcal{P}$ is fully determined by money demand ($v\mathcal{M} = \mathcal{P}Y$)
  - but the fiscal authority is "dominant": sets *deficits* independently of monetary policy actions
- SW emphasize seigniorage from money creation
  - fiscal needs determine the total present value of *seigniorage*
  - if monetary authority provides less now, it will be forced to provide more later
- Similarity with FTPL: SW also emphasize importance of fiscal policy for inflation
- Differences to FTPL:
  - seigniorage plays important role in SW but irrelevant for FTPL
  - FTPL about tax backing (primary surpluses), SW about funding deficits (negative surpluses)
  - SW about consistency of policy choices along an equilibrium path (no off-equilibrium actions)
  - price level determination in SW based on money demand, doesn't work with *i*-policy

Illustrating SW: Model Extension with Money

- Add money as a third asset to the model
  - nominal quantity $\mathcal{M}_t$, evolution $d\mathcal{M}_t = \mu_t^{\mathcal{M}} \mathcal{M}_t dt$
  - initial stock $\mathcal{M}_0 > 0$ given, $\mu_t^{\mathcal{M}} \geq 0$ controlled by monetary authority
  - does not pay interest
  - real value $q_t^M := \mathcal{M}_t / \mathcal{P}_t$
- Households face a payment constraint in production: $v m_t^i \geq \mathcal{P}_t y_t^i$ $(v > \rho)$

  (as in Merkel (2020) – isomorphic to consumption cash-in-advance constraint but formally simpler)
  - if binding, then $\mathcal{P} = v\mathcal{M}$ in the aggregate $\rightarrow$ tight link between money and price level
- Monetary authority transfers seigniorage $\sigma_t := \mu_t^{\mathcal{M}} q_t^M$ to fiscal authority
- Budget constraint of fiscal authority

$$i_t \mathcal{B}_t = \mu_t^{\mathcal{B}} \mathcal{B}_t + \mathcal{P}_t \tau_t + \mathcal{P}_t \sigma_t \Rightarrow \mu_t^{\mathcal{B}} = i_t - \frac{\tau_t + \sigma_t}{q_t^B}$$

$\rightarrow$ as before if we replace τ_t with $\tau_t + \sigma_t$

Model Solution for Binding Payment Constraint

- Let's assume that in equilibrium
  1. the payment constraint is always binding
  2. taxes satisfy $\tau_t = -\delta$, $\delta \geq 0$ (constant deficit)

- Then nominal wealth shares must satisfy

$$
\begin{aligned}
\vartheta_t^M &:= \frac{q_t^M}{q_t^M + q_t^B + q_t^K} = \rho / v \\
\vartheta_t^B &:= \frac{q_t^B}{q_t^M + q_t^B + q_t^K} = \int_t^\infty \rho e^{-\rho(s-t)} \left(\tau_s + \sigma_s\right) ds \\
&= \int_t^\infty \rho e^{-\rho(s-t)} \sigma_s ds - \delta
\end{aligned}
$$

A Fiscally Dominant Regime

- Suppose after some time $T < \infty$ the fiscal authority can take control of $\mu_t^{\mathcal{M}}$
- It chooses seigniorage to keep debt constant, i.e.

$$\sigma_t = \hat{\sigma}(\vartheta_T^B) := \delta + \vartheta_T^B, \qquad t \geq T$$

(there are limits on feasible seigniorage but let's ignore this for simplicity)

- For $t \leq T$, the monetary authority chooses (constant) $\mu^{\mathcal{M}}$ independently
  - then also $\sigma_t = \mu^{\mathcal{M}} q_t^M = \mu^{\mathcal{M}}/v =: \sigma$ is controlled by the monetary authority
- Proposition ("unpleasant arithmetic"): tight money now means higher inflation eventually
  - specifically: the (constant) inflation rate over $[T, \infty)$ is strictly decreasing in μ^M over $[0, T]$

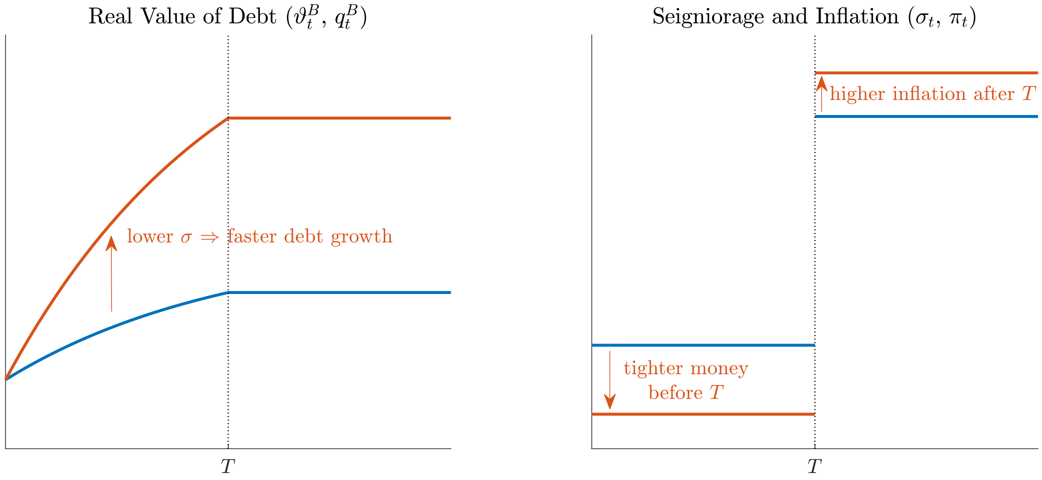
Why Does the Sargent-Wallace Proposition Hold?

- Iterating government budget constraint forward in time and dividing by total wealth yields

$$\vartheta_T^B = \vartheta_0^B + \int_0^T \rho e^{-\rho t}(\delta - \sigma)dt$$

- Tigher money over $[0, T]$ $\Rightarrow$ lower seigniorage transfers σ $\Rightarrow$ debt grows faster
- Higher debt at T: need larger seigniorage thereafter to cover interest payments
  - recall $\hat{\sigma}(\vartheta_T^B) = \delta + \vartheta_T^B$ is increasing in ϑ_T^B

Illustration of Unpleasant Arithmetic



Monetary Dominance

- Suppose $T = \infty$: monetary authority is always in control of the money supply
- Is there an equilibrium? (suppose also $\sigma \neq \vartheta_0^B + \delta$)
  - not with constant deficit $\tau_t = -\delta$
  - but: a constant deficit is not necessarily feasible policy
- Two cases
  1. if $\sigma > \vartheta_0^B + \delta$, $\tau_t = -\delta$ remains feasible
     - but fiscal authority will absorb money over time, effective money supply is smaller than $\mathcal{M}_t$
     - fiscal authority controls inflation (e.g. if real debt is kept constant, outcomes as if $\sigma = \vartheta_0^B + \delta$)
  2. if $\sigma < \vartheta_0^B + \delta$, τ_t has to rise to avoid default on nominal bonds
     - fiscal authority effectively faces an "intertemporal budget constraint"
     - e.g. smallest constant tax is $\tau = \vartheta_0^B - \sigma$
- *Remark*: Here, gov. debt is like real/foreign currency debt – very different from FTPL

Outline

FTPL with Government Debt Bubble

- When $r \leq g$, rational bubbles may appear, including on government debt
- The valuation equation
$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = \mathbb{E}_t[PV(\text{primary surpluses})]$$
emphasized by the FTPL no longer makes sense: may have a bubble term
- In addition: can raise seigniorage from "mining the bubble" even without money demand
- Questions:
  - (how) does the FTPL work in such environments?
  - how can the government ensure that the bubble is attached to its debt?
  - (in paper: when is bubble mining welfare-improving?)

Modified Model with Idiosyncratic Risk

- Return to the model without money, but introduce idiosyncratic risk
  - capital k_t^i of household i evolves according to

$$\frac{dk_t^i}{k_t^i} = \underbrace{d\Delta_t^{k,i}}_{\text{trading}} + \underbrace{\tilde{\sigma} d\tilde{Z}_t^i}_{\text{idio. shocks}}$$

  - then bonds represent safe assets (provide service flows from re-trading)
- If σ sufficiently large: rational bubbles may emerge
  - $r = \rho - (\tilde{\sigma}^c)^2$ lowered by precautionary savings motive
  - for $(\tilde{\sigma}^c)^2 \geq \rho$, we obtain $r \leq 0 (= g)$, so that bubbles can exist

Does the FTPL still work?

- Recall: two FTPL ingredients emphasized previously
  1. fiscal policy affects portfolio demand for nominal government debt (relative asset prices)
  2. wealth effects on goods market determine price level (level of asset prices)
- Both ingredients are still present here
- Portfolio choice in this model yields

$$\vartheta_t = \underbrace{\int_t^\infty \rho e^{-\rho(s-t)} \tau_s ds}_{\text{“cash flow”}} + \underbrace{\int_t^\infty \rho e^{-\rho(s-t)} \frac{(1-\vartheta_s)^2 \tilde{\sigma}^2}{\rho} \vartheta_s ds}_{\text{“service flow”}}$$

$\rightarrow \vartheta_t$ jointly determined by fiscal policy and safe asset demand

- Link between ϑ_t and $\mathcal{P}_t$ as before

$$\mathcal{P}_t = \frac{\rho}{\vartheta_t} \mathcal{B}_t$$

FTPL as a Selection Device

- Previous discussion: FTPL link between fiscal policy and price level *in any given equilibrium* is still present
- But can fiscal policy also resolve equilibrium multiplicity?
  - two sources of multiplicity: (1) bubble multiplicity; (2) nominal indeterminacy
  - FTPL arguments can resolve both
  - off-equilibrium fiscal backing is sufficient
  - but requires credibility and fiscal capacity to promise off-equilibrium surpluses (otherwise: vulnerability to bubble crashes)

Resolving Equilibrium Multiplicity

- For "active" fiscal policy with $\tau > 0$:
  - there is a unique solution for ϑ (as in $\tilde{\sigma} = 0$-case)
  - standard FTPL argument applies: unique $\mathcal{P}_t$ consistent with equilibrium
  - but then $r > g$ and there is no bubble in equilibrium
- Resolving multiplicity with an equilibrium bubble:
  - more challenging: continuum of bubble values consistent with the same tax path

    $\Rightarrow$ exogenous tax sequence insufficient for uniqueness
    (and exogenous negative τ is also not feasible policy)
  - contingent policy can select the bubble equilibrium
    - primary deficits on the equilibrium path (bubble mining)
    - switch to $\tau > 0$ if inflation breaks out

Some Formal Details: Differential Equation for Bond Wealth Share

- Differential version of previous portfolio choice equation (ODE)

$$\dot{\vartheta}_t = f(\vartheta_t)\vartheta_t - \rho\tau_t, \qquad f(\vartheta) := \rho - (1-\vartheta)^2\tilde{\sigma}^2$$

- Note that $f(\vartheta)$ is strictly increasing in $\vartheta \in [0,1]$
- Interpretation: conceptually, this is a backward equation
  - which expectations about path for ϑ_t rationalize today's value?
  - only paths that remain in $[0,1]$ can be consistent with an equilibrium
- Let's next contrast two simple policies:
  - a. constant $\breve{\mu}^{\mathcal{B}}$ ($\breve{\mu}^{\mathcal{B}}_t := \mu^{\mathcal{B}}_t - i_t$), implies $\tau_t = -\breve{\mu}^{\mathcal{B}} \frac{\vartheta_t}{\rho}$

    $\rightarrow$ continuum of equilibria with $\vartheta_t \in [0, \vartheta^*]$ and $\dot{\vartheta}_t \leq 0$
  - b. threshold policy: constant $\breve{\mu}^{\mathcal{B}}$ if $\vartheta_t \geq \underline{\vartheta}$, constant *positive* taxes $\tau > 0$ otherwise

    $\rightarrow$ unique equilibrium with $\vartheta_t = \vartheta^*$

Illustration: Constant $\breve{\mu}^{\mathcal{B}}$

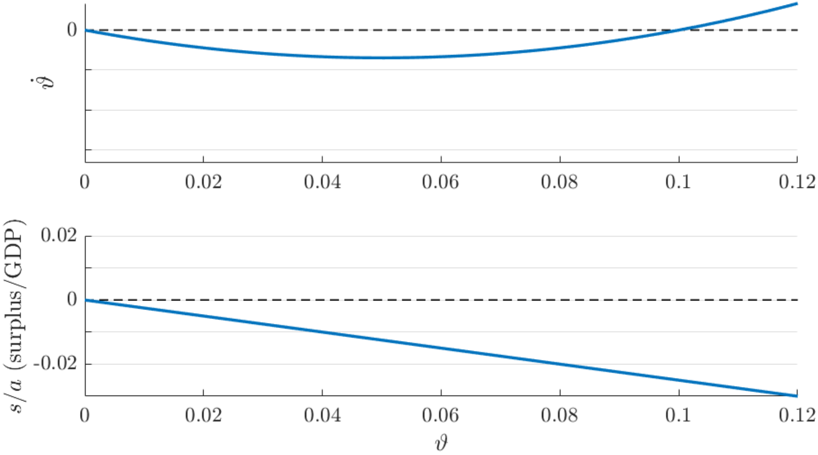
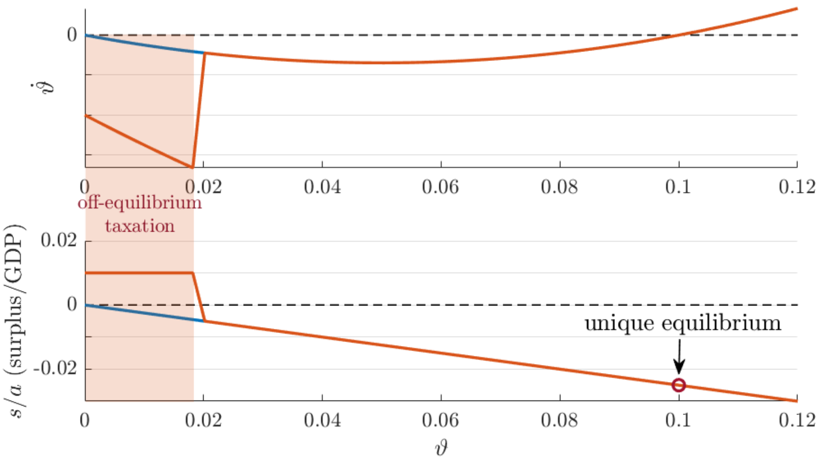


Illustration: Constant $\breve{\mu}^{\mathcal{B}}$ & Threshold Policy



Additional Uniqueness Arguments in Paper

1. Equilibrium selection still works in presence of alternative bubbly assets
   - households can also trade "cryptocoins" instead of government bonds to self-insure against idiosyncratic risk
2. Equilibrium selection can work even under limited commitment
   - even government with a short horizon wants to tax to create safe assets when ϑ is low
   - hence, off-equilibrium taxation may be credible even if governments can't commit
3. Alternative policies to defend the bubble:
   - insolvency law, restrictions on using alternative assets, financial repression
   - these can complement off-equilibrium tax backing

Summary

- Fiscal Theory of the Price Level:
  - links portfolio demand for *nominal* government bonds to the general price level
  - within given equilibrium: relationship between fiscal variables and price level
  - equilibrium selection: "active" policy can render the nominal side of the economy determinate
- Relationship to Sargent Wallace (1981)
  - also about fiscal-monetary linkages but focuses on seigniorage from money
  - focused on restrictions on joint fiscal-monetary policy along equilibrium path
- FTPL with a bubble
  - bubble mining: government can extract seigniorage from debt growth directly
  - FTPL mechanisms still present
  - uniqueness requires off-equilibrium tax backing