Debt, deficits and inflation

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Motivation for the paper

- Blanchard (2019), in his presidential address, explored the implications of persistent low interest rates on government debt.

- If the real interest rate on government debt remains below the growth rate of the economy, debt can increase steadily, with no increase in primary surpluses, and with no increase in the debt to GDP ratio.

But there is a tradeoff nonetheless

- In a perfect foresight equilibrium model, the transversality of private agents who discount the future at the rate β generally delivers something like this:

$$b_t = \int_0^\infty e^{-\beta s}(\text{revenue}_{t+s} - \text{spending}_{t+s} + \text{seigniorage}_{t+s})\, ds\ .$$

This is derived from the government's current budget constraint and private agent's transversality condition (an implication of the private sector's optimizing behavior). The seigniorage is the gap between the agent's discount rate and the return on government debt, times the amount of government debt. It represents a kind of tax: spending foregone by the private agent or agents in order to hold government debt rather than some other asset.

The Friedman rule

- Friedman argued that, since fiat money can be produced at zero cost, it must be optimal to expand the amount outstanding until the real return on money and on other assets is the same. "Saturate the demand for money"

- That we are assuming a non-zero nominal interest rate on what we call B does not alter Friedman's logic.

Super-simple model where Friedman's rule is questionable

$$\text{GBC}: \quad \frac{-\dot{M}}{P} = \tau = -\frac{\dot{M}}{M}\frac{M}{P}$$

The rate of return on real balances is $-\dot{P}/P$, which is likely in steady state to match $-\dot{M}/M$. To provide a positive real return on real balances requires positive taxes. If saturating demand requires high M/PC, the required taxes may be large. If τ is distorting, might a zero-tax, fixed M be better?

Implications of the tradeoff

- What allows "zero cost" debt is just the gap between the return on debt and the discount rate. Whether there is real growth or not does not matter for the existence of the tradeoff.

- And the cost is not zero. It shows up in seigniorage.

- It might or might not be a good idea to finance spending by increasing seigniorage rather than direct taxes.

Three related papers

- Woodford (1990), in one of the many models in his survey, shows that in a model similar to that in this paper there can be a steady state in which, because other available taxes are distortionary, it is optimal to make use of the inflation tax. That is, an optimal steady state preserves a gap between the rate of return on money and the discount rate.

- Calvo (1978) shows that in a model where it is feasible to implement a steady state with the rate of return on money matching the discount rate (i.e., "implement the Friedman rule"), an optimizing benevolent government nonetheless chooses to create inflation early, raise taxes later, and converge to a steady state where the Friedman rule is not implemented.

- Chari and Kehoe (1999) show in a class of models that includes that in this paper, so long as real balances and consumption enter utility or the transactions technology homothetically (meaning that their relative marginal utilities depend only on their ratio), the Friedman rule is always optimal.

Results in this paper, in relation to Chari and Kehoe

Though quite a few economists I've talked to thought Chari and Kehoe had shown that homotheticity by itself implies optimality of the Friedman rule, and thus that Woodford and Calvo's examples rely on non-homotheticity, this is not true. Chari and Kehoe's result relies on assuming the government can acquire privately issued bonds that pay a higher return than government bonds, so that increased debt can be financed without increased total tax revenue.

Relation to Woodford and Calvo

- This paper's model lies within the class that Woodford showed could imply use of seigniorage in an optimal steady state, and is in line with Woodford's conclusion. Unlike Woodford's example, here we impose homotheticity, to show that Woodford's result does not depend on non-homotheticity. We also investigate existence and uniqueness of equilibrium more thoroughly.

- In our different model, we verify Calvo's conclusion that a benevolent planner, constrained to rely on distorting taxes but able to credibly commit to the path of future policy, chooses to have initial high inflation and low taxes, and does not converge to a steady state in which the Friedman rule holds.

A simple equilibrium model with a liquidity premium on B and distorting tax

- Government debt in the budget constraint, as providing transaction services, so that it is return-dominated.
- Only input is labor, L.
- Just one tax: proportional tax on labor.
- Just one kind of government liability: nominal, duration zero.
- Government purchase or expropriation of private sector assets is not possible.

Private sector

$$\max_{C,L,B} \int_0^\infty e^{-\beta t}(\log C_t - L_t)\, dt$$

subject to

$$C \cdot (1 + \gamma v) + \frac{\dot{B}}{P} = (1 - \tau)L + \frac{rB}{P}$$
$$v = \frac{PC}{B}\,.$$

Private FOC's

$$\partial C: \qquad \frac{1}{C} = \lambda \cdot (1 + 2\gamma v)$$

$$\partial L: \qquad -1 = -\lambda \cdot (1 - \tau)$$

$$\partial B: \qquad \frac{-\dot{\lambda} + \lambda\beta + \lambda \dot{P}/P}{P} = \frac{r\lambda}{P} + \frac{\lambda\gamma v^2}{P}$$

$$TVC: \qquad \frac{\lambda B}{P} e^{-\beta t} = \frac{B e^{-\beta t}}{(1-\tau)P} \xrightarrow[t \to \infty]{} 0$$

Equation system

Solving private FOC's to eliminate λ:

$$C \cdot (1 + 2\gamma v) = 1 - \tau$$

$$\frac{\dot{C}}{C} + \frac{2\gamma \dot{v}}{1 + 2\gamma v} = \gamma v^2 - \beta - \frac{\dot{P}}{P} + r$$

Adding GBC and SRC:

$$SRC: \qquad C \cdot (1 + \gamma v) + G = L$$

$$GBC: \qquad \frac{\dot{B}}{P} + \tau L = G + \frac{rB}{P}\,.$$

Solving the GBC forward

To combine the GBC with the private TVC to get a valid equiibrium condition, we need to rearrange the GBC so that β appears in it (while at the same time writing it in terms of real debt $b = B/P$):

$$\dot{b} + b\frac{\dot{P}}{P} + \tau L = G - \frac{(\beta - r)B}{P} + \beta\frac{B}{P}\,.$$

We'll call the gap between the discount rate and the return on government debt, times real debt, seigniorage, σ. Then we can invoke the private TVC to solve the GBC forward as

$$b_t = \int_0^\infty e^{-\beta s}(\tau_{t+s}L_{t+s} + \sigma_{t+s} - G_{t+s})\,ds$$

Optimal steady state results

- Even when there is no need to tax, because $G = 0$, the optimal steady state involves some taxation in order to increase the real value of the debt and thereby reduce transactions costs.

- The amount of revenue raised by seigniorage is very small, even when, as in the $G = .8$ examples, the optimal steady state involves fairly high inflation.

- If the tax rate τ is held constant, then when equilibrium exists $(1 + \tau > 2G)$, it is unique and a steady state.

- There is no transition path: an unanticipated jump to a new permanent τ or a new γ creates a price level jump that implements a jump to the new steady state.

Example numerical solutions for optimal steady state

γ	C	b/y	L	$\dot{P}/P$	U	τ	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.001	0.97	1.31	0.97	0.0006	-1.00	0.03	-0.0254	0.0007	0.76
0.01	0.94	2.70	0.94	0.0012	-1.01	0.05	-0.0507	0.0035	0.37
0.1	0.87	5.27	0.88	0.0027	-1.02	0.10	-0.0912	0.0162	0.19
1	0.72	9.27	0.78	0.0061	-1.10	0.17	-0.1293	0.0722	0.11

Table 1: Optimal steady state with $G = 0$, $\beta = .02$, $i = .02$

γ: transactions cost parameter; C: consumption; b/y: debt/output; L: labor; $\dot{P}/P$: inflation rate; U: utility; τ: labor tax rate; σ: seigniorage revenue; $\gamma v/(1+\gamma v)$: proportion of consumption expenditure absorbed by transaction costs; P_0 : initial price level, assuming $B_0 = 1$; G non-productive government expenditure; β: discount rate.

Example numerical solutions for optimal steady state

γ	C	b/y	L	$\dot{P}/P$	U	τ	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.001	0.74	0.28	0.99	0.0069	-1.29	0.26	-0.0037	0.0026	3.56
0.01	0.72	0.83	0.98	0.0076	-1.30	0.27	-0.0103	0.0086	1.21
0.1	0.67	2.19	0.94	0.0107	-1.34	0.29	-0.0234	0.0296	0.46
1	0.55	4.68	0.86	0.0137	-1.46	0.32	-0.0295	0.1047	0.21

Table 2: Optimal steady state with $G = .25$, $\beta = .02$. $i = .02$

See notes to Table 1

Example numerical solutions for optimal steady state

γ	C	b/y	L	$\dot{P}/P$	U	τ	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.001	0.20	0.02	1.00	0.0823	-2.63	0.80	0.0013	0.0090	46.27
0.01	0.19	0.06	0.99	0.0855	-2.66	0.80	0.0042	0.0284	15.55
0.1	0.17	0.17	0.98	0.0955	-2.78	0.80	0.0129	0.0890	5.87
1	0.12	0.37	0.96	0.1137	-3.06	0.79	0.0342	0.2522	2.74

Table 3: Optimal steady state with $G = .8$, $\beta = .02$, $i = .02$

See notes to Table 1

Comparing to non-optimal steady states

G	τ	C	b/y	L	$\dot{P}/P$	U	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.20	0.22	0.77	1.00	0.97	0.0059	-1.238	-0.0142	0.0076	1.00
0.25	0.27	0.72	0.83	0.98	0.0076	-1.304	-0.0103	0.0086	1.21
0.20	0.27	0.73	2.51	0.93	0.0008	-1.247	-0.0481	0.0029	0.40
0.25	0.22	0.71	0.15	1.00	0.2281	-1.336	0.0310	0.0456	6.71

Table 4: Optimal and suboptimal financing of G

See notes to Table 1 for variable definitions. Lines 1 and 2 show solutions with optimal tax rates for the given G values. Lines 3 and 4 are solutions for given G and τ, with no optimization. Comparing lines 1 and 4 shows the change in going from $G = .20$ with optimal τ to $G = .25$ with unchanged τ. Comparing lines 2 and 3 shows the reverse case.

Comparing steady states with constant tax rate τ

- A sudden increase in γ, implying an increased demand for liquidity, will, with τ fixed, require sudden and large deflation, even though in this flexprice model it has no real effect.

- To avoid the sudden deflation, the government would have to run a very temporary, very large, flow deficit, financing a wealth transfer to the private sector, so that B/P can increase without a decrease in P.

- A permanent increase in G, with no accompanying increase in τ, requires a corresponding increase in seigniorage, which may require very high inflation and a large increase in the fraction of output absorbed by liquidity services.

The credibly commiting, optimizing planner

- If the representativce agent economy actually has identical agents, all facing a constant hazard rate of death, with replacement agents endowed with the government debt of the dying ones, and if they discount only because of the death rate, then arguably the planner should not discount, and the optimal steady state is optimal policy.

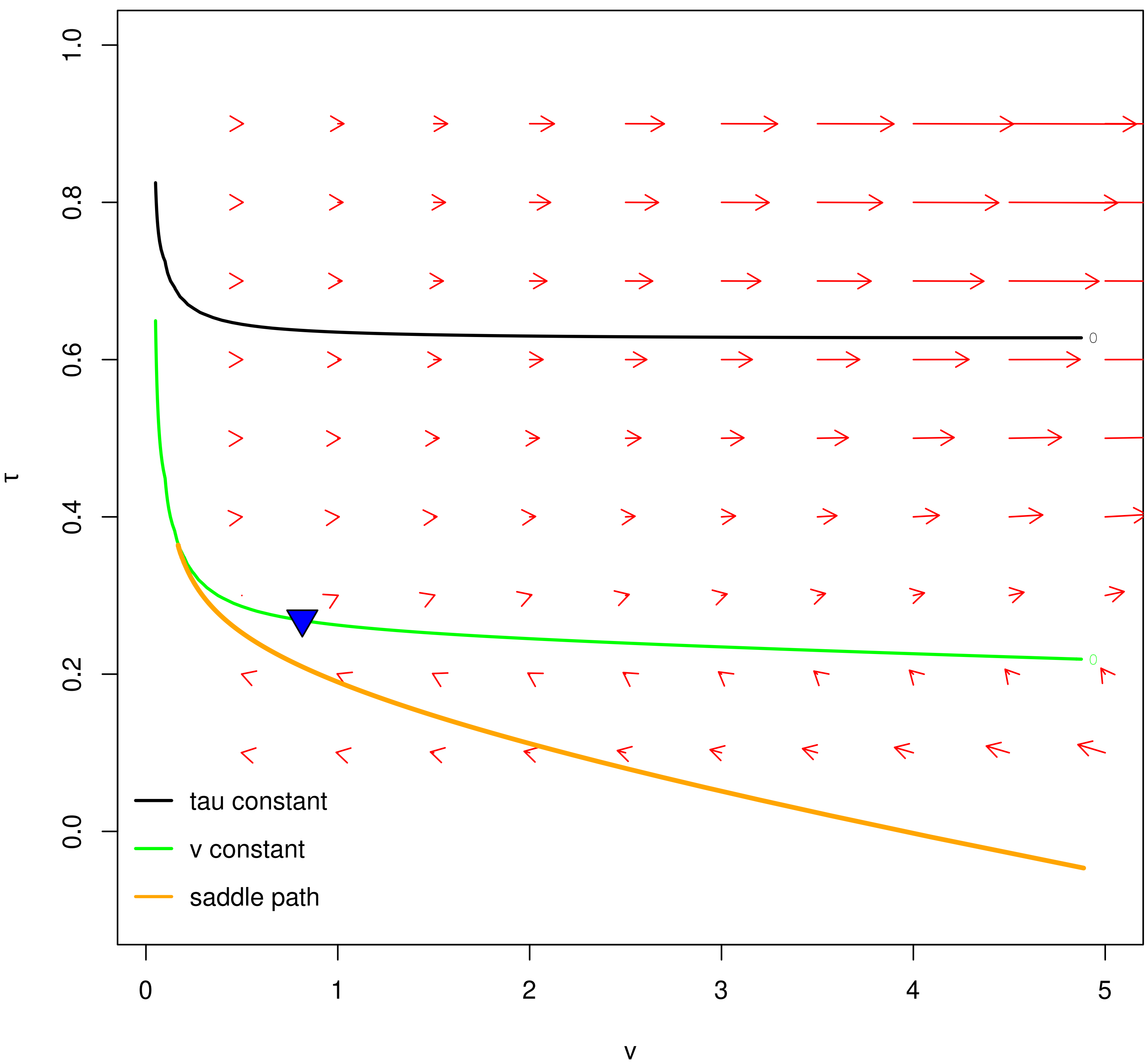
- But if the planner adopts the objective function of the representative agent, and therefore discounts at the rate β, The solution is quite different from the optimal steady state.

How to formulate and solve the problem

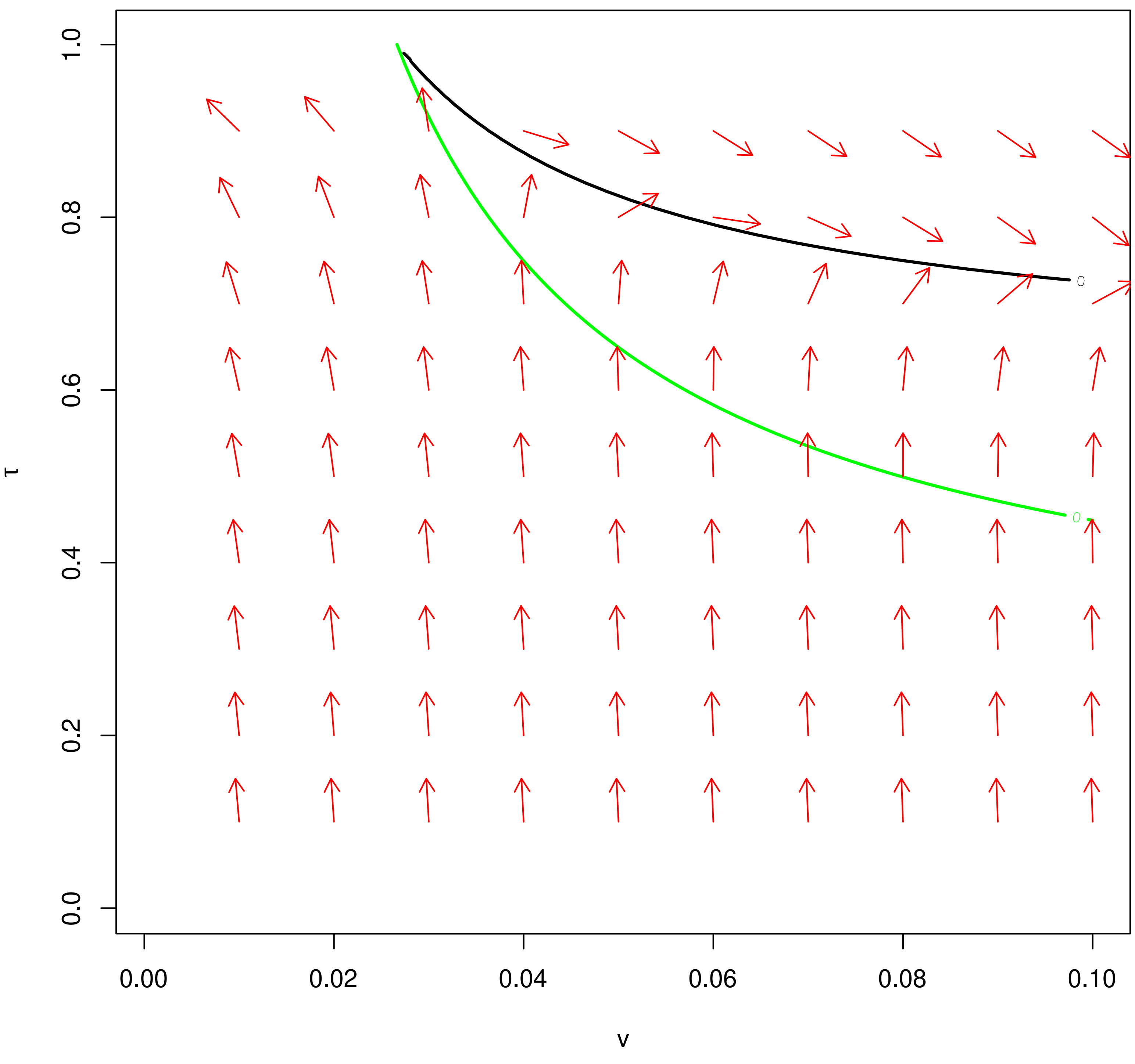
- The private sector's choices, given current and future τ and b, can be boiled down to a single differential equation expressing $\dot{v}$ as a function of v and τ.

- The planner's problem then maximizes the agent's objective function, taking this differential equation as the state equation and τ as the control variable. The planner with full commitment power can choose the initial value of v freely.

- Setting up the problem as a Hamiltonian dynamic system, one can derive the optimum path.



1.0
0.8
0.6
0.4
0.2
0.0
τ
0
1
2
3
4
5
v
0
0
tau constant
v constant
saddle path



1.0
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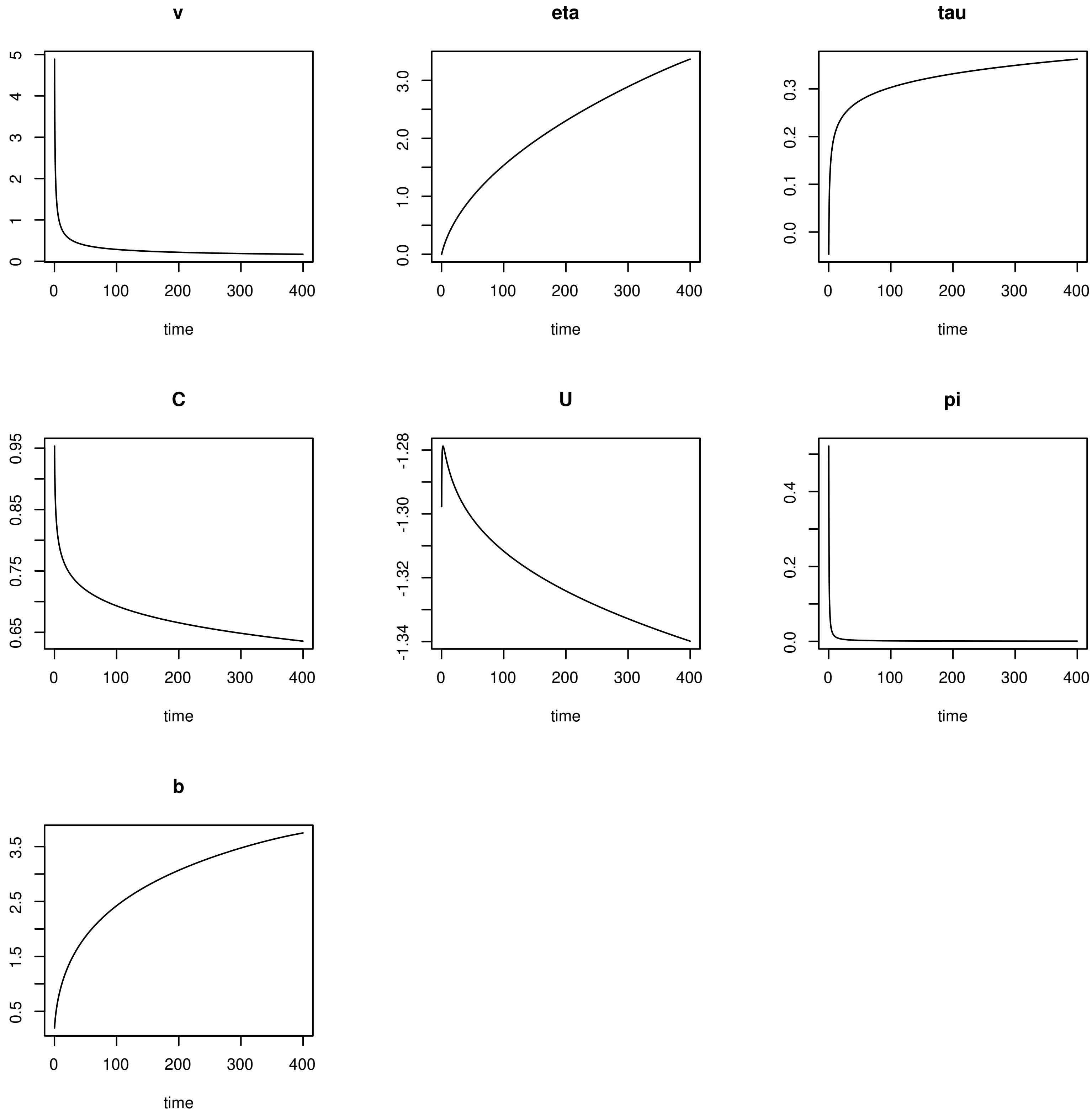
v
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0.3
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0.75
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0.95
-1.34
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-1.30
-1.28
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1.5
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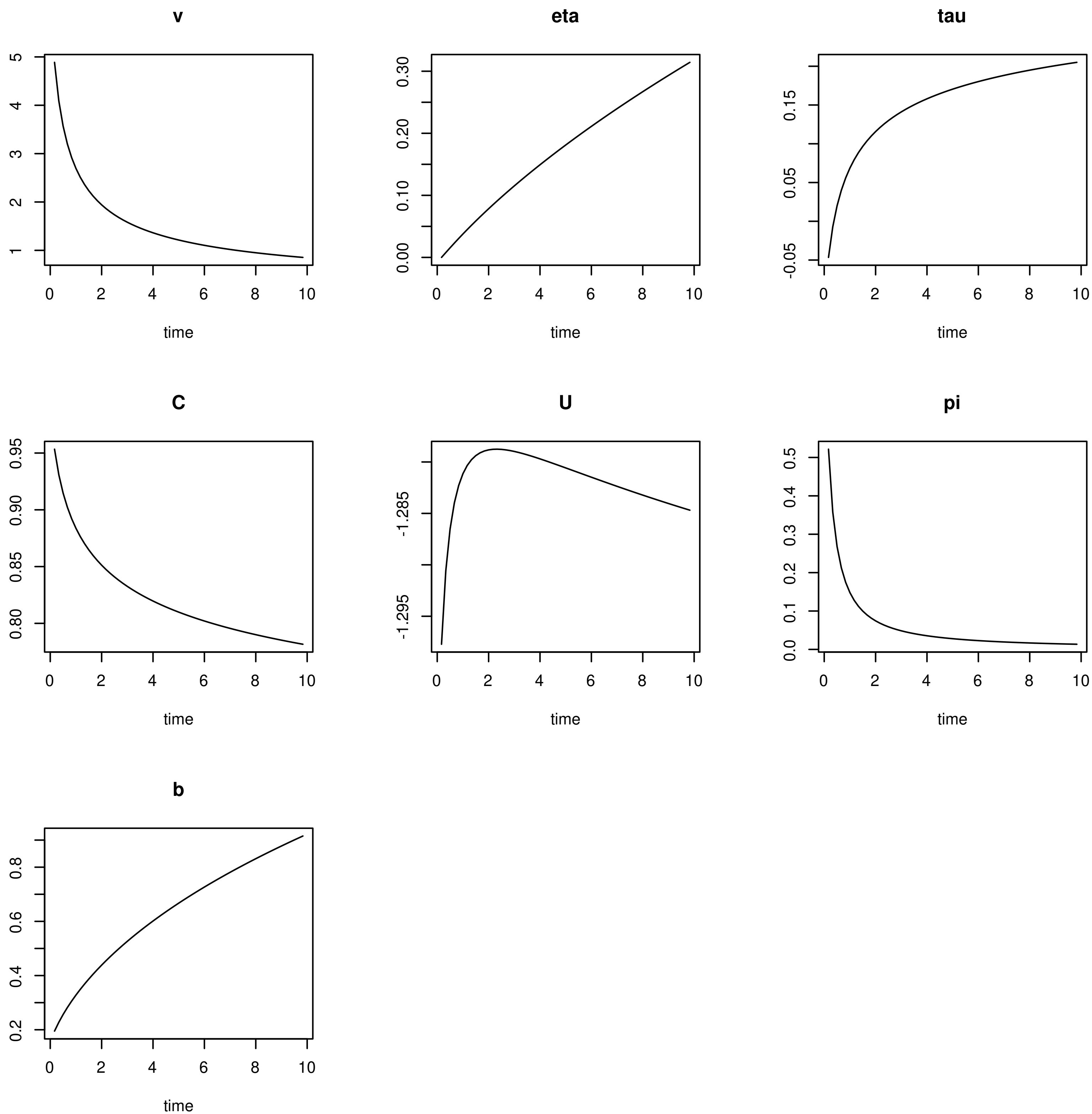
3.5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0 2 4 6 8 10
1 2 3 4 5
0.00 0.10 0.20 0.30
-0.05 0.05 0.15
0.80 0.85 0.90 0.95
-1.295 -1.285
0.0 0.1 0.2 0.3 0.4 0.5
0.2 0.4 0.6 0.8

Why?

- So long as the liquidity premium on government liabilities shrinks toward zero as debt increases relative to consumption, postponing taxation requires higher taxation later.

- But the need for compensating taxation later grows only at the real rate on government debt, not at the discount rate β.

- In a constant-τ equilibrium, it will at the margin be welfare improving to tax less today, somewhat more later.

- Therefore constant-τ can't deliver a full commitment equilibrium.

- Thus there is always an optimal τ_0 and a policy maker who sees current $\tau \neq \tau_0$ will be tempted to "restart" by jumping to τ_0.

Conclusion: Lessons from this exercise

- Thinking about fiscal-monetary policy interactions as analogous to choice of excise tax rates in a static framework is misleading.

- The analogy is not so misleading if we constrain choices to comparing steady states, but the incentives to deviate from a steady state are perhaps the central problem in setting policy.

- Sudden shifts in demand for liquidity can lead, with optimal policy, to correspondingly sudden shifts in the price level and/or the amount of outstanding debt.

- High debt to income ratios are sustained by high taxes and low inflation.

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