Princeton Initiative: Macro, Money, and Finance 2022
Comparing Monetary Models

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Some Questions

- What determines the price level and/or inflation?
- How does interest rate policy affect the economy?
- When and how are fiscal policy and government debt relevant for inflation?
- What is the role of equilibrium multiplicity and expectations coordination?
- What is the role of portfolio choice between nominal government liabilities and other assets?

This lecture: analyze these questions through the lens of three monetary models

Lecture partially (but very loosely) based on Li, Merkel (2022), "Flight to Safety in a New Keynesian Model"

Outline

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Baseline Model: AK Economy

- Household preferences ($i \in [0, 1]$)

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right]$$

- Each agent i manages capital k_t^i
  - production flow $y_t^i dt = a_t k_t^i dt$
  - no investment, no depreciation
  - traded on capital markets at (real) price q_t^K
- Aggregates and market clearing
  - normalize $K_t := \int k_t^i di = 1$
  - goods market clearing $C_t := \int c_t^i di = \int y_t^i di =: Y_t$

Equilibrium Solution

This model is trival to solve:

- Market clearing

$$C_t = a_t$$

- Log utility consumption rule $c_t^i = \rho q_t^K k_t^i$ tells us capital price

$$q_t^K = \frac{a_t}{\rho}$$

- Can recover interest rate from household Euler equation (and $c_t^i \propto C_t$):

$$\mathbb{E}_t[dC_t] = (r_t - \rho)\, C_t dt \qquad \Rightarrow \qquad r_t = \rho + \mu_t^a =: r_t^*$$

Accommodating Price Setting Frictions

- Later, want to study a version with nominal goods prices, possibly sticky
- Need two features to accommodate price setting frictions

1. elastic short-term supply (within dt-period)
    - at "wrong" prices, goods demand may be excessive or insufficient
    - markets can only clear if supply can adjust within the period

    $\rightarrow$ introduce variable capital utilization

2. individual price-setting firms cannot face perfectly elastic demand
    - Walrasian market: each agent faces a flat demand curve (price taker)
    - no meaningful price setting problem: $p + \varepsilon$: no demand, $p - \varepsilon$: infinite demand

    $\rightarrow$ introduce differentiated goods and monopolistic competition
    (but eliminate other distortions this creates with subsidy & profit redistribution)

Extended Model: Setup

- Household preferences ($i \in [0,1]$)

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\log c_t^i - \frac{\left(u_t^i\right)^{1+\nu}}{1+\nu}\right) dt\right]$$

- CES demand for goods

$$Y_t = \left(\int \left(y_t(j)\right)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \qquad \Rightarrow \text{demand for variety } j: \quad y_t(j) = \left(p_t(j)\right)^{-\epsilon} Y_t$$

- Household i rents out effective capital $\hat{k}_t^i = u_t^i k_t^i$ to firms at unit rental price p_t^R
- Firm j:
  - production function $y_t(j) = a_t \hat{k}_t(j)$
  - time-t profits

$$\Pi_t(j) = (1+\tau) p_t(j) y_t(j) - p_t^R \hat{k}_t(j)$$

  - profits net of subsidy payments redistributed to households in proportion to k_t^i holdings

Extended Model: Solution

- Firm price setting problem:
  - constant markup over unit marginal cost

$$p_t(j) = \frac{1}{1+\tau}\frac{\epsilon}{\epsilon - 1}\frac{p_t^R}{a_t}$$

  - in equilibrium: $p_t(j) = 1$ for all j, so this determines p_t^R
  - if $\tau = \frac{1}{\epsilon - 1}$, $p_t^R = a_t$ (assume this from now on)
- Household utilization decision:
  - first-order condition: $p_t^R k_t^i / c_t^i = \left(u_t^i\right)^\nu$
  - in equilibrium: $p_t^R = u_t^{1+\nu} a_t \Rightarrow u_t = 1 =: u^*$
- Conclusion: identical equilibrium as in baseline model

Remark: Isomorphic to Model with Labor

- Can relabel things:
  - utilization $u_t \rightarrow$ labor ℓ_t
  - rental price $p_t^R \rightarrow$ wage w_t
  - get rid of capital (or call it labor productivity)
- Then this is the real counterpart of a standard New Keynesian textbook model (e.g. Gali 2015)
- Why the (unconventional) capital formulation?
  - closer to other models you see this weekend
  - matters for safe asset model later (can trade capital, but not labor)

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A "Monetary" Version of the Same Model

Let's introduce dollars as a pure unit of account

- $\mathcal{P}_t > 0$ dollar price of aggregate good at time t
- limit attention to deterministic time paths

$$d\mathcal{P}_t = \pi_t \mathcal{P}_t dt$$

We connect dollars to our real economies in two ways

1. goods firms have to quote their prices in dollars ($\mathcal{P}_t(j)$)
   - by itself inconsequential: can still implement any real $p_t(j)$ by setting $\mathcal{P}_t(j) = p_t(j)\mathcal{P}_t$
   - does not impose any restrictions on equilibrium $\mathcal{P}_t$ paths without price setting frictions
2. add nominal bond in zero net supply
   - bond return

$$dr_t^{\mathcal{B}} = i_t dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = (i_t - \pi_t)\, dt$$

   - i_t set externally by policy (e.g. central bank)
   - in a very broad sense, generates a store of value role for dollars

Solving the "Monetary Model" (not in the most efficient way)

- Deriving the "IS equation"
  - start with the Euler equation (and $c_t^i = C_t$ to simplify notation)

$$\mathbb{E}_t[dC_t] = (r_t - \rho)\, C_t dt$$

  - $C_t = u_t a_t$ by goods market clearing, thus

$$\frac{\mathbb{E}_t[dC_t]}{C_t dt} = \frac{\mathbb{E}_t[du_t]}{u_t dt} + \frac{da_t}{a_t dt} = \frac{\mathbb{E}_t[du_t]}{u_t dt} + \mu_t^a$$

  - portfolio choice between real and nominal bonds yields the Fisher equation

$$r_t = i_t - \pi_t$$

  - combining the three (recall $r_t^* = \rho + \mu_t^a$)

$$\mathbb{E}_t[du_t] = (i_t - \pi_t - r_t^*)\, u_t dt$$

Solving the "Monetary Model" – continued (not in the most efficient way)

- Three equations for key variables $\mathcal{P}_t$, u_t, π_t

$$\begin{aligned} \mathbb{E}_t[d\mathcal{P}_t] &= \pi_t \mathcal{P}_t dt && \text{price level evolution} \\ \mathbb{E}_t[du_t] &= \left(i_t - \pi_t - r_t^*\right) u_t dt && \text{IS equation} \\ u_t &= u^* = 1 && \text{optimal price setting \& utilization} \end{aligned}$$

  - third equation fully determines u_t, implies $du_t = 0$
  - second equation determines π_t

$$\pi_t = i_t - r_t^*$$

  - $\mathcal{P}_0$ not pinned down by these equations (or any other equilibrium condition)

- Easy to recover rest of the model solution

$$p_t^R = a_t, \qquad C_t = a_t, \qquad q_t^K = \frac{a_t}{\rho}, \qquad r_t = r_t^*$$

Solution of the "Monetary Model"

- Real side: same as real model
- Nominal side:
  - inflation effectively determined by Fisher equation
  - $\mathcal{P}_0$ undetermined

→ For any given interest rate path $\{i_t\}_{t\geq 0}$ there is a continuum of equilibria indexed by $\mathcal{P}_0 \in (0, \infty)$

What is the Mechanism behind Fisher Equation Inflation Determination?

- Two interpretations how increase in i_t raises inflation:
  (also mixture between the two possible)
  1. Current price $\mathcal{P}_0$ drops
     - agents coordinate expectations (somehow) on some future price level $\mathcal{P}_T$
     - higher i_t depresses demand today $\rightarrow$ firms lower prices $\rightarrow$ $\mathcal{P}_0$ falls
  2. Future prices $\mathcal{P}_t$ rise
     - todays price remains fixed, higher interest rates moves up expectations of future $\mathcal{P}_t$
     - (somehow) behavior in future periods validates these beliefs
- This model does not really provide an answer
  - derivation based on IS equation (backward equation) suggestive of first interpretation
  - but there is no economic argument (within this model) why higher interest rates could not coordinate expectations on higher future prices

Outline

Adding Sticky Prices

- Sticky prices (quadratic adj. costs) replaces $u_t = 1$ with New Keynesian Phillips curve

$$\frac{\mathbb{E}_t\left[d\pi_t\right]}{dt} = \rho\pi_t - \kappa\left(\frac{p_t^R}{a_t} - 1\right) = \rho\pi_t - \kappa\left(u_t^{1+\nu} - 1\right)$$

- Adds a second forward-looking equation to the system
- Simpler to analyze, but similar conclusions: static Phillips curve

$$\pi_t = \kappa\left(u_t^{1+\nu} - 1\right)$$

$\rightarrow$ will work with this version here

- *Aside*: can "microfound" this in one of two (crude) ways
  1. let firms' time horizon and price stickiness go to zero in the right proportion
  2. assume that adjustment costs are relative to (lagged) aggreate price

Equilibrium in the Sticky Price Model

- Three equations for key variables $\mathcal{P}_t$, u_t, π_t

$$
\begin{aligned}
d\mathcal{P}_t &= \pi_t \mathcal{P}_t dt && \text{price level (state) evolution} \\
\mathbb{E}_t[du_t] &= \left(i_t - \pi_t - r_t^*\right) u_t dt && \text{IS equation} \\
\pi_t &= \kappa \left(u_t^{1+\nu} - 1\right) && \text{Phillips curve}
\end{aligned}
$$

- Remaining quantities can be backed out from static relationships

$$
C_t = u_t a_t, \qquad p_t^R = u_t^{1+\nu} a_t, \qquad r_t = i_t - \pi_t, \qquad q_t^K = u_t a_t / \rho
$$

- Differences from flexible price equilibrium:
  - $\mathcal{P}$ is now a state variable with given initial state $\mathcal{P}_0$
  - but IS equation plus Phillips curve leaves room for multiple equilibrium inflation rate paths (there is no terminal/transversality condition)

Equilibrium Multiplicity

- Let the (bounded) path of nominal rates $\{i_t\}_{t\geq 0}$ be given
- Substituting Phillips curve into IS equation yields

$$\mathbb{E}_t[du_t] = \left(i_t - \kappa\left(u_t^{1+\nu} - 1\right) - r_t^*\right) u_t dt$$

- Missing terminal condition leads to continuum of (bounded) solution paths:
  - fix expectation of u_T (or π_T) at some (arbitrary) time T
  - can solve backward: there is a unique solution path consistent with that expectation
  - but this works for any $u_T \in (0, \infty)$ (or any $\pi_T \in (-\infty, \infty)$)
  - note: there is also a unique forward solution after time T and because of the negative feedback, none of these explodes at $t \to \infty$

→ Conclusion: one-dimensional continuum of equilibria
can be indexed by expected inflation $\pi_T \in (-\infty, \infty)$ at some future date T

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Interest Rate Policy

- Recall combined IS-Phillips equation

$$\mathbb{E}_t[du_t] = \left(i_t - \kappa\left(u_t^{1+\nu} - 1\right) - r_t^*\right) u_t dt$$

- Suppose we fix ("anchor") expectations of future π_T (and thus u_T)
    - higher interest rates i_t over $i \in [0, T]$ raise expected growth rate of u_t
    - as u_T is fixed, this means a lower u_t path

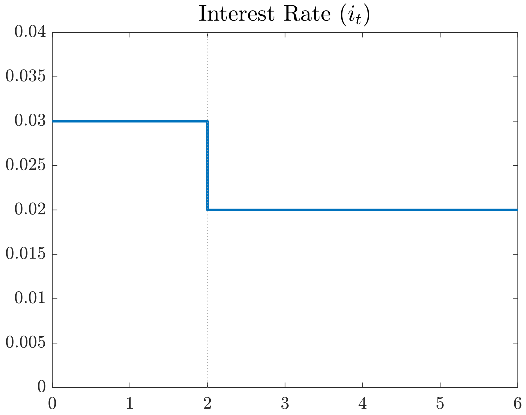
  $\rightarrow$ for fixed π_T expectation, higher interest rates cause output and inflation to fall

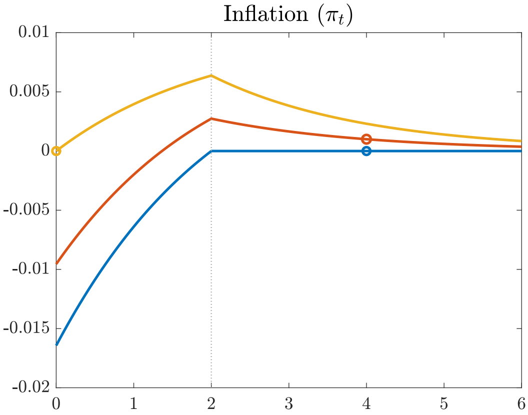
- But issue: could also use a different equilibrium selection

  e.g. higher interest rates cause π_T to drift upwards

  $\rightarrow$ neo-Fisherian conclusion that higher interest rates raise inflation

Effects of Interest Rate Hike





Taylor Rules

- Suppose next that policy follows a feedback rule

$$i_t = i_t^0 + \phi \pi_t, \qquad \phi \geq 0$$

- Plugging into combined IS-Phillips equation

$$\mathbb{E}_t[du_t] = \left(i_t^0 - (\phi - 1)\kappa \left(u_t^{1+\nu} - 1\right) - r_t^*\right) u_t dt$$

- How does this affect the solution structure?

$\phi < 1$ no change, continuum of bounded solutions

$\phi > 1$ all but one solutions are unbounded, unique bounded solution is "locally unique"

- If we limit attention to locally unique solutions, Taylor principle ($\phi > 1$) can select a unique equilibrium

- This even works if Taylor principle is only followed eventually (after some time T)

Do Taylor Rules Refine the Set of Equilibria?

- We have seen that given an equilibrium path $\{i_t\}_{t\geq 0}$ there is a continuum of equilibria
- Can any of these equilibria be selected by a suitable Taylor rule?
  - $\rightarrow$ Yes, pick any $\phi > 1$, $i_t^0 = i_t - \phi\pi_t$
- Thus, the criterion "can be selected by some Taylor rule" does *not* refine the set of possible equilibria

Summary: Inflation and Monetary Policy in this Model

- Inflation is guided by a purely forward-looking equation
- Monetary policy (i_t policy) is about managing expectations to implement a desired equilibrium
- There are two (logical) dimensions to this:
  1. anchor inflation expectation π_T at some future date T (e.g. by Taylor rule)
  2. choose (expected) interest rate sequence over $[0, T]$ to move private sector demand in way consistent with desired π_0 (and u_0)
- If future expectations are anchored, raising interest rates has the conventional effects
  - lower inflation
  - reduced economic activity (lower u_t)
- Optimal policy analysis: interest rate policy can implement the first best
  - set the interest rate on the equilibrium path to the natural rate, $i_t = r_t^*$
  - use equilibrium selection to select the zero inflation equation

  $\rightarrow$ leads to $r_t = r^*$, $\pi_t = 0$, $u_t = u^*$ (divine coincidence)

Outline

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Modified Model Setup with Nominal Government Debt

- Government issues nominal bonds
  - nominal face value $\mathcal{B}_t$, evolution $d\mathcal{B}_t = \mu_t^{\mathcal{B}} \mathcal{B}_t dt$
  - pays (floating) interest i_t
  - real value $q_t^B := \mathcal{B}_t / \mathcal{P}_t$
- Interest paid with new bonds or taxes τ_t on capital (equivalent to lump-sum tax)

$$i_t \mathcal{B}_t = \mu_t^{\mathcal{B}} \mathcal{B}_t + \mathcal{P}_t \tau_t K_t = \mu_t^{\mathcal{B}} \mathcal{B}_t + \mathcal{P}_t \tau_t \qquad \Rightarrow \qquad i_t = \mu_t^{\mathcal{B}} + \frac{\tau_t}{q_t^B}$$

- Household net worth evolves according to

$$dn_t^i = -c_t^i dt + \theta_t^i dr_t^{\mathcal{B}} + (1 - \theta_t^i) dr_t^K$$

with returns

$$dr_t^{\mathcal{B}} = (i_t - \pi_t)\, dt = \left(\frac{\tau_t}{q_t^B} + \mu_t^{q,B} \right) dt$$

$$dr_t^K = \left(\frac{p_t^R a_t u_t^i + (1 - p_t^R) a u_t - \tau_t}{q_t^K} + \mu_t^{q,K} \right) dt$$

Portfolio Choice between Bonds and Capital

- Imposing no arbitrage (portfolio choice) and $u_t = u_t^i$ yields

$$\frac{\tau_t}{q_t^B} + \mu_t^{q,B} = \frac{a_t u_t - \tau_t}{q_t^K} + \mu_t^{q,K}$$

$$\Rightarrow \mu_t^{q,B} - \mu_t^{q,K} = \frac{a_t u_t}{q_t^K} - \left(\frac{\tau_t}{q_t^B} + \frac{\tau_t}{q_t^K}\right)$$

- In terms of $\vartheta_t := q_t^B/(q_t^B + q_t^K)$, $\hat{\tau}_t := \tau_t/Y_t = \tau_t/(u_t a_t)$

$$\mu_t^{\vartheta} = \rho\left(1 - \vartheta_t^{-1}\hat{\tau}_t\right)$$

- Integrating forward in time yields for ϑ_{t_0} ($= \theta_{t_0}^i$ in equilibrium)

$$\vartheta_{t_0} = \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \rho e^{-\rho(t-t_0)}\hat{\tau}_t dt\right]$$

- *In words*: equilibrium portfolio weight on bonds is a (expected) weighted average of future surplus-output ratios

Portfolio Choice and Debt Valuation

- The portfolio choice condition

$$\vartheta_{t_0} = \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \rho e^{-\rho(t-t_0)}\hat{\tau}_t dt\right]$$

is equivalent to a debt valuation equation

$$\frac{\mathcal{B}_{t_0}}{\mathcal{P}_{t_0}} = q^B_{t_0} = \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \exp\left(-\int_{t_0}^{t} r_s ds\right)\tau_t dt\right]$$

- To derive latter: multiply by $q^B_{t_0} + q^K_{t_0}$ plus some algebra
- Interpretation: households willing to absorb any amount of bonds as long as they expect sufficient future primary surpluses to back them
- Note: first equation does not depend (explicitly) on interest rates or inflation, only on future surplus-output ratios $\hat{\tau}_t$

Price Level Determination under Flexible Prices

- Consider flexible price model ($\kappa \to \infty$ & $\mathcal{P}_0$ free)
- Have just seen: fiscal policy affects portfolio choice ϑ_t and thus relative asset valuations (q_t^K / q_t^B)
- What determines level of asset prices q_t^K, q_t^B?

  $\to$ consumption-savings choice and wealth effects (& goods market clearing)

  - goods market clearing (recall $u_t = u^* = 1$):

$$a_t = C_t = \rho(q_t^B + q_t^K) = \rho \frac{q_t^B}{\vartheta_t}$$

  - solving for q_t^B

$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = q_t^B = \vartheta_t \frac{a_t}{\rho}$$

- This is a condition for the equilibrium price level $\mathcal{P}_t$

  (because $\mathcal{B}_0$ is a pre-determined state variable)

Interpretation: Portfolio Choice can Determine the Price Level

- Previous result suggests: portfolio choice can determine the price level when there are nominal assets
- Economic logic, for given ϑ_t
  - $\mathcal{P}_t$ too high $\to$ total wealth q_t^B/ϑ_t too low $\to$ insufficient demand $\to$ firms lower prices
  - $\mathcal{P}_t$ too low $\to$ total wealth q_t^B/ϑ_t too high $\to$ excess demand $\to$ firms raise prices
- Key to this logic: some asset value is fixed in nominal terms (here bonds)
- Also: logic may break down if ϑ_t reacts to $\mathcal{P}_t$ (because future $\hat{\tau}_t$s do)
  (will come back to this later)

Fiscal Theory of the Price Level (FTPL)

- More conventional way of saying essentially the same: start from asset valuation equations

$$q_{t_0}^B = \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \exp\left(-\int_{t_0}^{t} r_s ds\right) \tau_t dt\right]$$

$$q_{t_0}^K = \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \exp\left(-\int_{t_0}^{t} r_s ds\right) (a_t - \tau_t)\, dt\right]$$

- Consumption demand is

$$C_{t_0} = \rho\left(q_{t_0}^B + q_{t_0}^K\right) = \underbrace{\rho \int_{t_0}^{\infty} \exp\left(-\int_{t_0}^{t} r_s ds\right) a_t dt}_{=a_{t_0}} + \rho\left(q_{t_0}^B - \int_{t_0}^{\infty} \exp\left(-\int_{t_0}^{t} r_s ds\right) \tau_t dt\right)$$

- Unless second term vanishes, government bonds net of tax liabilities represent (pos. or neg.) net wealth and affect demand
    - wealth effects on nominal government bonds can bring the goods price to equilibrium
    - this idea is called the *Fiscal Theory of the Price Level* (FTPL)

Inflation Determination under Sticky Prices

- Next consider sticky price model
- Same derivation as before applies (with u_t possibly different from 1)

$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = \vartheta_t \frac{u_t a_t}{\rho}$$

- But now $\mathcal{P}_t$ (and q_t^B) is a state variable $\rightarrow$ this can no longer determine the price level (it is already determined)
- Instead determines utilization u_t and inflation π_t
  - $\mathcal{P}_t$ too large $\rightarrow$ insufficient demand generates under-utilization and deflation
  - $\mathcal{P}_t$ too small $\rightarrow$ excess demand generates over-utilization and inflation
- What we really get is a "fiscal theory of inflation"

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Active Fiscal Policy

- Loosely speaking, fiscal policy is called "active", if surpluses do not react to stabilize debt (active/passive terminology is due to Leeper 1991)

- Under active fiscal policy, FTPL can determine a unique equilibrium
  - surpluses are (sufficiently) unresponsive to variations in ϑ_t (or q_t^B)
  - only one ϑ_t solution is consistent with portfolio choice and asset market clearing
  - equivalent: only one $\mathcal{P}_t$ is consistent with debt valuation equation

- Specific example: suppose path for surplus-GDP ratio $\{\hat{\tau}_t\}_{t\geq 0}$ is exogenous

  $\rightarrow$ portfolio choice: $\{\vartheta_t\}_{t\geq 0}$ is determined independently of $\{i_t\}_{t\geq 0}$, $\{\pi_t\}_{t\geq 0}$, and $\{u_t\}_{t\geq 0}$

$$\vartheta_t = \mathbb{E}_t\left[\int_t^\infty \rho e^{-\rho(s-t)}\hat{\tau}_s ds\right]$$

Remark: this is just a benchmark; $\hat{\tau}_t$ reacting to i_t, π_t, or u_t could still be active

Equilibrium Solution

Given ϑ_t, remaining model has closed-form solution

$$dq_t^B = \left(\left(i_t - \pi(q_t^B, \vartheta_t)\right) q_t^B - \hat{\tau}_t a_t u(q_t^B, \vartheta_t)\right) dt \qquad \text{state evolution}$$

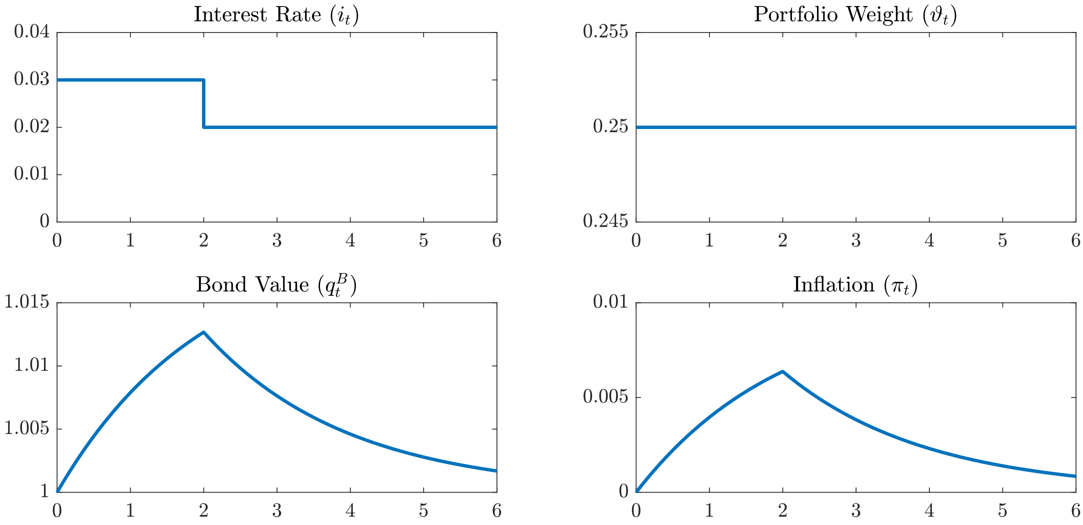
$$u(q_t^B, \vartheta_t) = \rho \frac{q_t^B}{a_t \vartheta_t} \qquad \text{market clearing}$$

$$\pi(q_t^B, \vartheta_t) = \kappa \left(\left(u(q_t^B, \vartheta_t)\right)^{1+\nu} - 1\right) \qquad \text{Phillips curve}$$

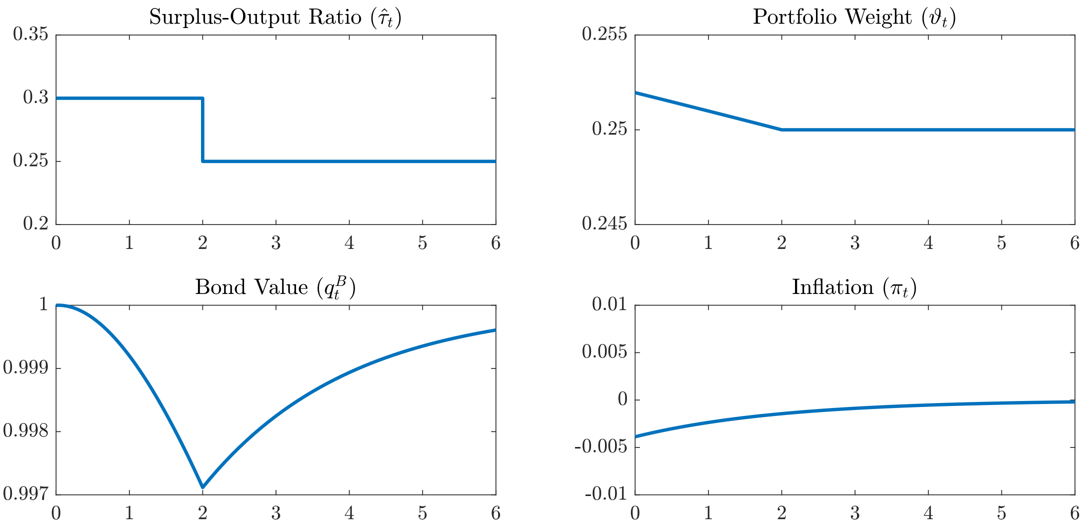
Where Did the IS Equation Go?

- IS equation was one of the key equations in model without bonds
- Here, it appears to be gone. How can that be?
- Answer: it is implicit in the consumption rule $c_t^i = \rho n_t^i$
  - IS equation is Euler equation (consumption-savings choice) combined with Fisher equation
  - all we need for FTPL is that higher wealth leads to higher consumption demand
    (plus exact value for c_t / n_t to compute demand)
  - beyond this, intertemporal substitution not key to any mechanism here
- What really matters is portfolio demand for nominal bonds (ϑ_t)

Effects of Interest Rate Hike under Active Fiscal Policy



Effects of Fiscal Tightening under Active Fiscal Policy



Limited Effectiveness of Interest Rate Policy

- Suppose we have a shock at $t = 0$ that moves either ϑ_0 or a_0
  - Under flexible prices, q_0^B would adjust
  - Under sticky prices, it is a state variable and adjusts only sluggishly
- Monetary policy cannot do anything to restore $u_0 = 1$ on shock impact:

$$u_t = \rho \frac{q_t^B}{a_t \vartheta_t}$$

- To correct demand: need fiscal policy to move portfolio weight ϑ_t
  (interest policy can merely manage the transition dynamics)

→ Monetary policy alone no longer able to implement first best allocation, even if it sets $i_t = r_t^*$ in equilibrium

- *Remark*: introducing long-term bonds restores some ability of i_t-policy to manage demand on impact, but perfect stabilization still infeasible

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Passive Fiscal Policy

- Fiscal policy is "passive" if it adjusts surpluses to rising debt levels

- Simple example of a passive rule (zero intercept is not crucial, but simplifies argument on next slide)

$$\tau_t = \alpha q_t^B, \qquad \alpha > 0$$

- Proposition: if fiscal policy is passive, then *any* initial portfolio weight ϑ_0 is consistent with equilibrium

    equivalent: any initial real value of debt q_0^B satisfies the debt valuation equation

- Significance of this result: portfolio choice/debt valuation indeterminacy brings back indeterminacy of price level/inflation

- *Aside*: this result is also of interest outside monetary economics, e.g. for debt sustainability tests (Bohn 1998)

Why Does Proposition Hold?

- Government flow budget constraint implies real debt evolution

$$dq_t^B = \left(r_t q_t^B - \tau_t\right) dt$$

- With passive surplus rule (specific example above)

$$dq_t^B = (r_t - \alpha)\, q_t^B\, dt \qquad \Rightarrow \qquad q_T^B = q_0^B \exp\left(\int_0^T (r_t - \alpha)\, dt\right)$$

$$\Rightarrow \qquad \exp\left(-\int_0^T r_t dt\right) q_T^B = q_0^B e^{-\alpha T} \to 0$$

  as $T \to 0$ for any q_0^B

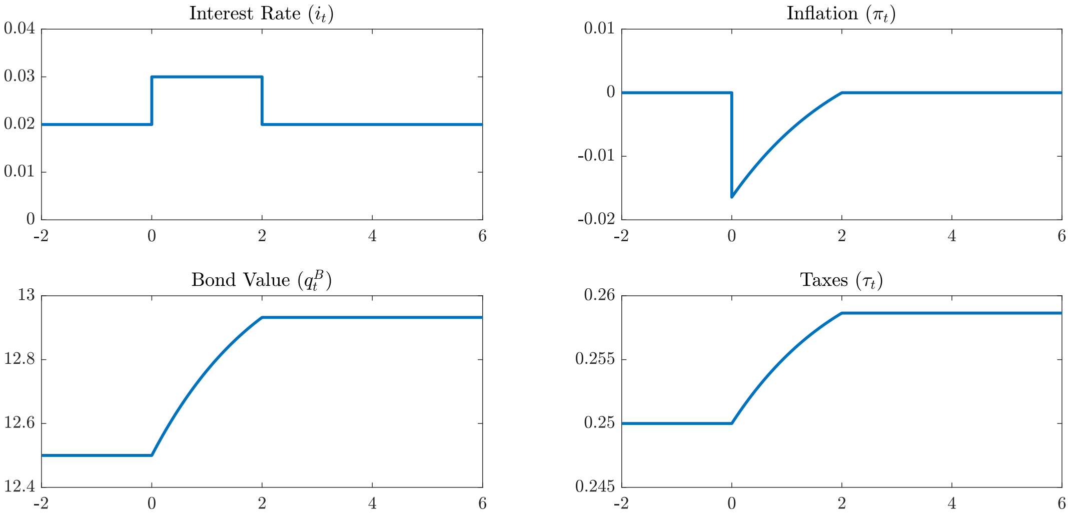
- So any q_0^B is consistent with debt evolution and household transversality condition
- How does this work?
  - as q_t^B grows surpluses rise at least linearly with q_t^B
  - prevents decay in "dividend yield" and keeps debt growth rate strictly below the interest rate

Equilibrium with Passive Fiscal Policy

- Under passive fiscal policy, the model works like our first model without bonds
  - price level/inflation is indeterminate
  - fiscal policy adjusts in the background to debt dynamics
- Fiscal shocks affect economy at most as a coordination device (due to multiplicity)
- If interest rate policy eliminates multiplicity with a Taylor rule: inflation and output gaps are insulated from fiscal shocks
  (key difference to active fiscal policy)
- Can then assign the task of inflation/output gap stabilization solely to monetary policy

→ Standard doctrine in New Keynesian economics:
  - let fiscal policy worry about stabilizing the debt
  - let monetary policy worry about inflation and output gaps
- *But note*: even under passive fiscal policy, the model has predictions for fiscal variables

Passive Policy: Fiscal Implications of Interest Rate Hike



Outline

Modified Model with Idiosyncratic Risk

- Consider again the model with government bonds and assume passive fiscal policy ($\alpha > 0$)
- Now add idiosyncratic risk as in Markus' earlier lecture
  - capital k_t^i of household i evolves according to

$$\frac{dk_t^i}{k_t^i} = \underbrace{d\Delta_t^{k,i}}_{\text{trading}} + \underbrace{\tilde{\sigma}_t d\tilde{Z}_t^i}_{\text{idio. shocks}}$$

  - $\tilde{\sigma}_t$ follows exogenous path
- In this model, bonds represent safe assets (provide service flows from re-trading)
- Portfolio choice implies

$$\mathbb{E}_t[d\vartheta_t] = \left(\rho - \alpha - (1 - \vartheta_t)^2 \tilde{\sigma}_t^2\right) \vartheta_t dt$$

Portfolio Choice Solution Structure

$$\mathbb{E}_t[d\vartheta_t] = \left(\rho - \alpha - (1 - \vartheta_t)^2 \tilde{\sigma}_t^2\right) \vartheta_t dt$$

- Assume $\alpha < \rho$ (so that government does not repay all debt eventually)
- There is a continuum of solutions ϑ_t
- But only one of them satisfies $\lim_{t \to \infty} \vartheta_t \in (0, \infty)$
- This solution is locally isolated, while all others have alternative solutions "nearby"
  (same conclusion also holds for implied equilibrium consumption risk and natural rate)

$\Rightarrow$ With safe asset demand, portfolio choice has "locally unique" solution for ϑ even under passive fiscal policy
(can also make it globally unique with off-equilibrium arguments, see "The Fiscal Theory with a Bubble")

Price Level/Inflation Determination from Safe Asset Demand

- Because ϑ_t uniquely determined: FTPL predictions even with passive fiscal policy
- Flexible prices:
  - safe asset portfolio demand determines the price level

$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = q_t^B = \vartheta_t \frac{a_t}{\rho}$$

- Sticky prices:
  - q_t^B state variable, adjusts only gradually

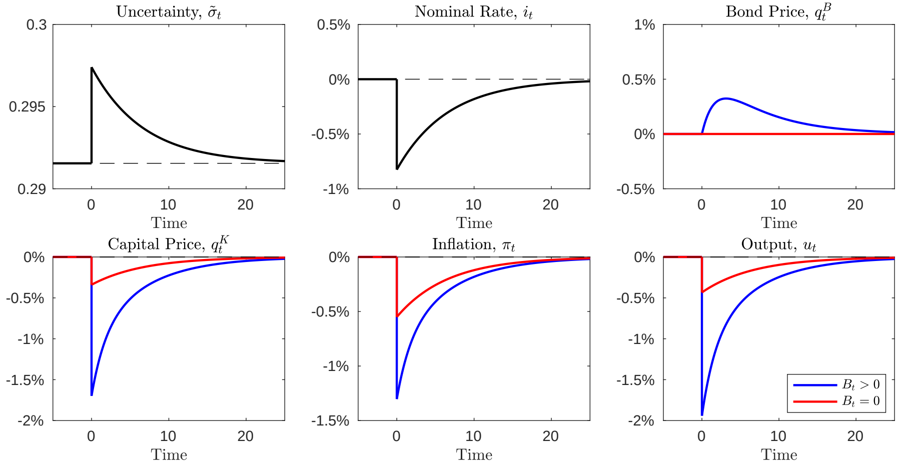
$$dq_t^B = (i_t - \pi_t + \alpha) q_t^B dt$$

  - variations in portfolio demand (e.g. due to higher $\tilde{\sigma}_t$) have demand effects

$$u_t = \frac{\rho q_t^B}{a_t \vartheta_t}$$

Do Effects from Safe Asset Demand Matter? We Believe So

Impulse Responses to an Uncertainty Shock



Source: Li, Merkel (2022)

Ineffectiveness of Interest Rate Policy Strikes Back

Also policy conclusions as in FTPL model with active policy:

- Interest rate policy cannot move the initial state, only manage the transition dynamics

  (and interest rate policy is neo-Fisherian, but can be fixed with long-term bonds...)

- Setting i_t to the natural rate r_t^* does not implement the flexible price allocation
- To move initial state, more aggressive fiscal adjustments are needed in response to shocks
  - can no longer rely on automatic stabilization to kick in eventually when $\alpha > 0$
  - instead, need α_t to respond to initial shock to move ϑ_0
- Suggests fiscal policy has to play a more active role in macro stabilization

Is there a "Super-Passive" Fiscal Regime? Is it Desirable?

- Could we design a "super-passive" fiscal regime that renders ϑ_t indeterminate?
  - possibly yes, but requires stronger than linear reaction to debt
- Does this bring us back to the conventional NK analysis?
  - with regard to aggregate demand and inflation stabilization: yes
    - $i_t = r_t^*$ plus Taylor rule selects zero inflation equilibrium
    - fiscal policy adjusts in the background to make portfolio choice consistent with it
  - but: this policy destroys the safe asset feature (negative β) of government debt
    - when safe asset demand rises, government makes bonds unattractive by lowering surpluses
    - mitigates flight to safety, bonds no longer appreciate in value
- Such a policy would not be optimal, at least in response to $\tilde{\sigma}_t$ shocks
  - want to allow for some flight to safety to improve risk sharing
  - optimal policy trades off aggregate demand stabilization with risk sharing

Summary

- Simple Money Model without Bonds in Positive Supply
  - key condition: IS equation (intertemporal substitution)
  - policy effects depend on equilibrium selection, stabilization policy = expectations management
  - can be achieved with interest rate policy (plus Taylor rule)
- Model with Nominal Government Bonds
  - portfolio demand for bonds can render nominal side determinate (FTPL)
  - when fiscal policy is active
    - bond value becomes a state variable
    - fiscal shocks matter for inflation and aggregate demand
    - interest rate policy alone cannot stabilize economy
  - passive fiscal policy restores intution of first model
- Safe Asset Model
  - even with passive policy, portfolio demand for bonds can matter for inflation and demand
  - interest policy again not sufficient for stabilization
  - aggressive fiscal policy can stabilize, but not necessarily optimal