Motivation

- What is a safe asset? What are its features: Retrading?

- How much government debt can the market absorb?
  - “Debt Laffer Curve” and debt sustainability analysis?
  - When can governments run a deficit without ever paying back its debt, like a Ponzi scheme?

- Why is there debt valuation puzzle for US, Japanese?

- This paper: safe asset nature of government debt
  - Model of government debt as a countercyclical safe asset (negative $\beta$)
  - Safe asset nature matters qualitatively and quantitatively for debt valuation
Valuating Government Debt

- Think of a representative agent holding all gov. debt
  - His cash flow is primary surplus
    \[ B_t = \mathbb{E}_t[PV_r(\text{primary surpluses})] + \ldots \]  
  - … link to inflation
  - Can surpluses be negative forever?

Japan: Govt primary balance

<table>
<thead>
<tr>
<th>% GDP</th>
</tr>
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<tbody>
<tr>
<td>4</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>0</td>
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<td>-2</td>
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<td>-4</td>
</tr>
<tr>
<td>-6</td>
</tr>
<tr>
<td>-8</td>
</tr>
<tr>
<td>-10</td>
</tr>
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</table>

Year: 60 63 66 69 72 75 78 81 84 87 90 93 96 99 02 05 08 11 14 17
Valuating Government Debt

- Think of a representative agent holding all gov. debt
  - His cash flow is primary surplus
    \[ B_t = E_t[PV_r(\text{primary surpluses})] + \textit{Bubble} \]  
    [FTPL]
  - ... link to inflation
  - Can surpluses be negative forever? Yes, if \( r < g \) (e.g. due to safe asset nature)

Japan: Govt primary balance

% GDP
Primary surplus, $r$ and $g$ for the United States

- Primary surplus/GDP
- Negative surplus in recession

- $g$ GDP growth
- $r$
- $r - g$
Negative primary surplus forever?

- without creating inflation (devaluing debt)?
- Yes, if \( r < g \)

\[
\frac{B_t}{\varphi_t} = E_t[PV_r(\text{primary surpluses})] + \lim_{T \to \infty} PV_r \frac{B_T}{\varphi_T}
\]

Discount at a different rate \( r > g \), so that

\[
\frac{B_t}{\varphi_t} = E_t[PV_r(\text{primary surpluses})] + E_t[PV_r(\text{service flow})]
\]

To determine real value of gov. debt and price level
FTPL equation is not enough
(goods market clearing and wealth effect)
Negative primary surplus forever?

- without creating inflation (devaluing debt)?
- Yes, if \( r < g \)

\[
\begin{align*}
\frac{B_t}{\varphi_t} &= E_t[PV \text{ (primary surpluses)}] + \lim_{T \to \infty} PV_r \frac{B_T}{\varphi_T} \\
&\text{discount at } r \quad \text{(agents' SDF)} \\
&\text{grows at } g \text{ with constant deficit/GDP} \\
&\to -\infty & \text{Bubble} & \to +\infty
\end{align*}
\]

- Valuing “liquidating bond portfolio”
  - negative payoff grows at \((g - \text{sell-off rate})\)
  - \(\Leftrightarrow\) discount with \((r + \text{sell-off rate})\)

\[
\begin{align*}
\frac{B_t}{\varphi_t} &= E_t[PV_r \text{(primary surpluses)}] + E_t[PV_r(\text{xxx})] \\
&\to -\infty & < +\infty
\end{align*}
\]
Negative primary surplus forever?

- without creating inflation (devaluing debt)?
- Yes, if \( r < g \)

\[
\frac{B_t}{\varphi_t} = E_t[PV_r(\text{primary surpluses})] \quad + \quad \lim_{T \to \infty} PV_r \frac{B_T}{\varphi_T} \quad \text{Bubble} \\
\text{grows at } g \text{ with constant deficit/GDP} \quad \rightarrow -\infty \quad \rightarrow +\infty
\]

- Valuing “liquidating bond portfolio”
  - negative payoff grows at \((g - \text{sell-off rate})\)
  - \(\Leftrightarrow \) discount with \((r + \text{sell-off rate})\)

\[
\frac{B_t}{\varphi_t} = E_t[PV_r(\text{primary surpluses})] \quad + \quad E_t[PV_{r'}(\text{xxx})] \\
\rightarrow -\infty \quad < \quad +\infty
\]

- Valuing “retrading portfolio” to replicate “safe asset trading strategy”
Negative primary surplus forever?

- without creating inflation (devaluing debt)?
- Yes, if \( r < g \)

\[
\mathbb{B}_t = E_t[PV_r(\text{primary surpluses})] + \lim_{T \to \infty} PV_r \frac{\mathbb{B}_T}{\mathbb{Q}_T}
\]

Both terms meaningful
- Discount rate \( r^{**} = \) representative agents’ risk-free rate \( \neq m \) (Reis)
What’s a Safe Asset? What is its Service Flow?

\[ \frac{B_t}{\delta_t} = E_t[PV_{r**} \text{(primary surpluses)}] + E_t[PV_{r**} \text{(service flow)}] \]
What’s a Safe Asset?  What is its Service Flow?

\[ \frac{B_t}{\varphi_t} = E_t[ PV_{r^{**}} (\text{primary surpluses}) ] + E_t[ PV_{r^{**}} (\text{service flow}) ] \]

Example: \( = 0 \)

![Diagram showing the portfolio of safe assets and their cash flow shocks.](image)
What’s a Safe Asset? What is its Service Flow?

- \( \frac{B_t}{\varphi_t} = E_t[PV_{r^{**}}(primary\ surpluses)] + E_t[PV_{r^{**}}(service\ flow)] \)

- Value come from re-trading
- Insures by partially completing markets
  Reduces \( Var_t[\tilde{g}_c] \)

- Can be = fragile
What’s a Safe Asset? What is its Service Flow?

\[
\frac{B_t}{\phi_t} = E_t[PV_{r^*}(primary\ surpluses)] + E_t[PV_{r^*}(service\ flow)]
\]

- Value come from re-trading
- Insures by partially completing markets
- Can be = fragile

In recessions:
- Risk is higher
  - Service flow is more valuable
  - Cash flows are lower
    (depends on fiscal policy)
Outline

- Model
  - Setup
  - Closed-Form Solution for Steady State
  - Debt Valuation - Two Perspectives

- Countercyclical Safe Asset and Valuation Puzzles
  - Calibrated Model Solution
  - Debt Valuation Puzzles

- Safe Asset and the Stock Market
Model Overview

- Continuous time, infinite horizon, one consumption good
- Continuum of agents
  - Operate capital with time-varying idiosyncratic risk, $AK$ production technology
  - Can trade capital and government bond, Extension: add diversified equity claims
- Government
  - Exogenous spending
  - Taxes output
  - Issues (nominal) bonds
- Financial Frictions: incomplete markets
  - Agents cannot trade idiosyncratic risk
  - Extension with equity: must retain skin in the game
- Aggregate risk: fluctuations in volatility of idio risk (& capital productivity)
Model with Capital + Safe Asset

- Each heterogenous citizen $\tilde{\iota} \in [0,1]$
  
  $$E\left[\int_0^\infty e^{-\rho t} \log c^\tilde{\iota}_t \, dt\right] \quad \text{s.t.} \quad \frac{dn^\tilde{\iota}_t}{n^\tilde{\iota}_t} = -\frac{c^\tilde{\iota}_t}{n^\tilde{\iota}_t} \, dt + dr^B_t + (1 - \theta^\tilde{\iota}_t)(dr^K_t(\iota^\tilde{\iota}_t) - dr^B_t)$$

- Each citizen operates physical capital $k^\tilde{\iota}_t$
  - Output (net investment) $y^\tilde{\iota}_t = (a_t - i^\tilde{\iota}_t)k^\tilde{\iota}_t \, dt$
  - Output tax $\tau_t a_t k^\tilde{\iota}_t \, dt$
  - $\frac{dk^\tilde{\iota}_t}{k^\tilde{\iota}_t} = (\Phi(\iota^\tilde{\iota}_t) - \delta) \, dt + \tilde{\sigma}_t d\tilde{Z}^\tilde{\iota}_t + d\Delta^k_{t,\tilde{\iota}}$
    - $d\tilde{Z}^\tilde{\iota}_t$ idiosyncratic Brownian
  - Aggregate risk: $\tilde{\sigma}_t, a_t, g_t$ exogenous process by aggregate Brownian $dZ_t$
  - Financial Friction: Incomplete markets: no $d\tilde{Z}^\tilde{\iota}_t$ claims
Government: Taxes, Bond/Money Supply, Gov. Budget

- **Policy Instruments** \( (K_t := \int k^\tilde{t} dt) \)
  - Government spending \( g_t K_t \) (with exogenous \( g_t \))
  - Proportional output tax \( \tau_t a_t K_t \)
  - Nominal government debt supply \( \frac{dB_t}{B_t} = \mu^B_t dt \)
  - Floating nominal interest rate \( i_t \) on outstanding bonds

- **Government budget constraint (BC)**

\[
\left( \mu^B_t - i_t \right) B_t + g_t K_t \left( \tau_t a_t - g_t \right) = 0
\]

\[
\bar{\mu}^B_t := \mu^B_t - i_t
\]

Primary surplus (per \( K_t \))
Distribution of “Seigniorage”

1. Proportionally to bond/money holdings
   ▪ No real effects, only nominal

2. Proportionally to capital holdings
   ▪ Bond/Money return decreases with $dB_t$ (change in debt level/money supply)
   ▪ Capital return increases
   ▪ Pushes citizens to hold more capital

3. Proportionally to net worth
   ▪ Fraction of seigniorage goes to capital - same as 2.
   ▪ Rest of seigniorage goes to money holders - same as 1.

4. Per capita
   ▪ No real effects:
     people simply borrow against the transfers they expect to receive
Optimality and market clearings

- Optimal real investment rate: (Tobin’s q)
  \[ \Phi(\iota) = \frac{1}{\phi} \log(1 + \phi \iota) \quad \iota_t = \frac{1}{\phi}(q_t^K - 1) \]

- Optimal consumption:
  \[ C_t = \rho N_t = (q^K_t + q^B_t)K_t \]

- Optimal portfolio choice \( \theta_t \):
  \[ E \left[ \frac{dr^K}{dt} - \frac{dr^B}{dt} \right] = \zeta_t \left( \sigma^q_{t}K - \sigma^q_{t}B \right) + \tilde{\zeta}_t \tilde{\sigma}_t \]
  \[ \frac{a_t(1-\tau) - \iota_t}{q^K_t} + \tilde{\mu}_t^B = \left[ \sigma^q_{t}B + \left( 1 - \theta_t \right) \left( \sigma^q_{t}K - \sigma^q_{t}B \right) \right] \left( \sigma^q_{t}K - \sigma^q_{t}B \right) + \left( 1 - \theta_t \right) \tilde{\sigma}_t^2 \]

  - price of aggregate risk
  - “excess aggregate volatility”
  - = net worth volatility

"excess aggregate volatility"
Optimality and market clearings

- Optimal real investment rate: (Tobin’s q)
  \[ \Phi(l) = \frac{1}{\phi} \log(1 + \phi l) \]
  \[ \iota_t = \frac{1}{\phi} (q^K_t - 1) \]

- Optimal consumption:
  \[ C_t = \rho \quad N_t = (a_t - \iota_t - g_t)K_t \]

- Optimal portfolio choice \( \theta_t \):
  \[ E \left[ \frac{dr^K}{dt} - \frac{dr^B}{dt} \right] = \zeta_t \left( \sigma^K_t - \sigma^B_t \right) + \tilde{\zeta}_t \tilde{\sigma}_t \]
  \[ \frac{a_t(1 - \tau) - \iota_t}{q^K_t} + \tilde{\mu}_t^B = \left[ \sigma^B_t + (1 - \theta_t) \left( \sigma^K_t - \sigma^B_t \right) \right] \left( \sigma^K_t - \sigma^B_t \right) + (1 - \theta_t) \tilde{\sigma}_t^2 \]

  price of aggregate risk
  = net worth volatility

  "excess aggregate volatility"
Optimality and market clearings \textit{in steady state} (and constant $\bar{\sigma}, \bar{a}$)

- Optimal real investment rate: (Tobin’s q)
  \[ \Phi(\iota) = \frac{1}{\phi} \log(1 + \phi \iota) \quad \iota_t = \frac{1}{\phi} (q^K_t - 1) \]

- Optimal consumption:
  \[ C_t = \rho N_t = (a_t - \iota_t - \varrho_t) K_t \]

- Optimal portfolio choice $\theta_t$:
  \[ E \left[ \frac{dr^K}{dt} - \frac{dr^B}{dt} \right] = \varsigma_t \left( \sigma_t^{q^K} - \sigma_t^{q^B} \right) + \tilde{\varsigma}_t \tilde{\sigma}_t \]

\[ \frac{a_t(1-\tau) - \iota_t}{q^K_t} + \tilde{\mu}_t^B = \left[ \sigma_t^{q^B} + \left( 1 - \theta_t \right) \left( \sigma_t^{q^K} - \sigma_t^{q^B} \right) \right] \left( \sigma_t^{q^K} - \sigma_t^{q^B} \right) + \left( 1 - \theta_t \right) \tilde{\sigma}_t^2 \]

- Price of aggregate risk
  \[ = \text{net worth volatility} \]

- “Excess aggregate volatility”
Two Stationary Equilibria \((\text{for } K_0 = 1 \text{ and constant } \tilde{\sigma}, \tilde{a})\)

<table>
<thead>
<tr>
<th>Gordon-Growth Formula</th>
<th>Closed Form Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q^K = \frac{(1-\tau)\tilde{a} - \iota}{E[dr^K]/dt - g})</td>
<td>(q^K = \frac{\sqrt{\rho + \tilde{\mu}^B} (1 + \phi \tilde{a})}{\sqrt{\rho + \tilde{\mu}^B} + \phi \tilde{\sigma} \rho})</td>
</tr>
<tr>
<td>(\frac{B}{\phi} = \frac{s}{E[dr^m]/dt - g} + \frac{(1 - \vartheta)^2 \tilde{\sigma}^2 B}{\phi} \frac{E[dr^m]/dt - g}{E[dr^m]/dt - g})</td>
<td>(q^B K_t = \frac{(\tilde{\sigma} - \sqrt{\rho + \tilde{\mu}^B})(1 + \phi \tilde{a})}{\sqrt{\rho + \tilde{\mu}^B} + \phi \tilde{\sigma} \rho} K_t)</td>
</tr>
<tr>
<td>(\iota = \frac{a \sqrt{\rho + \tilde{\mu}^B - \tilde{\sigma} \rho}}{\sqrt{\rho + \tilde{\mu}^B + \phi \tilde{\sigma} \rho}})</td>
<td></td>
</tr>
</tbody>
</table>

\(dr^n = \vartheta dr^B + (1 - \vartheta) dr^K\)

\(\vartheta = \begin{cases} 0 & \text{if } \theta \\ \theta & \text{otherwise} \end{cases}\)

- \(\rho\) time preference rate
- \(\phi\) adjustment cost for investment rate
- \(\tilde{\mu}_t^B = \mu_t^B - i\) bond issuance rate beyond interest rate
- \(\tilde{a} = a - g\) part of TFP not spend on gov.)
Safe Asset Valuation Equation: 2 Perspectives

- Agent $\tilde{\iota}$’s SDF, $\xi_t^\tilde{\iota}$: 
  \[ d\xi_t^\tilde{\iota} / \xi_t^\tilde{\iota} = -r_t^f dt - \zeta_t dZ_t - \tilde{\zeta}^\tilde{\iota} d\tilde{Z}_t \]

- **Buy and Hold Perspective:**
  \[ \frac{B_0}{P_0} = \lim_{T \to \infty} \left( E \left[ \int_0^T \xi_t^l s_t K_t dt \right] + E \left[ \xi_t^l B_T / P_T \right] \right) \]
  - Bubble is possible: 
    \[ \lim_{T \to \infty} E[\xi_t^l B_T / \bar{\rho}_T] > 0 \text{ if } r_t^f + \zeta_t \sigma_t q_t B \leq g_t \text{ (on average)} \]

- **Dynamic Trading Perspective:**
  - Value cash flow from individual bond portfolios, including trading cash flows
  - Integrate over citizens weighted by net worth share $\eta_t^i$
  - Bond as part of a dynamic trading strategy
  \[ \frac{B_0}{P_0} = E \left[ \int_0^\infty \left( \int \xi_t^l \eta_t^i d\xi \right) s_t K_t dt \right] + E \left[ \int_0^\infty \left( \int \xi_t^l \eta_t^i d\xi \right) (\tilde{\xi}_t^*)^2 B_t / P_t dt \right] \]

  - Discount rate $E[dr^\eta]/dt = r^f + \zeta \bar{\sigma}$
  - $\xi^i$ and $\eta^i$ are negatively correlated $\Rightarrow$ depresses weighted “Quasi-SDF” (higher discount rate)
Safe Asset Valuation Equation: 2 Perspectives

- **Buy and Hold Perspective:**
  Expected bond return
  \[
  E[r_b] = \rho + \mu^c - \{\sigma^c)^2 + (\bar{\sigma}\)^2\} + risk\ premium - \text{convience yield}
  \]
  Risk-free rate \( r^f = \)
  Discount rate

- **Dynamic Trading Perspective:**
  Expected bond return
  \[
  E[r_b] = \rho + \mu^c - \{\sigma^c)^2\} + risk\ premium - \{(\bar{\sigma}\)^2 + convience\ yield\}
  \]
  Risk-free rate \( r^{f*} = \) "Service Flow"
  Discount rate

- \( r^{f*} = \) “representative agent’s” risk-free rate
Outline

- Model
  - Setup
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- Countercyclical Safe Asset and Valuation Puzzles
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- Safe Asset and the Stock Market
Equity Markets (ETF) + Epstein-Zin Preferences

- **Equity Market**
  - Each citizen $\hat{i}$ can sell off a fraction $(1 - \chi)$ of capital risk to outside equity holders.
  - Return $dr_{t}^{E,\hat{i}}$
    - Same risk as $dr_{t}^{K,\hat{i}}$
    - But $\mathbb{E}_t[dr_{t}^{E,\hat{i}}] < \mathbb{E}_t[dr_{t}^{K,\hat{i}}]$ ... de to insider premium.
  - Prop.: Model equations as before but replace $\bar{\sigma}$ with $\chi\bar{\sigma}$.

- **Epstein-Zin preferences for calibration (EIS=1)**
  - Citizen $\hat{i}$ maximizes $V_0^{\hat{i}}$ where $V_t^{\hat{i}}$ is recursively defined by
    $$V_t^{\hat{i}} = \mathbb{E}_t \left[ \int_t^\infty (1 - \gamma) \rho V_s^{\hat{i}} \left( \log(c_s^{\hat{i}}) - \frac{1}{1 - \gamma} \log \left( (1 - \gamma) V_s^{\hat{i}} \right) \right) ds \right]$$
  - Needed to generate realistic prices of risk (Sharpe ratio).
Calibration

- Exogenous processes:
  - **Recessions** feature high idiosyncratic risk and low consumption
    - $\tilde{\sigma}_t$: Heston (1993) model of stochastic volatility
      \[
d\tilde{\sigma}_t^2 = -\psi(\tilde{\sigma}_t^2 - (\tilde{\sigma}^0)^2)dt - \sigma\tilde{\sigma}_t dZ_t
      \]
    - $a_t$: $a_t = a(\tilde{\sigma}_t)$ such that in equilibrium
      \[
      \frac{C}{K}(\tilde{\sigma}_t) = \alpha_0 - \alpha_1\tilde{\sigma}_t
      \]
    - $g_t = 0$

- Government (bubble-mining policy)
  \[
  \dot{\mu}_t^B = -\nu_0 + \nu_1\tilde{\sigma}_t
  \]

- Calibration to US data (1966-2019, period length is one year)
# Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{\sigma}$</td>
<td>$\bar{\sigma}_t$ stochastic steady state</td>
<td>0.29</td>
<td></td>
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<tr>
<td>$\psi$</td>
<td>$\bar{\sigma}^2_t$ mean reversion</td>
<td>0.15</td>
<td>idiosyncratic productivity shocks (Bloom et al. 2018)</td>
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<tr>
<td>$\sigma$</td>
<td>$\bar{\sigma}^2_t$ volatility</td>
<td>0.037</td>
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<tr>
<td>$\overline{\chi}$</td>
<td>undiversifiable idio. risk</td>
<td>0.5</td>
<td>share of private equity wealth (Angeletos 2007)</td>
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</tbody>
</table>

**calibration to match model moments**

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<th>parameter</th>
<th>description</th>
<th>value</th>
<th>target</th>
</tr>
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<tbody>
<tr>
<td>$\gamma$</td>
<td>risk aversion</td>
<td>7.5</td>
<td>chosen jointly to match (approximately)</td>
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<td>$\rho$</td>
<td>time preference</td>
<td>0.17</td>
<td>volatility of output, consumption, surplus-output ratio</td>
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<tr>
<td>$\alpha_0$</td>
<td>$C/K$ intercept</td>
<td>0.59</td>
<td>average consumption-output, capital-output, debt-output ratios</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>negative of $C/K$ slope</td>
<td>0.2</td>
<td>mean equity premium</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>negative of $\bar{\mu}_B$ intercept</td>
<td>0.085</td>
<td>equity Sharpe ratio</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>$\bar{\mu}_B$ slope</td>
<td>0.25</td>
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**other parameters**

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<td>$\phi$</td>
<td>capital adjustment cost</td>
<td>6</td>
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<tr>
<td>$\delta$</td>
<td>irrelevant for all results</td>
<td></td>
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### Quantitative Model Fit

<table>
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<tr>
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<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>output volatility</td>
<td>0.019</td>
<td>0.014</td>
</tr>
<tr>
<td>$\sigma(C)$</td>
<td>consumption volatility</td>
<td>0.010</td>
<td>0.008</td>
</tr>
<tr>
<td>$\sigma(S/Y)$</td>
<td>surplus volatility</td>
<td>0.004</td>
<td>0.009</td>
</tr>
<tr>
<td>$\rho(Y, C)$</td>
<td>correlation of output and consumption</td>
<td>0.977</td>
<td>0.826</td>
</tr>
<tr>
<td>$\rho(Y, S/Y)$</td>
<td>correlation of output and surpluses</td>
<td>0.927</td>
<td>0.471</td>
</tr>
<tr>
<td>$E[C/Y]$</td>
<td>average consumption-output ratio</td>
<td>0.667</td>
<td>0.615</td>
</tr>
<tr>
<td>$E[S/Y]$</td>
<td>average surplus-output ratio</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>$E[q^K/Y]$</td>
<td>average capital-output ratio</td>
<td>3.206</td>
<td>$\approx 3$</td>
</tr>
<tr>
<td>$E[q^B/K/Y]$</td>
<td>average debt-output ratio</td>
<td>0.672</td>
<td>0.578</td>
</tr>
<tr>
<td>$E[dr^E - dr^B]$</td>
<td>average equity premium</td>
<td>5.6%</td>
<td>$\approx 6.4%$</td>
</tr>
<tr>
<td>$\frac{E[dr^E - dr^B]}{\sigma(dr^E - dr^B)}$</td>
<td>equity sharpe ratio</td>
<td>0.436</td>
<td>$\approx 0.5$</td>
</tr>
</tbody>
</table>

Notes: $\sigma(x)$ denotes the standard deviation of $x$ and $\rho(x, y)$ denotes the correlation of $x$ and $y$, both at a quarterly frequency. Inputs $x$ and $y$ are HP-filtered with smoothing parameter $1600$. For $x, y \in \{ Y, C \}$, we take logarithms before filtering. $E[x]$ denotes expectations over the ergodic model distribution, inputs $x$ are not HP-filtered. $Y$: (aggregate) output, $C$: consumption, $S$: primary surplus, $q^K$, $q^B$, $dr^B$, $dr^E$ are defined as in Section 2.
Two Debt Valuation Puzzles

- Properties of US primary surpluses
  - Average surplus $\approx 0$
  - Procyclical surplus ($> 0$ in booms, $< 0$ in recessions)

- Two valuation puzzles from standard perspective:
  (Jiang, Lustig, van Nieuwerburgh, Xiaolan, 2019, 2020)
  1. “Public Debt Valuation Puzzle”
     - Empirical: $E[PV(\text{surpluses})] < 0$, yet $\frac{B}{\delta} > 0$
     - Our model: bubble/service flow component overturns results
  2. “Gov. Debt Risk Premium Puzzle”
     - Debt should be positive $\beta$ asset, but market don’t price it this way
     - Our model: can be rationalized with countercyclical bubble/service flow
Bond and Capital Value for time-varying idiosyncratic risk $\tilde{\sigma}_t$

Gov. debt value rises in recessions

Capital price
Safe Asset – Cash flow and Service flow

- Asset Price = \( E[PV(\text{cash flows})] + E[PV(\text{service flows})] \)
Debt Laffer Curve ≠ MMT  Debt Sustainability Analysis 1

- Issue bonds at a faster rate $\hat{\mu}^B$ (esp. in recessions)
  - $\Rightarrow$ tax precautionary self insurance $\Rightarrow$ tax rate
  - $\Rightarrow$ real value of bonds, $\frac{B}{\phi}$, $\Rightarrow$ “tax base”
- Less so in recession due to flight-to-safety
Outline

- Model
  - Setup
  - Closed-Form Solution for Steady State
  - Debt Valuation - Two Perspectives

- Countercyclical Safe Asset and Valuation Puzzles
  - Calibrated Model Solution
  - Debt Valuation Puzzles

- Safe Asset and the Stock Market
Why Does Safe Asset Survive in Presence of ETFs?

- Diversified stock portfolio is free of idiosyncratic risk
  - Trading in stocks (ETF) can also self-insure idiosyncratic risk
    - Good friend in idiosyncratically bad times

- But: poor hedge against aggregate risk, losses value in recessions
  - Positive $\beta$
    - Bad friend in aggregate bad times

- Why positive $\beta$? (after all $r_f$ goes down in recessions, lowers discount rate)
  - Equity are claims to capital, but marginal capital holder is insider
  - Insider bears idiosyncratic risk, must be compensated
  - $\tilde{\sigma}_t \uparrow \Rightarrow$ insider premium $E_t[dr^K_t] - E_t[dr^E_t] \uparrow \Rightarrow$ payouts to stockholders fall
Stock Market Volatility due to Flight to Safety

- "Aggregate Intertemporal Budget Constraint:

\[
q_t^K K_t + q_t^B K_t = \mathbb{E}_t \left[ \int_t^\infty \frac{\int \xi_s \eta_s^i di}{\int \xi_t \eta_t^i di} C_s ds \right]
\]

\[
= \mathbb{E}_t \left[ \int_t^\infty \frac{\xi_s^{**}}{\xi_t^{**}} C_s ds \right]
\]

- Lucas-type models: \( q^B = 0 \) (also \( C_t = Y_t \), no idiosyncratic risk)
  - Value of equity (Lucas tree) = PV of consumption claim
  - Volatility equity values require volatile RHS of (*)

- This model: even for constant RHS of (*), \( q_t^K K_t \) can be volatile due to flight to safety:
  - Increase in \( \sigma_t \) \( \Rightarrow \) Portfolio reallocation from capital to bonds, \( q_t^K K_t \downarrow, B_t/\phi_t \uparrow \)

- Quantitatively relevant? Yes
  - Excess return volatility
    - 2.9% in equivalent bondless model (\( s = 0 \) and no bubble)
    - 12.9% in our model
Loss of Safe Asset Status – Equilibrium selection

- When government debt has a (stationary) bubble, other equilibria possible
  - Stationary no bubble equilibrium
  - Nonstationary equilibria that converge to the no bubble equilibrium

- Implies fragility: bubbles may pop, loss of safe asset status

Are there policies to prevent a loss of safe asset status?

1. Create a “fundamentally safe asset”
   - Raise (positive) surpluses to generate safe cash flow component $q_t^{B,CF}$
   - If surpluses always exceed a (positive) fraction of total output, no bubble
   - But: gives up revenues from bubble mining

2. Off-equilibrium tax backing
   - Sufficient to (credibly) promise policy 1 off equilibrium
   - See “FTPL with a Bubble”
Conclusion

- **Safe Asset = good friend**
  - **Individually:** allows self-insurance through retrading
  - **Aggregate:** appreciates in bad times (negative $\beta$)

- **Fiscal Debt Sustainability Analysis**
  - Gov can “mine the bubble” within limits (max 2% of GDP)
  - Extra space, but Debt Laffer Curve ($\neq$ MMT)
  - Bubble can pop: loss of safe asset status

- **Asset pricing with safe assets**
  - Service Flow term $>>$ convenience yield
  - Flight to Safety creates
    - Countercyclical safe asset valuation
    - Large stock market volatility

- **Remark: Competing Safe Assets**
  - Within country private bonds are partial safe assets
  - Across countries $\Rightarrow$ Spillover of US Monetary Policy