International Monetary Theory:
A Risk Perspective

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Outline

- Money and frictions
  - individuals receive idiosyncratic and country-wide income shocks
  - incomplete asset market (no consumption smoothing across idiosyncratic states, costly or no against country-wide shocks)
- But there is consumption smoothing across time (Bewley)
  - save currency after positive shocks, spend after negative
- Two currencies: international $ and local peso

Equilibrium

- Money and frictions
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  - incomplete asset market (no consumption smoothing across idiosyncratic states, costly or no against country-wide shocks)
- But there is consumption smoothing across time (Bewley)
  - save currency after positive shocks, spend after negative
- Two currencies: international $ and local peso
- Value of peso endogenous, ∩-shaped in idiosyncratic risk
- Sudden stops: small shock, large effect on peso value
- Dollar monetary policy spillovers on small open economy
- Dollar debt / borrowing limit and exchange rates

Optimal Policy Results (for SOE)

- Policy
  - Supply of local public debt / fiscal backing of local currency
  - Dollar reserves
- Optimal outcome vs. equilibrium outcome (big difference)
  - Equilibrium: capital flows South to North (instead of reverse)
  - Policy strengthens home bias
  - Under optimal policy, country-wide risk is the key determinant of optimal dollars holdings (vs. in equilibrium, flight to safety: dollar demand rises in idiosyncratic risk)
- Spillovers (from dollar monetary policy) are large even with policy response

Some literature

- Money as insurance against undiversifiable idiosyncratic risk:
  - Bewley (1980) (money), Aiyagari and McGratten (1997) (public debt)
- Brunnermeier and Sannikov (2016) – The I Theory of Money
- Countries with different idiosyncratic risk, global imbalances / capital flows: Mendoza, Quadrini and Ruis-Roll (2000), Angeletos
- Kray-Ventura (2000) (response to transitory shocks, cts time)
- Public debt as a safe store of value (tax smoothing) Angeletos, Collar and Dellas (2016)
- Mundell (1963), Flemming (1962) (capital controls and flexibility of monetary policy)

Model Outline

- Small country: individuals can produce goods from capital, face idiosyncratic risk (as in Bewley economies)
- As in a Bewley economy, individuals can use money / local currency (or local public debt) to save against idiosyncratic risk
- There are also aggregate country-wide shocks, individuals can save or borrow dollars to absorb these shocks
- Here it gets interesting – dollar savings also protect individuals from idiosyncratic risk – hence the two currencies compete and the risk-free rate is determined endogenously

Small Open Economy

Aggregate capital in SOE

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta)dt - d\underline{\Delta}_t^K + d\overline{\Delta}_t^K$$

purchases abroad, price $\overline{q} > \underline{q}$

investment, $\Phi' > 0$, $\Phi'' \leq 0$ capital sales abroad at price $\underline{q}$

Capital k_t^i used by individual i produces consumption output

$$dy_t^i = (a - \iota_t)k_t^i dt + \underbrace{\sigma k_t^i dZ_t}_{\text{country-wide}} + \underbrace{\tilde{\sigma} k_t^i d\tilde{Z}_t^i}_{\text{idiosyncratic}}$$

Normalize 1 dollar = 1 unit of output (numeraire). Dollar risk-free return is r

Aggregate dollar savings

$$d\$_t = (r\$_t - C_t)dt + dY_t + (\underline{q}\ K_t d\underline{\Delta}_t^K - \overline{q}\ K_t d\overline{\Delta}_t^K)$$

Frictions: individuals can't (1) insure idiosyncratic shocks with anyone
(2) country-wide shocks with foreigners (can be relaxed), and
(3) friction to trading capital internationally

Individual optimization problem

- Max

$$E\left[\int_0^\infty e^{-\rho t}\frac{c_t^{1-\gamma}}{1-\gamma}dt\right]$$

by choosing a portfolio of capital, pesos and \$

- Returns: \$ has risk-free real return r (for exposition, easy to relax)
- Capital price $q_t \in [\underline{q}, \overline{q}]$ has expected dividend yield $(a-\iota_t)/q_t$, countrywide risk σ/q_t, idiosyncratic risk $\tilde{\sigma}/q_t$
- All pesos (held only by locals) have value $p_t K_t$
- No policy: fixed nominal quantity pesos
- Budget constraint

$$\frac{dn_t}{n_t} = \theta_t^{\$} r\,dt + \theta_t^{\hbar}\frac{d(p_t K_t)}{p_t K_t} + (1-\theta_t^{\$}-\theta_t^{\hbar})\left(\frac{a-\iota_t}{q_t}dt + \frac{d(q_t k_t)}{q_t k_t} + \frac{\sigma}{q_t}dZ_t + \frac{\tilde{\sigma}}{q_t}d\tilde{Z}_t\right) - \frac{c_t}{n_t}dt$$

Rebalancing

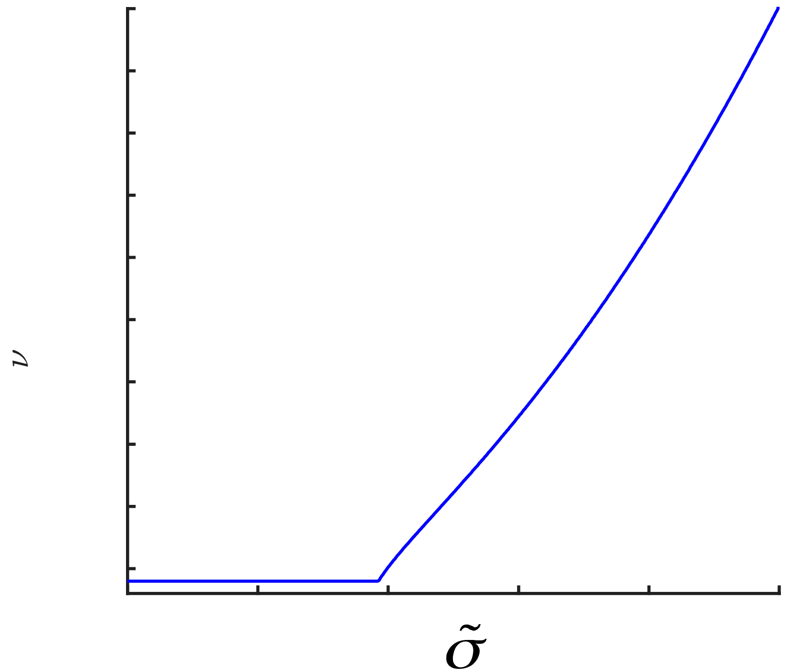
- Individual: bad idiosyncratic shock dy_t^i, sell some k_t for currencies to maintain portfolio weights
- If dy_t^i goes up, buy capital (grow the "firm")
- Country-wide: bad aggregate cash flow shock dY_t, cannot rebalance if $q_t \in (\underline{q}, \overline{q})$, prices react so optimal portfolio weights don't change
- everyone wants to adjust but cannot, adjust gradually (cut consumption, etc.)
- prices q_t, p_t clear the markets

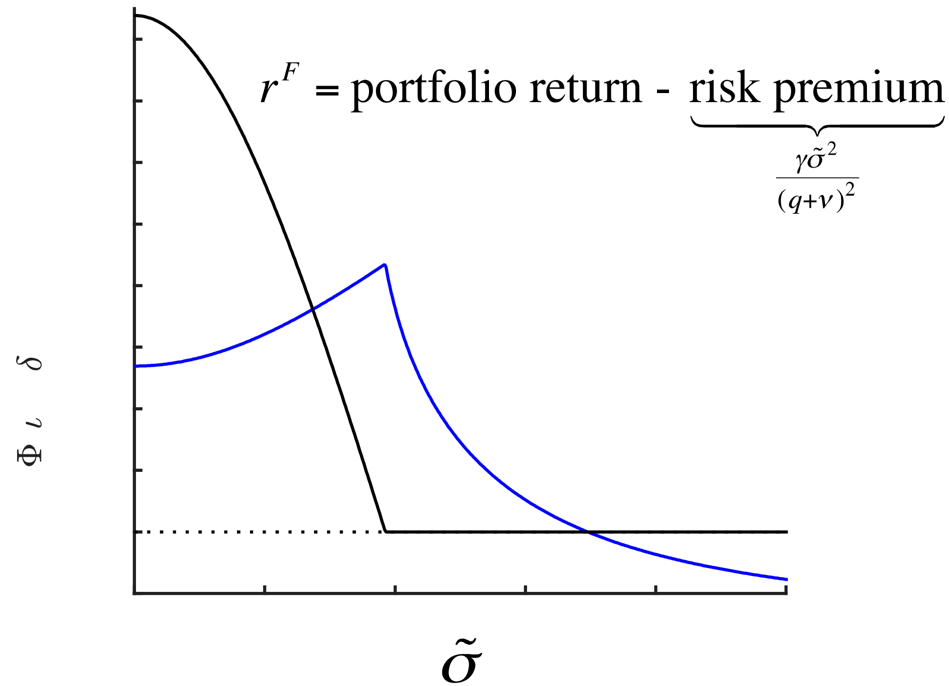
Roadmap

- Only idiosyncratic risk: dollar vs. peso (room for only 1 currency)
- Policy space (one definition): optimal policy ≠ equilibrium
- Only country-wide shocks: equilibrium = optimal policy
- General solution methodology
- General results

Some observations: only idiosyncratic risk

- Without pesos (p = 0), locals choose an optimal portfolio of capital and dollars (up to a borrowing limit of $\underline{q}$ per unit of capital)
- higher $\tilde{\sigma} \Rightarrow$ higher ν := $/K.
- Adjust slowly to the optimal portfolio





$$\rho = 4\%,\ r = 2\%,\ a = 0.14,\ \gamma = 2,\ \kappa = 2,\ \delta = 2\%,\ \underline{q} = 0.6,\ \sigma = 0$$

Some observations: only idiosyncratic risk

- Without pesos (p = 0), locals choose an optimal portfolio of capital and dollars (up to a borrowing limit of $\underline{q}$ per unit of capital)
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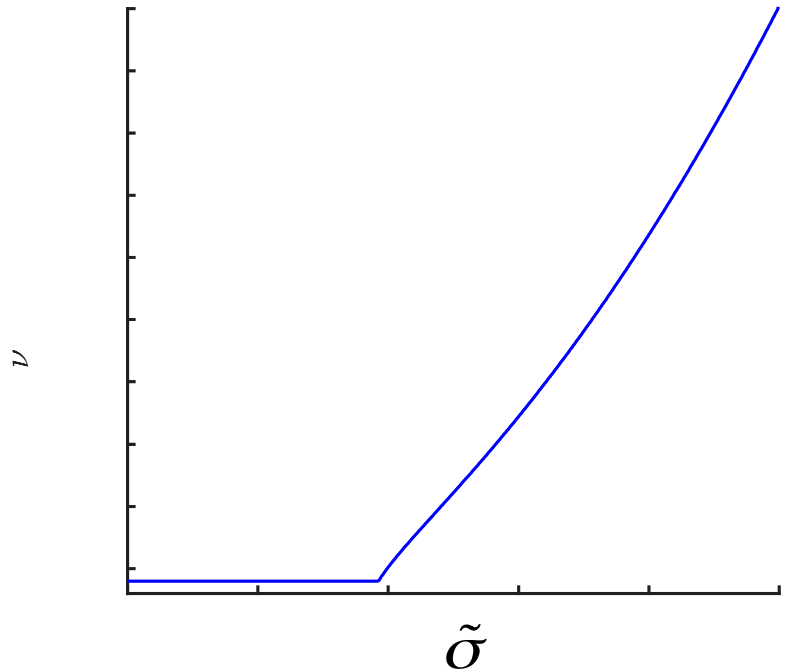
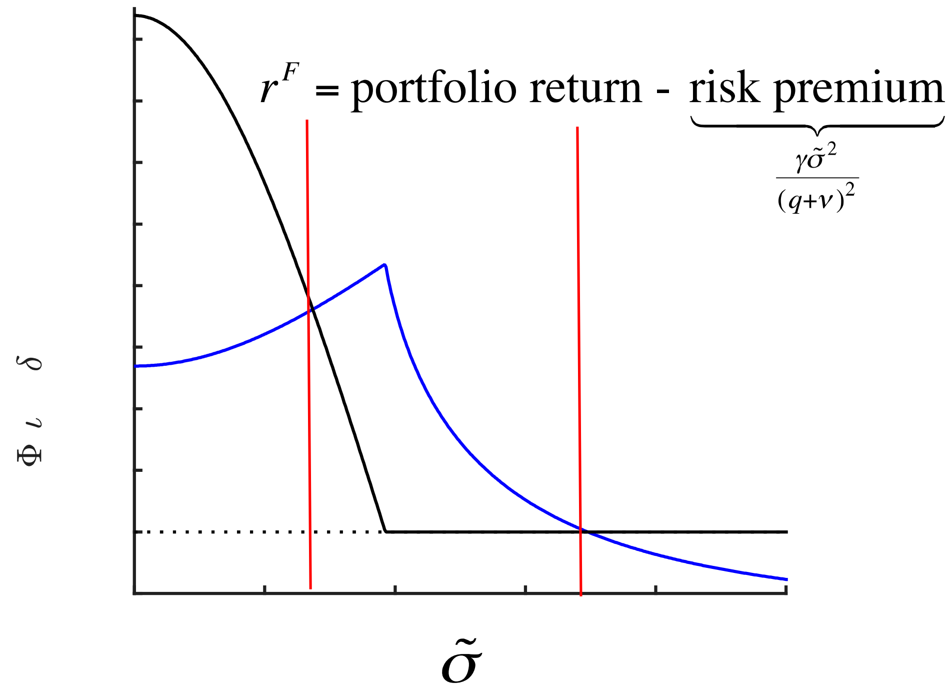
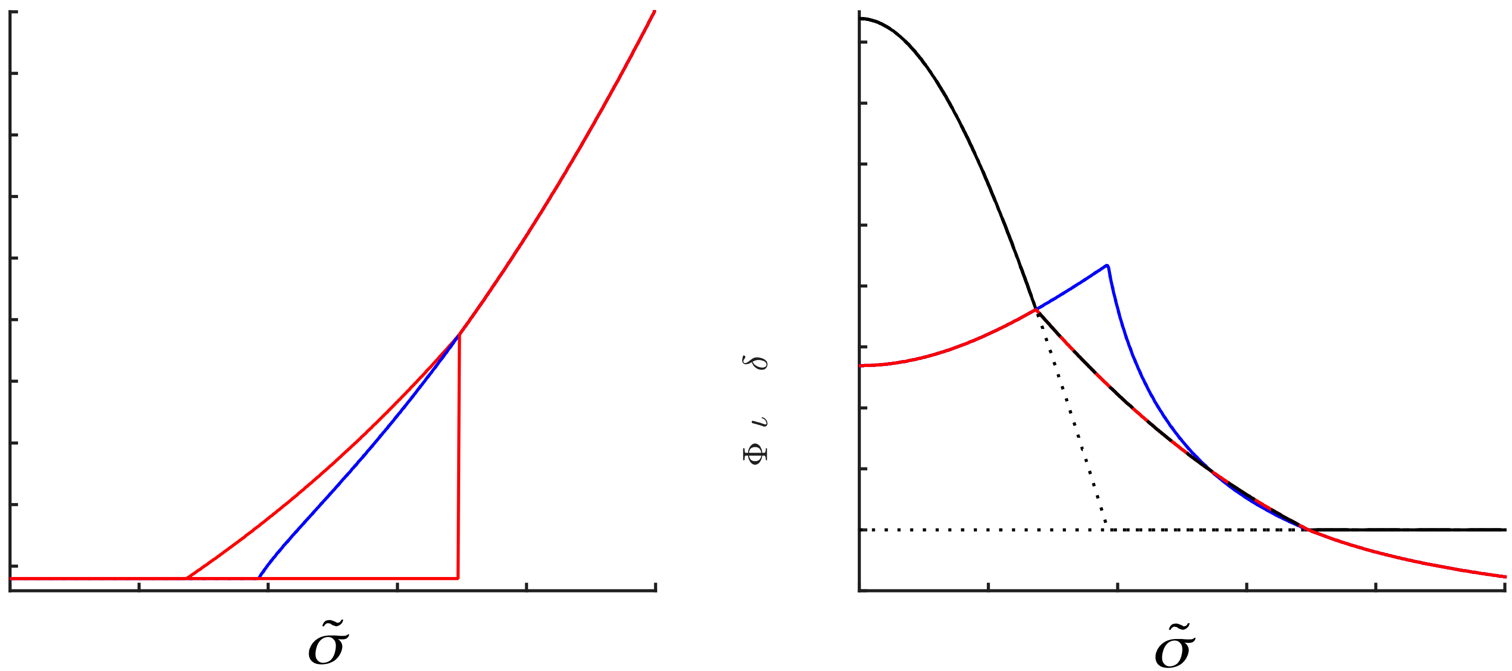




- When Φ(ι) – δ > r^F, the economy is ``bubble-prone”:
- a bubble asset can grow at rate Φ(ι) – δ and have positive value
- Local currency can have value $pK_t > 0$ without fiscal backing

Some observations: only idiosyncratic risk

- Bubble local currency can have value if $\tilde{\sigma}$ is large enough but not too…
- Steady state, r = 2%, ρ = 4%, Φ(ι) = log(κι+1)/κ, with κ = 2, δ = 2%, γ = 2, a = 0.14



- Room for only <u>one</u> currency if σ = 0
- Sudden stop: $\tilde{\sigma}$ unexpectedly goes above 0.35. Peso instantaneously worthless, takes time to save \$ to replace them

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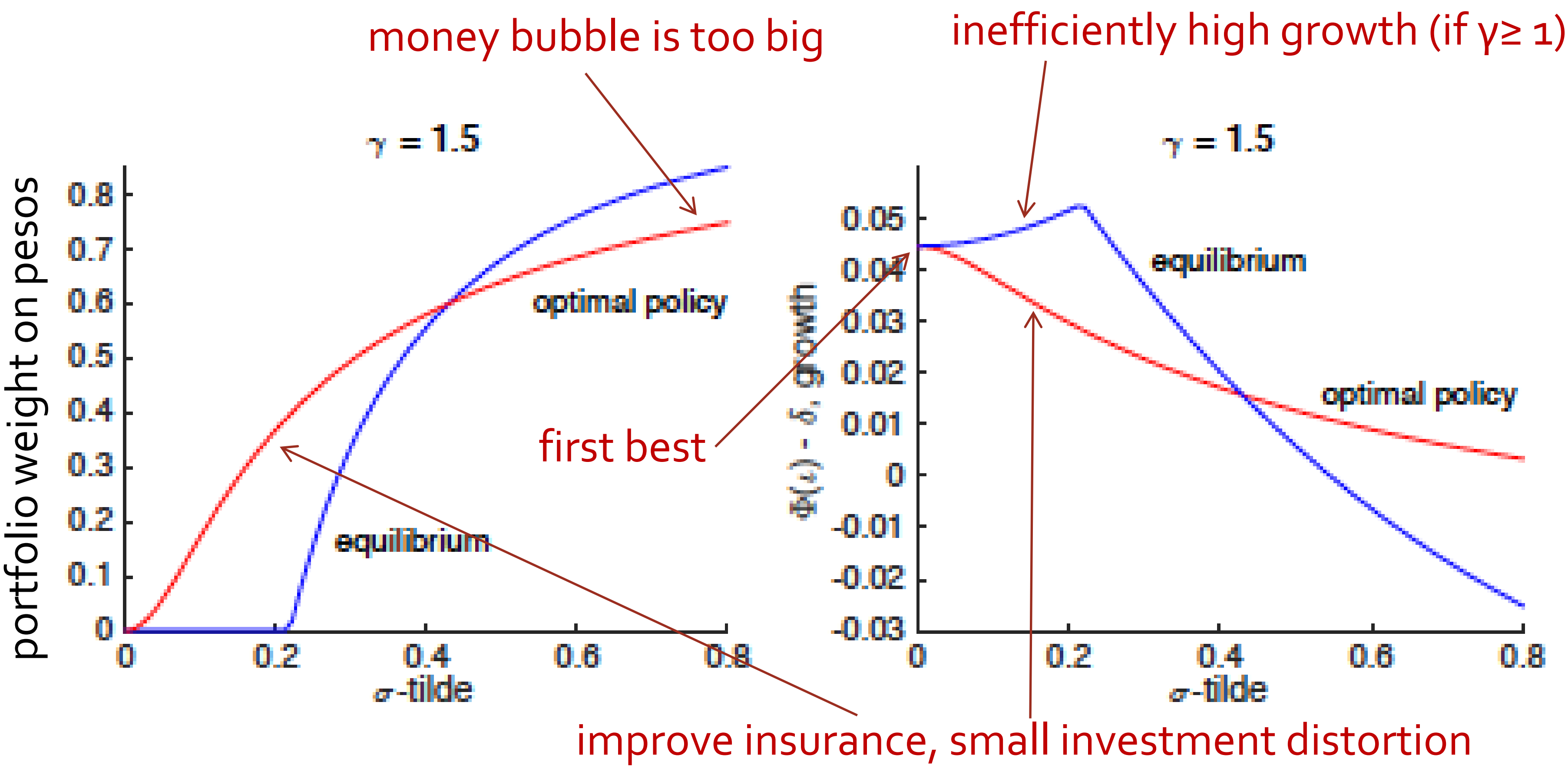
Policy space

- Control $\$_t$ (dollars held in small country) – capital controls
- Pesos (or local government debt) backed by taxes (on capital, or transfers to capital), reserves $\$_t$
  - this is a bit extreme, a smaller policy space without capital controls is also interesting to look at
- #1: trade-off (quantity of local currency): insurance vs investment. Taxing production to back money / gov't debt: raise p (insurance), but lower q (investment)
- #2: amount of $ reserves

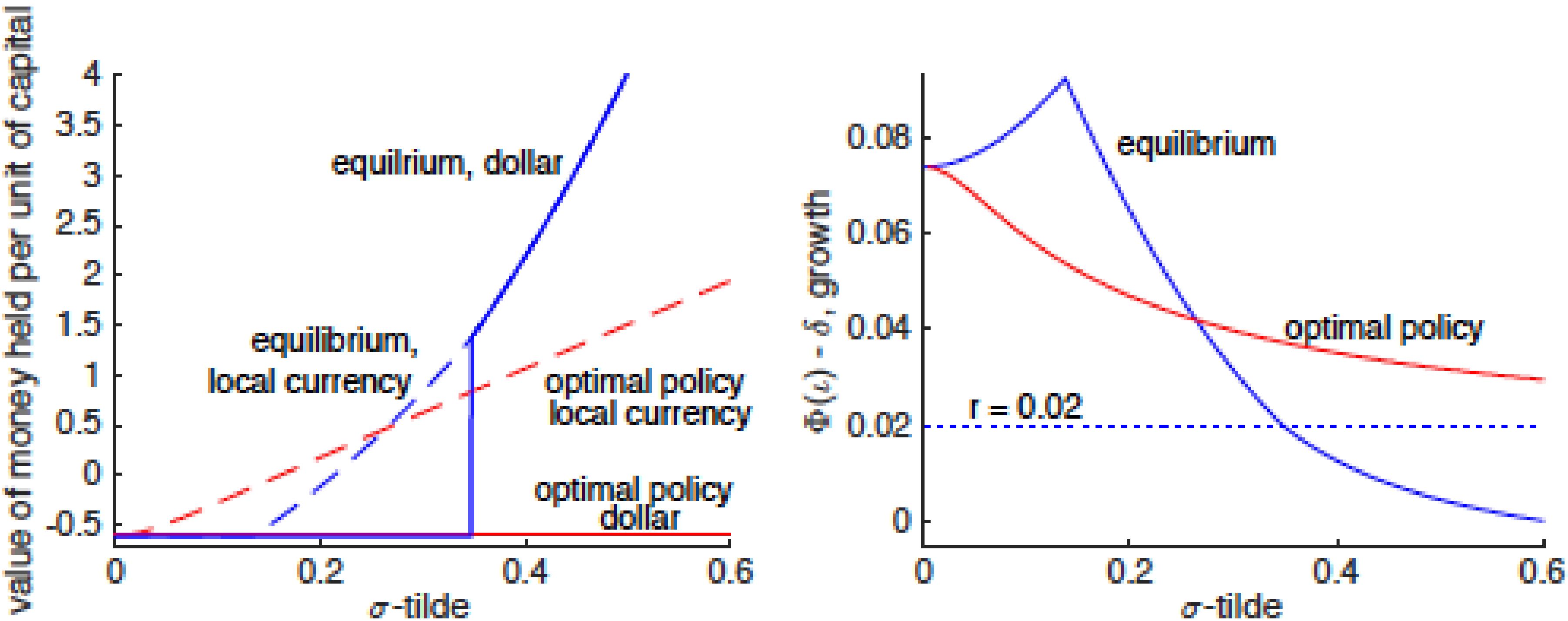
Optimal supply of pesos (isolated economy)

Brunnermeier and Sannikov (2016) "On the optimal inflation rate."



Only idiosyncratic risk, dollar vs. local currency

- Inefficient to use \$ for idiosyncratic risk insurance (country can get similar benefit from pesos and spend the \$)
  - although \$ creates a small perpetuity benefit if r > growth



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Recap

- Idiosyncratic risk creates demand for money (store-of-value)
- Equilibrium: dollar, r vs. peso (growth)
- Local currency (bubble) has value only on a range of $\tilde{\sigma}$
- Planner money supply differs from equilibrium
- Individuals hold more dollars than socially optimal

Country-wide shocks and optimal \$ savings

- With country-wide, but no idiosyncratic shocks, it's a single-agent decision problem (as in Bolton, Chen and Wang 2013)
- Choose ι_t, C_t, capital purchases/sales to maximize utility s.t.

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta)dt - d\underline{\Delta}_t^K + d\bar{\Delta}_t^K, \qquad d\$_t = (r\$_t - C_t)dt + dY_t + (\underline{q}\, K_t d\underline{\Delta}_t^K - \bar{q}\, K_t d\bar{\Delta}_t^K)$$

$$d\nu_t = \nu_t(r - \Phi(\iota_t) + \delta)dt + (a - \iota_t - \underbrace{\zeta_t}_{C_t/K_t})dt + \sigma dZ_t + (\underline{q} + \nu_t)d\underline{\Delta}_t^K - (\bar{q} + \nu_t)d\bar{\Delta}_t^K$$

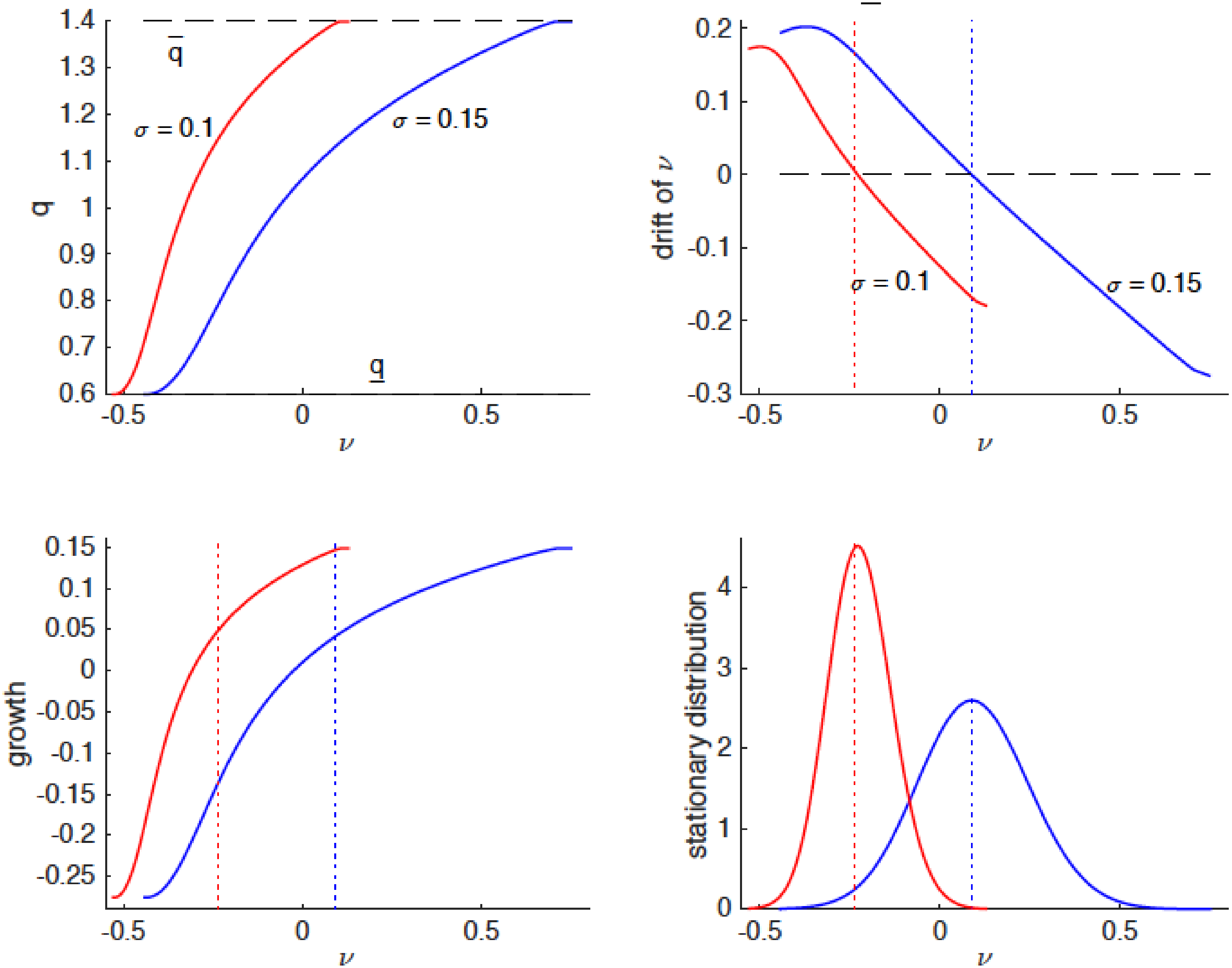
- Value function $\quad f(\nu_t)\frac{K_t^{1-\gamma}}{1-\gamma}, \quad \nu_t \geq -\underline{q}$
- HJB equation $\quad \rho f(\nu_t)\frac{K_t^{1-\gamma}}{1-\gamma}dt = \max_{\zeta_t, \iota_t, d\underline{\Delta}_t^K, d\bar{\Delta}_t^K} \frac{C_t^{1-\gamma}}{1-\gamma}dt + E_t\left[d\left(f(\nu_t)\frac{K_t^{1-\gamma}}{1-\gamma}\right)\right]$
- FOC wrt ι shadow price of capital $\quad q_t = (1-\gamma)\frac{f(\nu_t)}{f'(\nu_t)} - \nu_t$

Country-wide shocks and optimal $ savings

- **Proposition (useful benchmark): without idiosyncratic shocks, the equilibrium is optimal**
- Differences between the equilibrium and optimal policy are due to the idiosyncratic risk friction

Country-wide shocks and optimal $ savings

$$\rho = 4\%,\ r = 2\%,\ a = 0.14,\ \gamma = 2,\ \kappa = 2,\ \delta = 2\%,\ \underline{q} = 0.6,\ \overline{q} = 1.4,\ \tilde{\sigma} = 0$$



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Country-wide shocks and optimal $ savings

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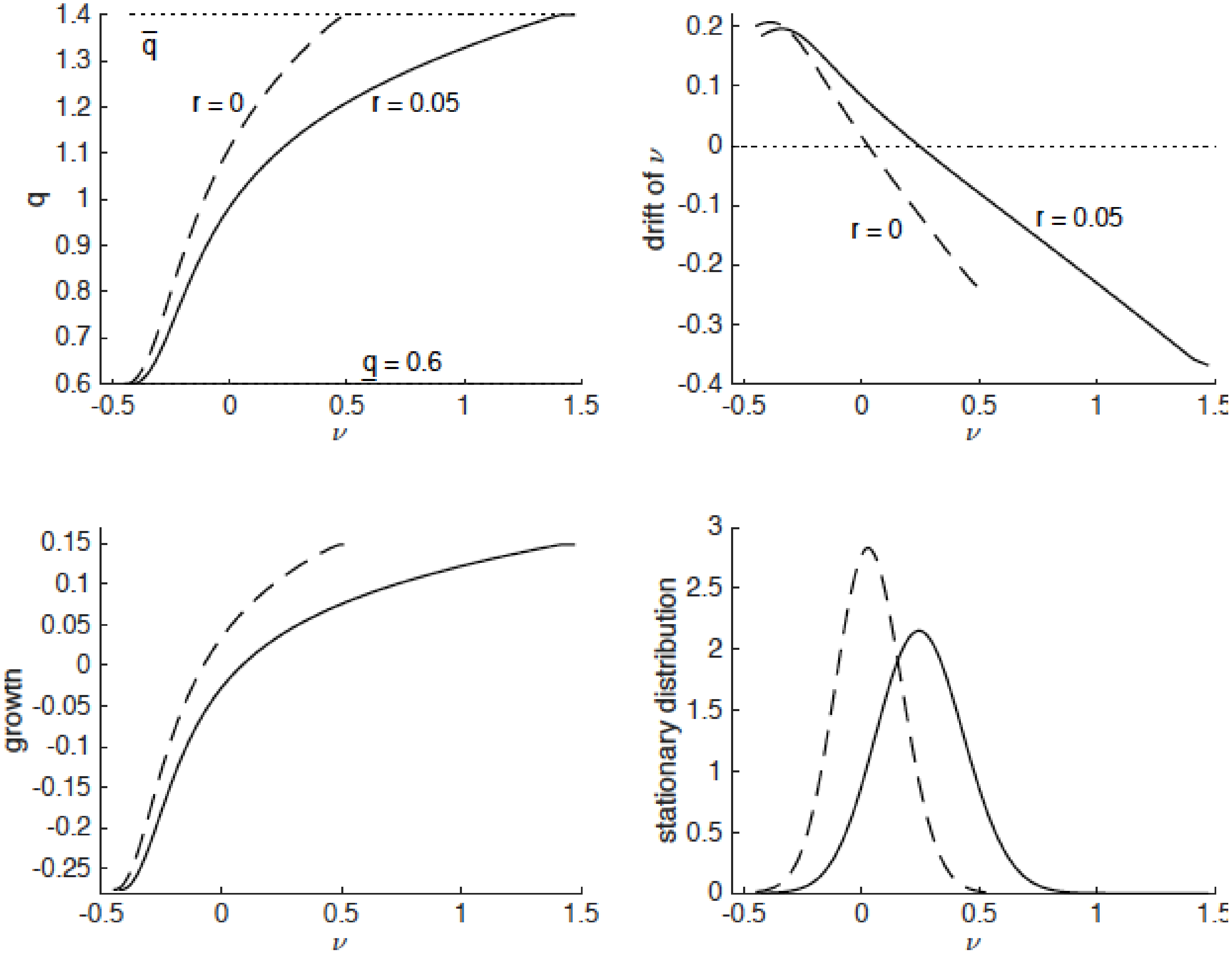


Solving the full model

- Equilibrium: portfolio optimization and market clearing
- portfolio weights

$$\theta_t^{\$} = \frac{\nu_t}{q_t + p_t + \nu_t} \qquad \theta_t^{\hbar} = \frac{p_t}{q_t + p_t + \nu_t} \qquad \theta_t^{k} = \frac{q_t}{q_t + p_t + \nu_t}$$

consumption rate $\zeta_t = C_t/K_t$

$$d\nu_t = \nu_t(r - \Phi(\iota_t) + \delta)dt + (a - \iota_t - \underbrace{\zeta_t}_{C_t/K_t})dt + \sigma dZ_t + \underbrace{(\underline{q} + \nu_t)d\underline{\Delta}_t^K}_{>0 \text{ when } q_t = \underline{q}} - \underbrace{(\bar{q} + \nu_t)d\bar{\Delta}_t^K}_{>0 \text{ when } q_t = \bar{q}}$$

- Next: characterize equilibrium using 3 equation solved via value function iteration

Equilibrium conditions

Value functions $f(v_t)\frac{k_t^{1-\gamma}}{1-\gamma}$

Diff-ate (wealth per unit of k $\frac{f(v_t)k_t^{-\gamma}}{p_t+q_t+v_t}$) $\Rightarrow$ opt. cons. $c_t^{-\gamma}=\frac{f(v_t)k_t^{-\gamma}}{p_t+q_t+v_t}$

SDF $\frac{e^{-\rho t}f(v_t)k_t^{-\gamma}}{p_t+q_t+v_t}$ should price any asset / portfolio

- Whole portfolio (value $(p_t + q_t + v_t)k_t$)

$$\frac{\zeta_t}{p_t+q_t+v_t}+E_t[\text{growth}(e^{-\rho t}f(v_t)k_t^{1-\gamma})]=0$$

- Pesos (value p_tK_t)

$$E_t\left[\text{growth}\left(e^{-\rho t}\frac{f(v_t)p_t}{p_t+q_t+v_t}K_tk_t^{-\gamma}\right)\right]=0$$

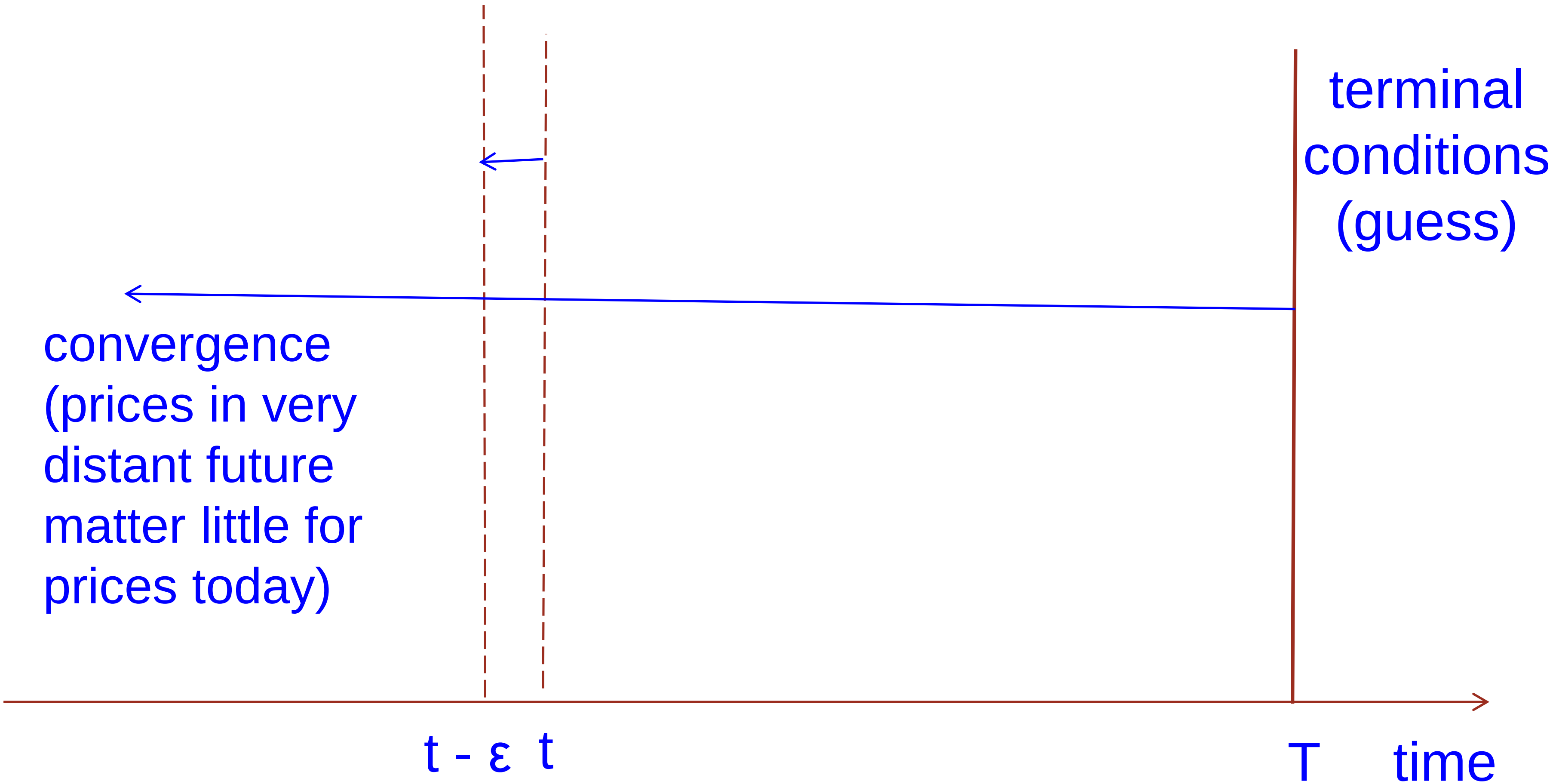
- Dollars (value K_t, dividend $r - \Phi(\iota) + \delta$)

$$r-\Phi(\iota_t)+\delta+E_t\left[\text{growth}\left(e^{-\rho t}\frac{f(v_t)}{p_t+q_t+v_t}K_tk_t^{-\gamma}\right)\right]=0$$

same form as HJB, solved by value iteration, as a system

- Finally, $\iota_t=\arg\max_{\iota}\ \Phi(\iota)q_t-\iota$

Computation



Equilibrium equations: summary

$$\frac{\zeta_t}{p_t + q_t + v_t} + E_t[\text{growth}(e^{-\rho t} f(v_t) k_t^{1-\gamma})] = 0$$

$$E_t\left[\text{growth}\left(e^{-\rho t} \frac{f(v_t) p_t}{p_t + q_t + v_t} K_t k_t^{-\gamma}\right)\right] = 0$$

$$r - \Phi(\iota_t) + \delta + E_t\left[\text{growth}\left(e^{-\rho t} \frac{f(v_t)}{p_t + q_t + v_t} K_t k_t^{-\gamma}\right)\right] = 0$$

$$\iota_t = \arg\max_{\iota} \ \Phi(\iota) q_t - \iota \qquad \zeta_t^{-\gamma} = \frac{f(v_t)}{p_t + q_t + v_t}$$

Policy problem

policy maker maximizes the value function, but can affect prices
decisions at any time point (investment / consumption) are functions of prices

$$\max_{p,q} \quad \frac{\zeta_t}{p_t+q_t+v_t} + E_t[\text{growth}(e^{-\rho t} f(v_t) k_t^{1-\gamma})] = 0$$

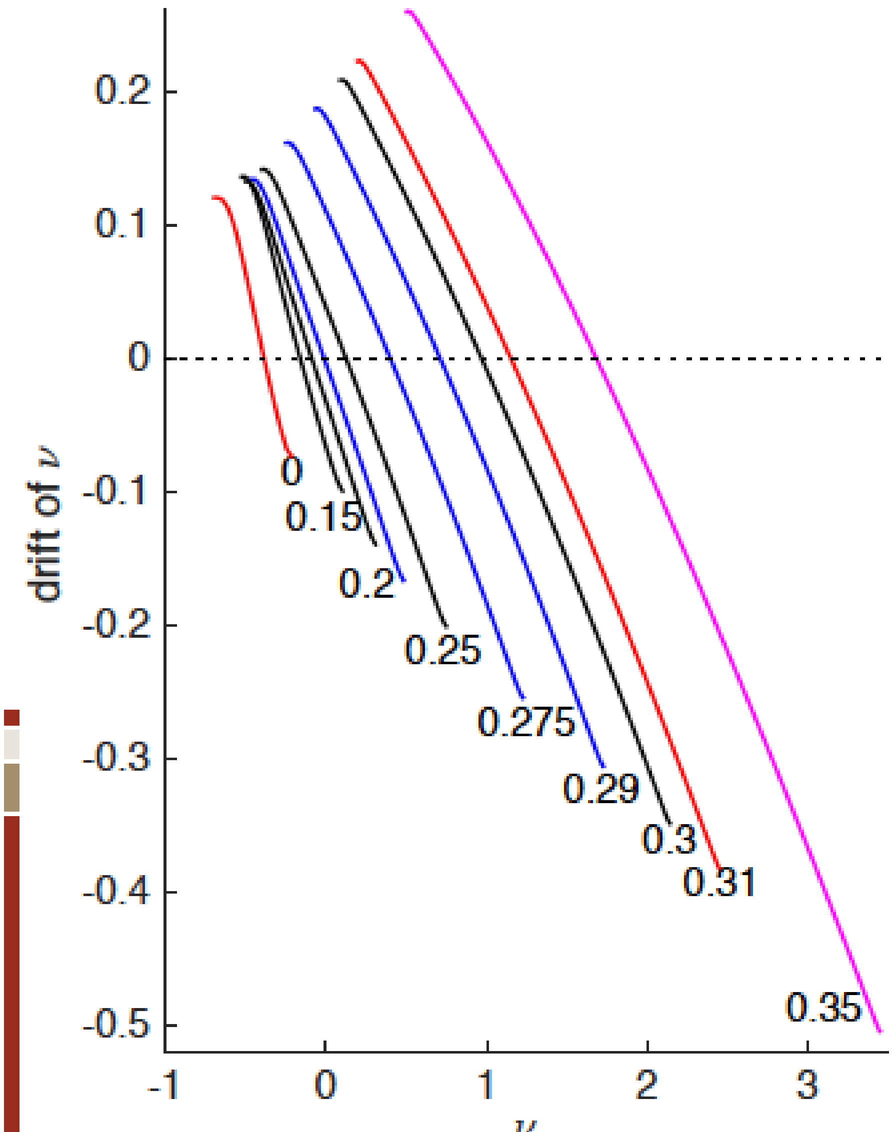
$$E_t\left[\text{growth}\left(e^{-\rho t}\frac{f(v_t)p_t}{p_t+q_t+v_t}K_t k_t^{-\gamma}\right)\right] = 0$$

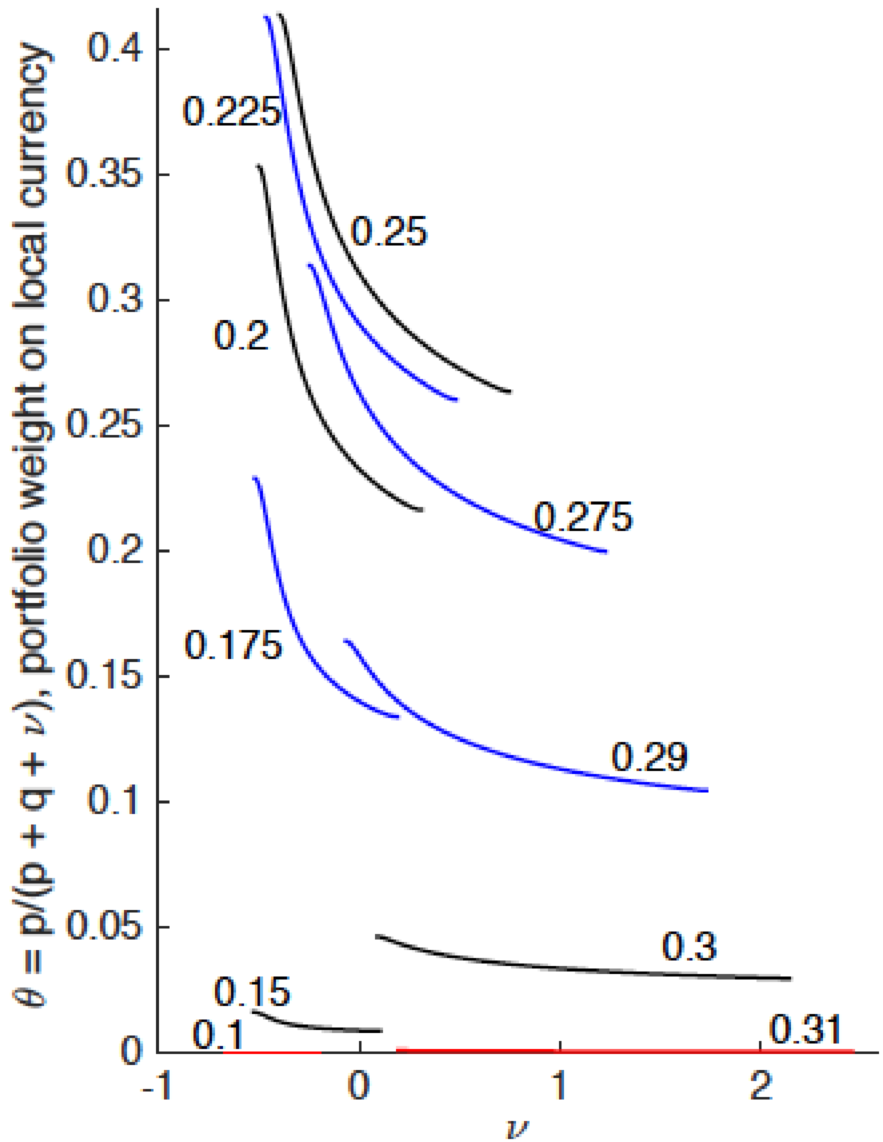
$$r - \Phi(\iota_t) + \delta + E_t\left[\text{growth}\left(e^{-\rho t}\frac{f(v_t)}{p_t+q_t+v_t}K_t k_t^{-\gamma}\right)\right] = 0$$

s.t. $\quad \iota_t = \arg\max_{\iota} \; \Phi(\iota)q_t - \iota \qquad \zeta_t^{-\gamma} = \dfrac{f(v_t)}{p_t+q_t+v_t}$

Dollar savings, Peso value as functions of $\tilde{\sigma}$

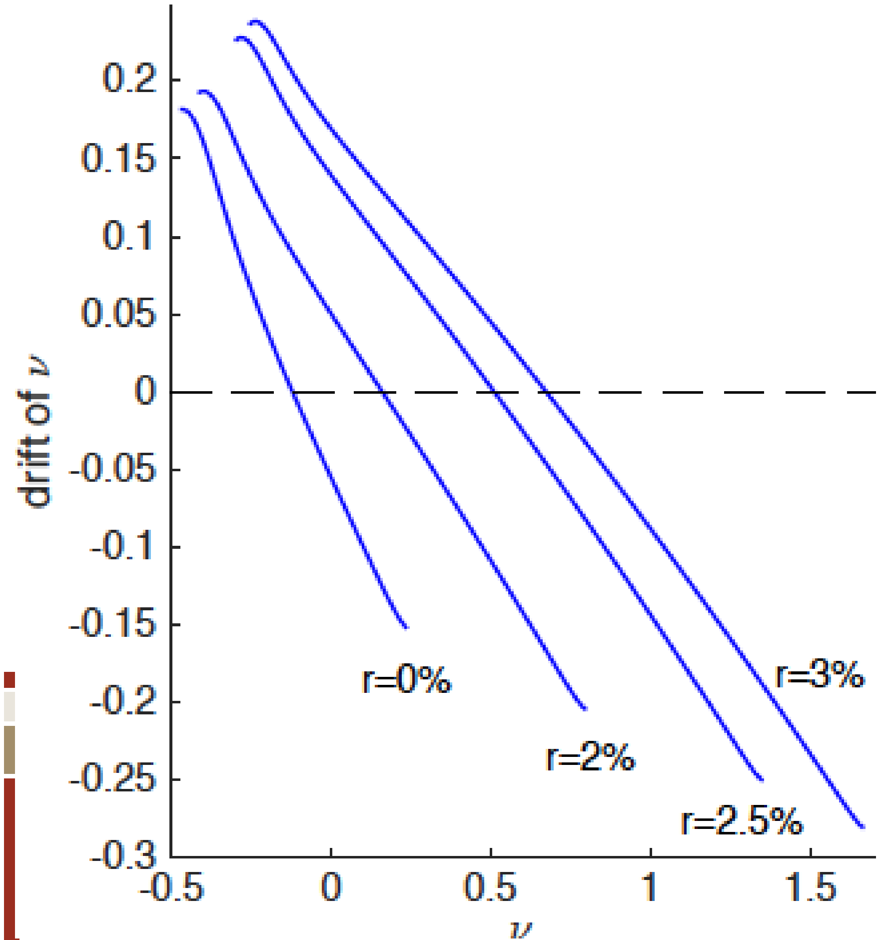
$\rho = 4\%,\ r = 2\%,\ a = 0.14,\ \gamma = 2,\ \kappa = 2,\ \delta = 2\%,\ \underline{q} = 0.8,\ \bar{q} = 1.3,\ \sigma = 0.1$

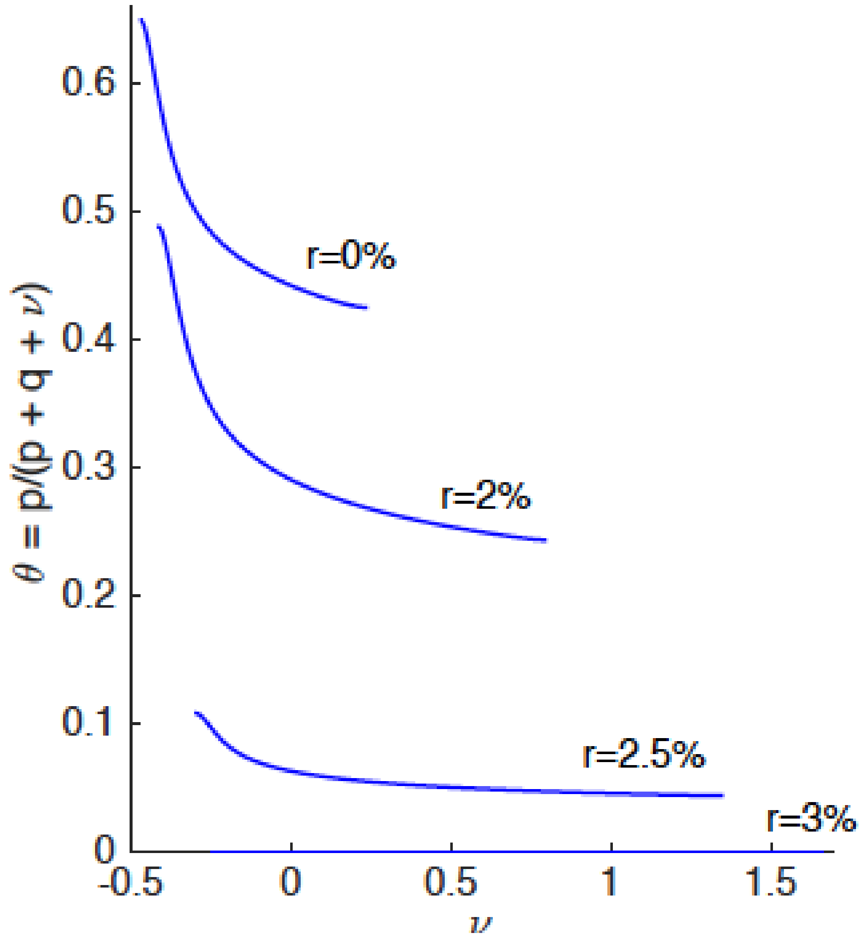




Spillovers (from dollar rates)

$$\rho = 4\%,\ a = 0.14,\ \gamma = 2,\ \kappa = 2,\ \delta = 2\%,\ \underline{q} = 0.8,\ \overline{q} = 1.3,\ \sigma = 0.1,\ \tilde{\sigma} = 0.25$$





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Equilibrium vs. optimal policy

- Recall (from models with only one type of a shock)

(1) optimal vs. equilibrium peso supply depends on $\tilde{\sigma}$

(2) individuals hold more dollars than socially optimal

(3) when $\tilde{\sigma} = 0,$ equilibrium outcome is socially optimal

Equilibrium vs. optimal policy

- Recall (from models with only one type of a shock)

(1) optimal vs. equilibrium peso supply depends on $\tilde{\sigma}$

(2) individuals hold more dollars than socially optimal

(3) when $\tilde{\sigma} = 0,$ equilibrium outcome is socially optimal

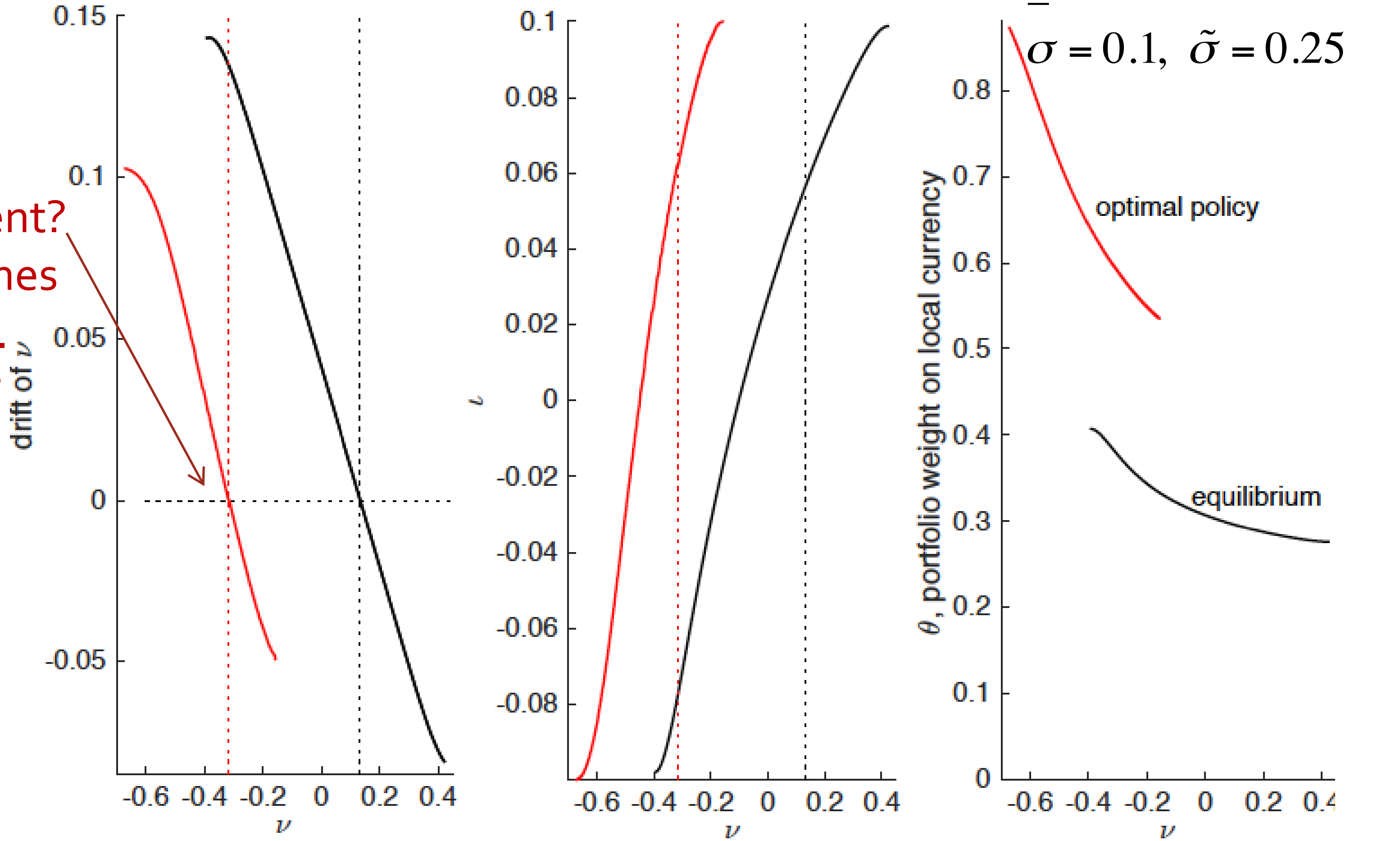
$\rho = 4\%, \ a = 0.14$

$\gamma = 2, \ \kappa = 2, \ \delta = 2\%$

$\underline{q} = 0.8, \ \bar{q} = 1.2$

$\sigma = 0.1, \ \tilde{\sigma} = 0.25$

Is this imprudent? what determines socially opt. $ holdings?



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Equilibrium vs. optimal policy and $\tilde{\sigma}$

- Optimal \$ holdings determined primarily by σ (not $\tilde{\sigma}$)

low sensitivity to $\tilde{\sigma}$.

$\rho = 4\%$

$r = 2\%$

$a = 0.14$

$\gamma = 2$

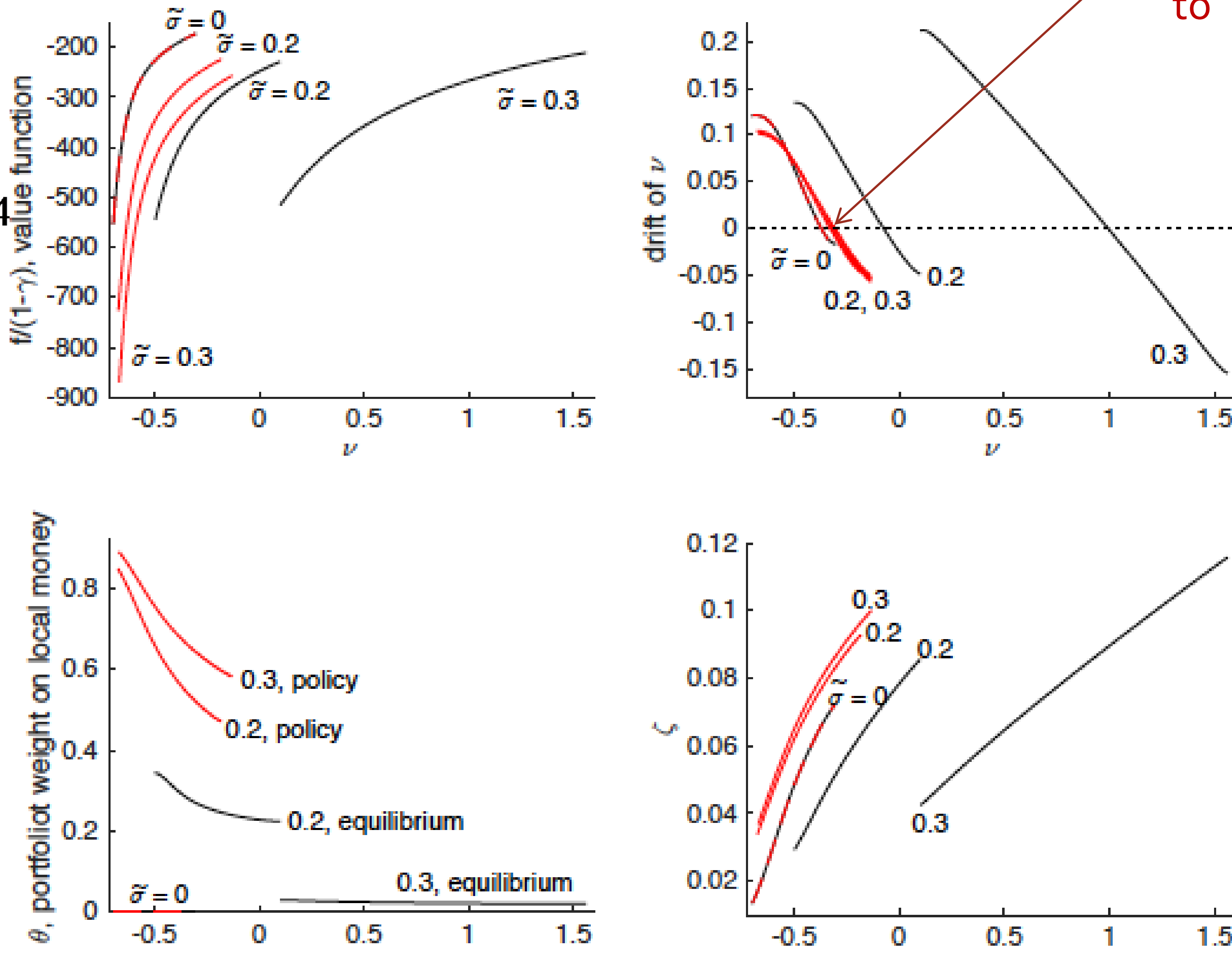
$\kappa = 2$

$\delta = 2\%$

$\underline{q} = .8$

$\bar{q} = 1.2$

$\sigma = .1$



Some other issues

- Without capital controls: we haven't solved for the optimal policy, but fiscal backing of local currency can discourage locals hoarding too many $, improve welfare partway
- The assumption that idiosyncratic risk is uninsurable is easier to digest than one for country-wide risk… if that risk can be hedged (at a price), then individuals still need to hold dollars to absorb the risk they choose to retain – equilibrium should be similar (a conjecture)… but policy can also distort the amount of aggregate risk country chooses to hedge (interesting)
- Policy, to be effective, has to be accompanied with restrictions (individuals can trade to undo the policy – irrelevance result)

Conclusions

- Endogenous value of money
  - local currency: insurance against idiosyncratic risk
  - global currency: insurance for country-wide and idiosyncratic risk
  - exchange rate is endogenous
- Equilibrium effects
  - currencies are imperfect substitutes (for hedging idiosyncratic risk)
  - value of local currency $\cap$ - shaped in idiosyncratic risk
  - large country's monetary policy can have strong spillovers on small economy and the value of its currency
- Optimal policy
  - without idiosyncratic risk, equilibrium is optimal
  - optimal dollar savings increasing in σ, idiosyncratic risk matters little
  - equilibrium dollar savings are rising in idiosyncratic risk (render local currency worthless at some point)

Thank you!

Conclusion

- Endogenous value of money in 2 countries
  - Local currency: better hedge for idiosyncratic risk (non-tradeable consumption)
  - Global currency: hedge against ToT + export productivity shocks
  - Exchange rate endogenous
- Spillover effects from foreign monetary policy
- Nuanced Mundell-Fleming Trilemma
  - Local and global money have different risk profile (imperfect substitutes)
    ⇒ increases MoPo space
  - Too high inflation:
    local citizens substitute local currency for global currency
    ⇒ limits MoPo space
- Central Bank's foreign reserves holding: Irrelevance Result
- Optimal Monetary Policy
  - Idiosyncratic risk – correct pecuniary externality (real interest rate)
  - International savings

Foreign Currency Reserves

- Irrelevance Theorem: If central bank holds global money (reserves) citizens in small country will hold accordingly less

- Remarks:
  - If central banks holds more $-reserves than citizens would like to hold, then agents borrow foreign currency from abroad
  - If local money is worthless (without foreign reserves), then the value of local money derives only from the reserves
  - If government borrows $, small country citizens will hold accordingly more $
  - With fiscal backing of the local money, there are more nuances

Some intuition: only idiosyncratic risk…

- Scale invariance ⇒ state variable $\nu_t = \$_t/K_t$. Portfolio weights

$$\theta_t^{\$} = \frac{\nu_t}{q_t + p_t + \nu_t} \qquad \theta_t^{\hbar} = \frac{p_t}{q_t + p_t + \nu_t} \qquad 1 - \theta_t^{\$} - \theta_t^{\hbar} = \frac{q_t}{q_t + p_t + \nu_t}$$

$$\text{risk} = 0 \qquad 0 \qquad \frac{\tilde{\sigma}}{q_t} d\tilde{Z}_t$$

- Overall portfolio has risk $\dfrac{\tilde{\sigma}}{q_t + p_t + \nu_t} d\tilde{Z}_t$

- Price of risk $= \dfrac{\gamma\tilde{\sigma}}{q_t + p_t + \nu_t}$

$$\text{risk-free rate} = \text{return on capital} - \frac{\gamma\tilde{\sigma}}{q_t + p_t + \nu_t}\frac{\tilde{\sigma}}{q_t} = \max(\Phi(\iota) - \delta, r)$$

Some intuition

- Scale invariance ⇒ state variable $v_t = \$_t/K_t$. Portfolio weights

$$\theta_t^{\$} = \frac{v_t}{q_t + p_t + v_t} \qquad \theta_t^{\hbar} = \frac{p_t}{q_t + p_t + v_t} \qquad 1 - \theta_t^{\$} - \theta_t^{\hbar} = \frac{q_t}{q_t + p_t + v_t}$$

$$\text{risk } = 0 \qquad \text{some country-wide risk} \qquad \text{risk} \sim \frac{\sigma}{q_t} dZ_t \text{ and } = \frac{\tilde{\sigma}}{q_t} d\tilde{Z}_t$$

- Overall portfolio has risk $\sim \frac{\sigma}{q_t + v_t} dZ_t$ and $\frac{\tilde{\sigma}}{q_t + p_t + v_t} d\tilde{Z}_t$

- Price of risk $\sim \frac{\gamma\sigma}{q_t + v_t}$ and $= \frac{\gamma\tilde{\sigma}}{q_t + p_t + v_t}$

Only idiosyncratic risk…

- Scale invariance ⇒ state variable $v_t = \$_t/K_t$. Portfolio weights

$$\theta_t^{\$} = \frac{v_t}{q_t + p_t + v_t} \qquad \theta_t^{\hbar} = \frac{p_t}{q_t + p_t + v_t} \qquad 1 - \theta_t^{\$} - \theta_t^{\hbar} = \frac{q_t}{q_t + p_t + v_t}$$

risk $= 0$ some country-wide risk risk $\sim \frac{\sigma}{q_t} dZ_t$ and $= \frac{\tilde{\sigma}}{q_t} d\tilde{Z}_t$

- Overall portfolio has risk $\sim \frac{\sigma}{q_t + v_t} dZ_t$ and $\frac{\tilde{\sigma}}{q_t + p_t + v_t} d\tilde{Z}_t$

- Price of risk $\sim \frac{\gamma \sigma}{q_t + v_t}$ and $= \frac{\gamma \tilde{\sigma}}{q_t + p_t + v_t}$

$$\text{risk-free rate} = \text{return on capital} - \frac{\gamma \tilde{\sigma}}{q_t + p_t + v_t} \frac{\tilde{\sigma}}{q_t} = \max(\Phi(\iota) - \delta, r)$$

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Motivation/Results

- Two currencies, exogenous exchange rate
  - Local currency: insure idiosyncratic risk (as in Bewley)
  - Global $-currency: save against country-wide risk
- Value of local currency ∩-shaped in idiosyncratic risk
  - No idiosyncratic risk no role for local currency
  - No country-wide risk no role for coexistence of 2 currencies
- Dollar monetary policy has significant spillovers on SOE
- UIP violation (due to risk premia)
- Sudden Stop
  - Increase in idiosyncratic risk or rise in $ interest rate