



Lecture 04

A Symmetric International Model with Runs/Sudden Stops

Markus Brunnermeier & Yuliy Sannikov

Overview

■ So far

- Multi/two-type/sector model **asymmetric**
 - Experts: more productive $a^e k_t$ less patient ρ^e
 - Households: less productive $a^h k_t$ more patient $\rho^e < \rho^h$
- Focus on equilibrium without jumps/runs

■ Now

- Two type/sector model **symmetric**
 - Symmetric productivity – 2 goods, one country more productive in one
 - Same preference discount rates
- 2 Brownian shocks
- $A(\psi)$ is micro-founded with 2 good economy
- Asset price run \neq depositor run a la Diamond-Dybvig

Role of (international) financial markets

1. Better allocation of physical capital/resources
 - ψ
2. Better allocation of risk (sharing)
 - χ

depends on future

- Complete markets
 - (1) and (2) can be controlled separately
 - Pecuniary externalities have 2nd order w-effects
- Frictions/incomplete markets $F(\psi, \chi) \leq 0$, e.g. $\psi = \chi$
 - (1) and (2) are interlinked
 - Pecuniary externalities have welfare effects

First Best

Second Best

||| Pecuniary Externalities: Some Literature

- Constrained inefficiency, pecuniary/firesale externalities
 - Incomplete markets:
 - Stiglitz 1982, Newsbury & Stiglitz 1984, Geanakoplos & Polemarchakis 1986, He & Kondor 2013
 - Debt collateral constraint (that depends on price):
 - Stiglitz & Greenwald, Lorenzoni 2005, Bianchi 2011, Bianchi & Mendoza 2012, Jeanne & Korinek 2012, Stein 2012,
 - Davilia & Korinek 2017, ...
- “terms of trade hedge”
 - Cole & Obstfeld 1991, Martin 2010

International Macro Interpretation

- Old “Washington consensus” in decline
 - Free trade: flow of goods/services intratemporal
 - Free finance: flow of capital intertemporal
- When does full capital account liberalization reduce (capital controls/macropu regulation improve) welfare?

Brunnermeier, Markus & Yuliy Sannikov.
“International Credit Flows and Pecuniary Externalities”.
American Economic Journal: Macroeconomics 71 (2015): , 7, 1, 297-338.

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 1. **Sudden stop** including **runs** due to liquidity mismatch
 - Technological illiquidity: irreversibility (adjustment costs)
 - Market illiquidity: redeployability/specificity – not this paper
 - Funding illiquidity: **short-term debt, “hot money”**
 - Type of capital flow matters: FDI, portfolio flows (equity), long-term debt

Liability Asset side

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 - Funding illiquidity: short-term debt, “hot money”
 - Type of capital flow matters: FDI, portfolio flows (equity), long-term debt
 2. **“Terms of trade hedge”** (Cole-Obstfeld) can be undermined when
 - Industry’s output is not easily substitutable. Consumers cannot easily find substitutes
 - No strong competitors in other countries
 - Natural resources: oil, copper for Chile,
 - Hard drives in Thailand, Bananas in Ecuador

Model setup - symmetric

- Preferences

$$E \left[\int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt \right]$$

- Same preference discount rate ρ – “saving out of constraint”

- Two output goods y^a and y^b - imperfect substitutes

$$y_t = \left[\frac{1}{2} (y_t^a)^{\frac{s-1}{s}} + \frac{1}{2} (y_t^b)^{\frac{s-1}{s}} \right]^{s/(s-1)}$$

- (Comparative) advantages:

| | Good a | Good b |
|-----------|--------------------|--------------------|
| Country A | $\bar{a}k_t$ | $\underline{a}k_t$ |
| Country B | $\underline{a}k_t$ | $\bar{a}k_t$ |

Two country/sector/type model

- World capital shares:

$$\psi_t^{Aa} + \psi_t^{Ab} + \psi_t^{Ba} + \psi_t^{Bb} = 1$$

- World supply of (output) goods:

$$Y_t^a = (\psi_t^{Aa}\bar{a} + \psi_t^{Ba}\underline{a})K_t \quad Y_t^b = (\psi_t^{Bb}\bar{a} + \psi_t^{Ab}\underline{a})K_t$$

- Plug into $y_t = \left[\frac{1}{2}(y_t^a)^{\frac{s-1}{s}} + \frac{1}{2}(y_t^b)^{\frac{s-1}{s}} \right]^{s/(s-1)}$ to get $A(\psi_t)$

Two country/sector/type model

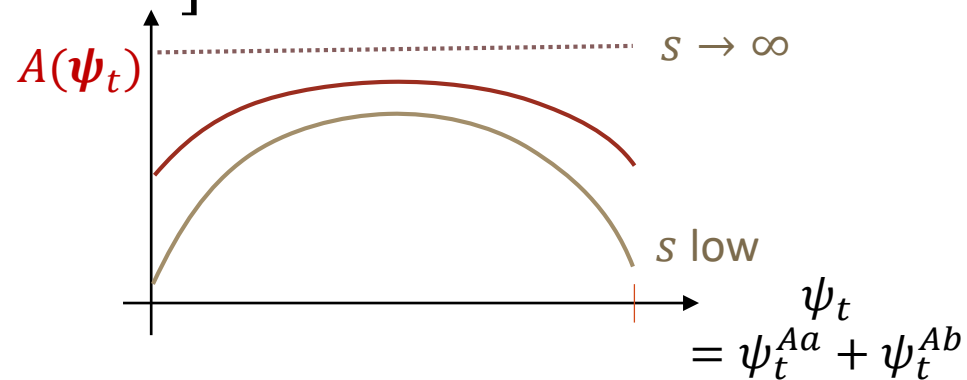
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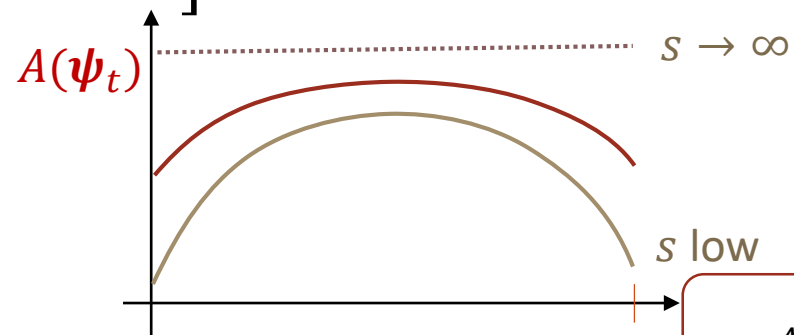
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- Price of output goods a and b in terms of price of y

$$P_t^a = \frac{1}{2} \left(\frac{Y_t}{Y_t^a} \right)^{1/s} \quad \text{and} \quad P_t^b = \frac{1}{2} \left(\frac{Y_t}{Y_t^b} \right)^{1/s}$$

- Terms of trade P_t^a / P_t^b

Two country/sector/type model

■ Capital evolution for

- $dk_t = (\Phi(l_t) - \delta)k_t dt + \sigma^A k_t dZ_t^A$ in country A

- $dk_t = (\Phi(l_t) - \delta)k_t dt + \sigma^B k_t dZ_t^B$ in country B

- Φ concavity – technological illiquidity

- Single type of capital

- Investment in composite good

Now there are
2 independent
Brownian

■ Shocks are

- Two dimensional

- Affect global capital stock $dZ_t^A + dZ_t^B$

- Redistributive (initial shock + amplification) \Rightarrow affects wealth share, η_t

- Example: Apple vs. Samsung lawsuit

Market structures

Trade

Finance

| Markets | Output y^a, y^b | Physical capital K | Debt | Equity |
|---|----------------------|-------------------------|------|--------|
| Complete Markets Full integration/First Best | X | X | X | X |
| Open credit account (equity home bias) | X | X | X | |
| Closed credit account | X | X | | |

Add taxes/capital controls

intra-temporal

inter-temporal

Solving MacroModels Step-by-Step

0. Postulate aggregates, price processes & obtain return processes
1. For given SDF processes *static*
 - a. Real investment ι , (portfolio θ , & consumption choice of *each agent*)
 - *Toolbox 1: Martingale Approach*
 - b. Asset/Risk Allocation *across types/sectors* & asset market clearing
 - *Toolbox 2: “price-taking social planner approach” – Fisher separation theorem*
2. Value functions *backward equation*
 - a. Value fcn. as fcn. of individual investment opportunities ω
 - *Special cases*
 - b. De-scaled value fcn. as function of state variables η
 - *Digression: HJB-approach (instead of martingale approach & envelop condition)*
 - c. Derive ζ -risk premia, C/N -ratio from value fcn. envelop condition
3. Evolution of state variable η *forward equation*
 - *Toolbox 3: Change in numeraire to total wealth (including SDF)*
 - (“Money evaluation equation” μ^ϑ)
4. Value function iteration & goods market clearing
 - a. PDE of de-scaled value fcn.
 - b. Value function iteration by solving PDE

0. Postulate price process & derive returns

- $dk_t/k_t = (\Phi(l_t) - \delta)dt + \sigma^A dZ_t^A$

■ Postulate

- $dq_t/q_t =$

- $d\xi_t^A/\xi_t^A =$

- $d\xi_t^B/\xi_t^B =$

Poll 15: Do these postulated processes depend on

a) For $\frac{d\xi_t^A}{\xi_t^A}$ on dZ^A

b) For $\frac{d\xi_t^B}{\xi_t^B}$ on dZ^B

c) On both Brownians

0. Postulate price process & derive returns

- $dk_t/k_t = (\Phi(l_t) - \delta)dt + \sigma^A dZ_t^A$

■ Postulate

- $dq_t/q_t = \mu_t^q dt + \sigma_t^{qA} dZ_t^A + \sigma_t^{qB} dZ_t^B$

- $d\xi_t^A/\xi_t^A = -r_t^{A,F} dt - \zeta_t^{AA} dZ_t^A - \zeta_t^{AB} dZ_t^B$

- $d\xi_t^B/\xi_t^B = -r_t^{B,F} dt - \zeta_t^{BA} dZ_t^A - \zeta_t^{BB} dZ_t^B$

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- $d\xi_t^B/\xi_t^B = -r_t^{B,F} dt - \zeta_t^{BA} dZ_t^A - \zeta_t^{BB} dZ_t^B$

■ Returns from holding physical capital

- $dr_t^{Aa} = \left(\frac{\bar{a}P_t^a - l_t}{q_t} + \mu_t^q + \Phi(l_t) - \delta + \sigma^A \sigma_t^{qA} \right) dt +$

$$+ (\sigma^A + \sigma_t^{qA}) dZ_t^A + \sigma_t^{qB} dZ_t^B$$

- $dr_t^{Ab} = \left(\frac{\underline{a}P_t^b - l_t}{q_t} + \mu_t^q + \Phi(l_t) - \delta + \sigma^A \sigma_t^{qA} \right) dt +$

$$+ (\sigma^A + \sigma_t^{qA}) dZ_t^A + \sigma_t^{qB} dZ_t^B$$

Ito product rule:
 $d(X_t Y_t) = dX_t Y_t + X_t dY_t + \sigma_X \sigma_Y dt$

Step 1a. Optimal Reinvestment Rate ι

- Tobin's q -- as before a simple static problem

$$\Phi'(\iota_t) = 1/q_t$$

- All agents $\iota^i = \iota$
- Special functional form:
 - Quadratic adjustment cost
 - Investment rate of $\iota = \Phi + \frac{1}{\kappa} \Phi^2$
generates new capital at rate Φ
 - $\Phi(\iota) = \frac{1}{\kappa} (\sqrt{1 + 2\kappa\iota} - 1)$
 - Alternative specification: $\Phi(\iota) = \frac{1}{\kappa} \log(\kappa\iota + 1) \Rightarrow \kappa\iota = q - 1$

Step 1a. Asset pricing equations

■ Martingale approach

- recall discrete time analog
- $\xi_t^A p_t = E_t[\xi_{t+s}^A (p_{t+s} + d_{t+s})]$ follows a martingale

■ Pricing of self-financing asset X :

If wealth ϵ_t is invested in X , s.t. $\frac{d\epsilon_t}{\epsilon_t} = dr_t^X$, $\xi_t^A \epsilon_t$ must follow a

- Martingale $E_t[\xi_{t+s}^A \epsilon_{t+s}] = \xi_t^A \epsilon_t$ if portfolio position > 0
- Supermartingale $E_t[\xi_{t+s}^A \epsilon_{t+s}] \leq \xi_t^A \epsilon_t$ if portfolio position $= 0$

■ Risk premium

- $\frac{E[dr_t^{Aa}]}{dt} - r_t^{F,A} = \zeta_t^{AA} (\sigma_t^{qA} + \sigma^A) + \zeta_t^{AB} \sigma_t^{qB}$
- $\frac{E[dr_t^{Ab}]}{dt} - r_t^{F,A} \leq \zeta_t^{AA} (\sigma_t^{qA} + \sigma^A) + \zeta_t^{AB} \sigma_t^{qB}$ (equality if $\psi^{Ab} > 0$)
- Analog for citizens in country B

■ Risk free rate

- Drift of $d\xi_t/\xi_t$

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Market structures

1. Complete markets \Rightarrow First best
2. Incomplete markets (equity home bias)
 - Levered short-term debt financing
 - Sudden stops: (varying technological illiquidity)
 - Amplification
 - Runs due to sunspots
3. No equity, no debt
 - Closed capital account: capital controls)
4. Welfare analysis

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Step 1b. First Best: Planner's allocation

- First-best:
 - Capital allocation ψ s and
 - Risk allocation χ s
can be chosen independently

1. Complete markets: First Best

1. Perfect specialization

- Investment rate equalization
- Full specialization
- Output equalization

$$l_t^A = l_t^B$$

$$\psi_t^{Aa} = \psi_t^{Bb} = 1/2$$

$$y_t^a = y_t^b \quad Y_t = \bar{a} \frac{K_t}{2}$$

2. Perfect risk sharing

- Consumption (intensity) shares where λ^A and λ^B are Pareto weights

$$C_t^A = \lambda^A \bar{C}_t, C_t^B = \lambda^B \bar{C}_t$$

- $\frac{dZ_t^A + dZ_t^B}{\sqrt{2}} \equiv dZ_t$ (standard Brownian)

- Global capital evolution

$$dK_t = [\Phi(l_t) - \delta]K_t dt + \frac{\sigma}{\sqrt{2}} K_t \underbrace{\frac{dZ_t^A + dZ_t^B}{\sqrt{2}}}_{:=dZ_t}$$

1. First Best Prices

- SDF: $\xi_t^A = e^{-\rho t} \left(\frac{C_t^A}{C_0^A}\right)^{-\gamma} = e^{-\rho t} \left(\frac{K_t}{K_0}\right)^{-\gamma} = e^{-\rho t} \left(\frac{C_t^B}{C_0^B}\right)^{-\gamma}$

$$\frac{d\xi_t^A}{\xi_t^A} = \frac{d\xi_t^B}{\xi_t^B} = \underbrace{\left\{ -\rho - \gamma[\Phi(l_t) - \delta] + \frac{\gamma(\gamma + 1)\sigma^2}{4} \right\}}_{=E\left[\frac{d\xi_t}{\xi_t dt}\right]} dt - \frac{\gamma\sigma}{\sqrt{2}} dZ_t$$

- Risk-free rate: $r^F = \rho + \gamma[\Phi(l) - \delta] - \frac{\gamma(\gamma+1)\sigma^2}{4}$

- Price of capital:

- Since we know r^F and have constant investment opportunities, we can use

$$\frac{c_t}{n_t} = \rho + \frac{\gamma-1}{\gamma} \left(r^F - \rho + \frac{\zeta^2}{2\gamma} \right) \text{ (from slide 45 in lecture 03).}$$

- Together with goods market clearing condition yields

Gordon Growth

Formula $\frac{d}{r-g}$

$$q = \frac{\bar{a} - l_t}{r_t^F + \frac{\gamma}{2}\sigma^2 - [\Phi(l) - \delta]}$$

1. Complete markets: First Best Remarks

- Perfect capital allocation + perfect risk sharing
- Prices are constant and independent of shocks
- Economy shrinks/expands with (multiplicative) shocks
- Elasticity of substitution, s , has no impact on prices

Market structures

1. Complete markets \Rightarrow First best
2. Incomplete markets (equity home bias)
 - Levered (short-term) debt financing
 - Sudden stops: (varying technological illiquidity, irreversibility)
 - Amplification
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Step 1b. Price Taking Planners Problem

A-risk premia

■ max

$$E[dr_t^{\bar{N}}](\psi_t) - \underbrace{(\zeta_t^{AA} \chi_t^{AA} + \zeta_t^{BA} \chi_t^{BA}) \sigma_t^{\bar{N}A}(\psi_t) - (\zeta_t^{AB} \chi_t^{AB} + \zeta_t^{BB} \chi_t^{BB}) \sigma_t^{\bar{N}B}(\psi_t)}_{B\text{-risk premia}}$$

B-risk premia

- Where $\sigma_t^{\bar{N}A}(\psi_t) = \psi_t^A \sigma^A + \sigma_t^{qA}$, $\sigma_t^{\bar{N}B}(\psi_t) = \psi_t^B \sigma^B + \sigma_t^{qB}$

■ Frictions: no outside equity issuance

- $\chi_t^{AA} = \frac{\psi_t^A (\sigma^A + \sigma_t^{qA})}{\sigma_t^{\bar{N}A}(\psi_t)}$

- $\chi_t^{BB} = \frac{\psi_t^B (\sigma^B + \sigma_t^{qB})}{\sigma_t^{\bar{N}B}(\psi_t)}$

- $\chi_t^{AB} = \frac{\psi_t^A \sigma_t^{qB}}{\sigma_t^{\bar{N}A}(\psi_t)}$

- $\chi_t^{BA} = \frac{\psi_t^B \sigma_t^{qA}}{\sigma_t^{\bar{N}B}(\psi_t)}$

Step 2. Get ζ s from Value Function Envelop

As in previous lecture, but with 2 Brownian

■ A's value function

B's value function

$$v_t \frac{K_t^{1-\gamma}}{1-\gamma}$$

■ To obtain $\frac{\partial V_t^A(n^A)}{\partial n_t^A}$ use $K_t = \frac{N_t^A}{\eta_t q_t} = \frac{n_t^A}{\eta_t q_t}$

$$V_t^A(n_t^A) = v_t^A \frac{(n_t^A)^{1-\gamma} / (\eta_t q_t)^{1-\gamma}}{1-\gamma}$$

■ Envelop condition $\frac{\partial V_t^A(n^A)}{\partial n_t^A} = \frac{\partial u(c_t^A)}{\partial c_t^A}$

$$v_t^A \frac{(n_t^A)^{-\gamma}}{(\eta_t q_t)^{1-\gamma}} = (c_t^A)^{-\gamma}$$

■ Using $K_t = \frac{n_t^A}{\eta_t q_t}$, $C_t^A = c_t^A$

$$\frac{v_t^A}{\eta_t q_t} (K_t^A)^{-\gamma} = (C_t^A)^{-\gamma}$$

$$\sigma_t^{vA} - \sigma_t^{\eta A} - \sigma_t^{qA} - \gamma \psi_t^A \sigma^A = -\gamma \sigma_t^{cA} = -\zeta_t^{AA}$$

$$\sigma_t^{vB} - \sigma_t^{\eta B} - \sigma_t^{qB} - \gamma \psi_t^B \sigma^B = -\gamma \sigma_t^{cB} = -\zeta_t^{AB}$$

...
Analogous for B
...

...

Markov equilibrium

- Equilibrium is a map

Histories of shocks

$$\{Z_s^A, Z_s^B, s \leq t\}$$

prices allocation

$$q_t, \psi_t^{Aa} \dots, l_t^A, l_t^B, \zeta_t^A, \zeta_t^B$$

wealth distribution

$$\eta_t = \frac{N_t^A}{q_t K_t} \in (0,1) \quad \text{A's wealth share}$$

Step 3. μ^η Drift of Wealth Share

- Martingale condition (relative to benchmark asset)

$$\mu_t^\eta + \frac{C_t^A}{N_t^A} - r_t^M = (\zeta_t^{AA} - \sigma_t^{\bar{N}A})(\sigma_t^{\eta A} - \underbrace{\sigma_t^{MA}}_{=0}) + (\zeta_t^{AB} - \sigma_t^{\bar{N}B})(\sigma_t^{\eta B} - \sigma_t^{MB})$$

- Add up across types (weighted),
(capital letters with bars are aggregates for total world economy)

$$\underbrace{(\eta_t \mu_t^\eta + (1 - \eta_t) \mu_t^{1-\eta})}_{=0} + \frac{\bar{C}_t}{\bar{N}_t} - r_t^M = \eta_t (\zeta_t^{AA} - \sigma_t^{\bar{N}A}) \sigma_t^{\eta A} + (1 - \eta_t) (\zeta_t^{BA} - \sigma_t^{\bar{N}A}) \sigma_t^{1-\eta, A} + \eta_t (\zeta_t^{AB} - \sigma_t^{\bar{N}B}) \sigma_t^{\eta B} + (1 - \eta_t) (\zeta_t^{BB} - \sigma_t^{\bar{N}B}) \sigma_t^{1-\eta, B}$$

- Subtract from each other yields **wealth share drift**

$$\begin{aligned} \mu_t^\eta &= (1 - \eta_t) (\zeta_t^{AA} - \sigma_t^{\bar{N}A}) \sigma_t^{\eta A} \\ &\quad - (1 - \eta_t) (\zeta_t^{BA} - \sigma_t^{\bar{N}A}) \sigma_t^{1-\eta, A} \\ &\quad + (1 - \eta_t) (\zeta_t^{AB} - \sigma_t^{\bar{N}B}) \sigma_t^{\eta B} \\ &\quad - (1 - \eta_t) (\zeta_t^{BB} - \sigma_t^{\bar{N}B}) \sigma_t^{1-\eta, B} - \left(\frac{C_t}{N_t} - \frac{C_t + \underline{C}_t}{q_t K_t} \right) \end{aligned}$$

Step 3. $\sigma^{\eta A}, \sigma^{\eta B}$ Volatility of Wealth Share

■ From previous slide

$$\sigma_t^{\eta A} = \frac{1}{\eta_t} [(1 - \eta_t)(\psi_t^{Aa} + \psi_t^{Ab})\sigma^A + ((\psi_t^{Aa} + \psi_t^{Ab}) - \eta_t)\sigma_t^{qA}]$$

$$\sigma_t^{\eta B} = \dots$$

■ Note also,

$$\eta_t \sigma_t^{\eta A} + (1 - \eta_t) \sigma_t^{1-\eta, A} = 0 \Rightarrow \sigma_t^{1-\eta, A} = -\frac{\eta_t}{1 - \eta_t} \sigma_t^{\eta A}$$

2. Three regions of state variable η

- Wealth share η
 - Three regions

| | | Full specialization | |
|------------|--------|---------------------|--------|
| A produces | a | a | a, b |
| B produces | a, b | b | b |

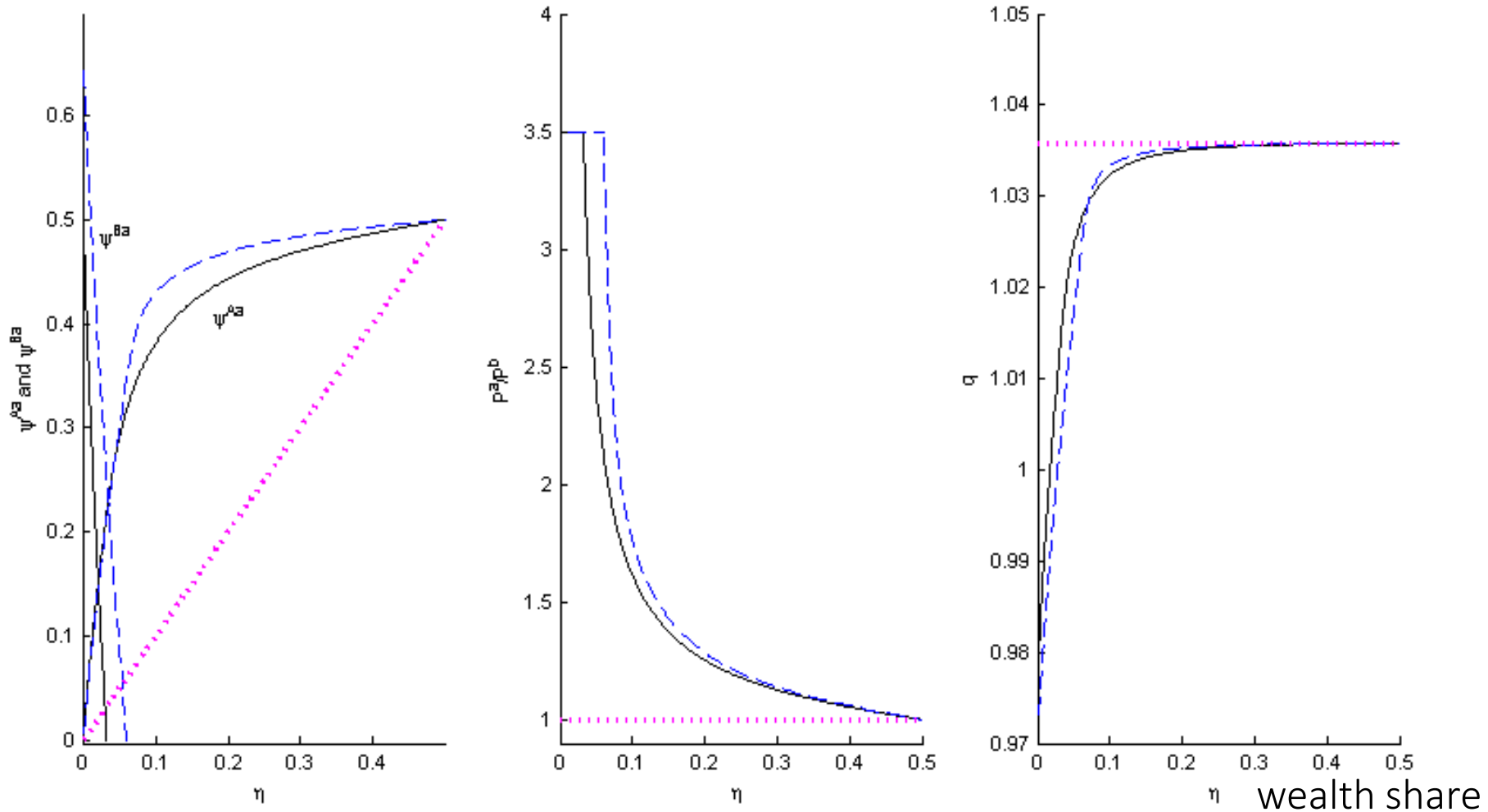
0 $1/2$ 1 η

- Symmetric

$$\begin{aligned} \psi_t^{Aa} &= \eta_t \\ \psi_t^{Bb} &= 1 - \eta_t \\ \psi_t^{Ba} &= \psi_t^{Ab} = 0 \end{aligned}$$

2. Capital share, terms of trade, price of capital

- Numerical: $\rho = 5\%$, $\gamma = 1$, $\bar{a} = 14\%$, $\underline{a} = 4\%$, $\delta = 5\%$, $\kappa = 2$, $\sigma^A = \sigma^B = 10\%$



- Three different elasticities of substitution: $s = \{.5, 1, \infty\}$

2. TOT: Supply vs. demand shock

- Supply versus demand shock

TOT improve for A as η_t declines for $\eta_t \in [\bar{\eta}, .5)$
can be due to

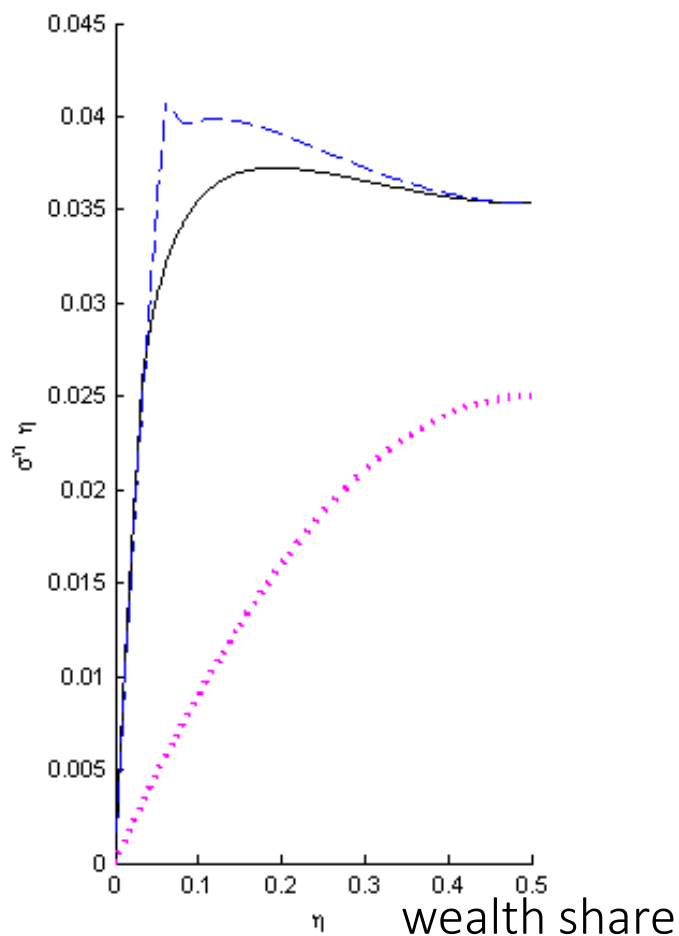
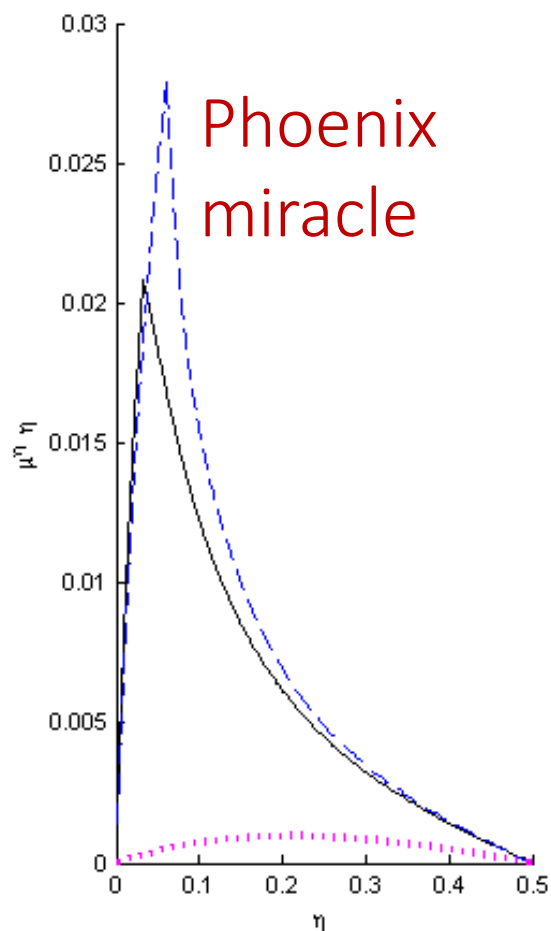
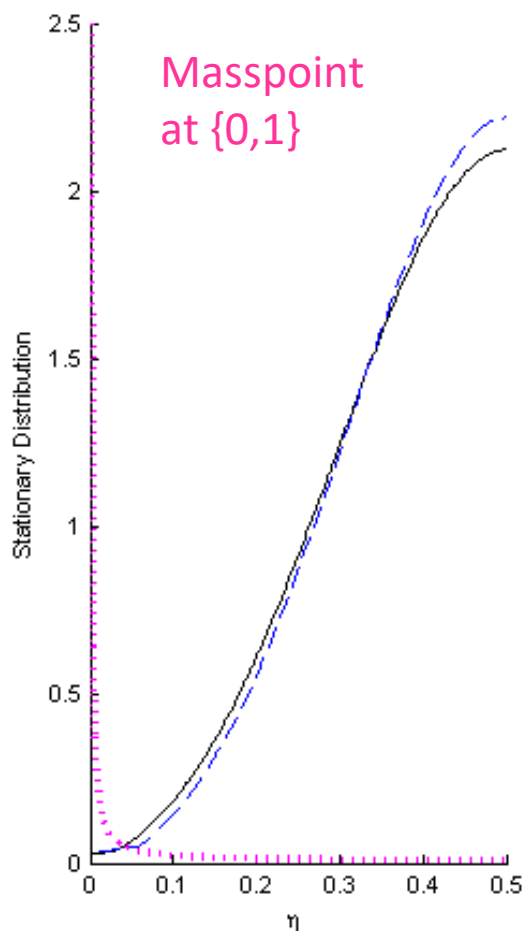
- $dZ^A < 0$: Negative supply shock World recession
- $dZ^B > 0$: Positive demand shock World boom

- TOT: Output price

- ...but fire-sale of (physical) capital stock k_t

2. Stability, Phoenix Miracle for different s

- Stationary distribution drift volatility



- Three different elasticities of substitution: $s = \{.5, 1, \infty\}$
- Difference to Cole & Obstfeld 1991: persistence of capital, $\delta < \infty$

Overview

1. Complete markets \Rightarrow First best
2. Incomplete markets (equity home bias)
 - Levered short-term debt financing
 - Sudden stops: (varying technological illiquidity)
 - Amplification
 - Runs due to sunspots
3. No equity, no debt
 - Closed capital account: capital controls
4. Welfare analysis

2. Amplification

$$\sigma_t^{\eta A} = \frac{\frac{\psi_t^{Aa}}{\eta_t} (1 - \eta_t)}{1 - \frac{\psi_t^{Aa} - \eta_t}{\eta_t} \frac{q'(\eta_t)}{q(\eta_t)/\eta_t}} \sigma^A$$

asset-equity ratio

Market illiquidity
(price impact elasticity)

Leverage: debt-equity ratio

- Leverage effect $\psi_t^{Aa} / \eta_t, (\psi_t^{Aa} - \eta_t) / \eta_t$
-

2. Amplification

$$\sigma_t^{\eta A} = \frac{\frac{\psi_t^{Aa}}{\eta_t} (1 - \eta_t)}{1 - \frac{\psi_t^{Aa} - \eta_t}{\eta_t} \frac{q'(\eta_t)}{q(\eta_t)/\eta_t}} \sigma^A$$

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- Leverage effect

$$\psi_t^{Aa} / \eta_t, (\psi_t^{Aa} - \eta_t) / \eta_t$$

- Loss spiral

$$1 / \left\{ 1 - \frac{\psi_t^{Aa} - \eta_t}{\eta_t} \frac{q'(\eta_t)}{q(\eta_t)/\eta_t} \right\} \quad \text{(infinite sum)}$$

2. Amplification

$$\sigma_t^{\eta A} = \frac{\frac{\psi_t^{Aa}}{\eta_t} (1 - \eta_t)}{1 - \frac{\psi_t^{Aa} - \eta_t}{\eta_t} \frac{q'(\eta_t)}{q(\eta_t)/\eta_t}} \sigma^A$$

asset-equity ratio

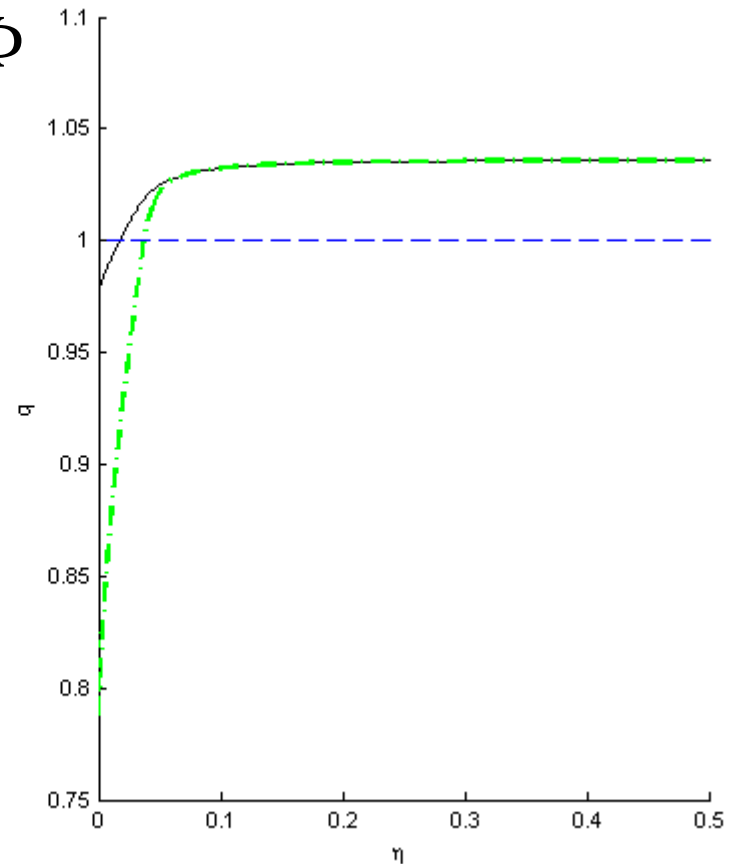
Market illiquidity (price impact elasticity)

Leverage: debt-equity ratio

- Leverage effect $\psi_t^{Aa} / \eta_t, (\psi_t^{Aa} - \eta_t) / \eta_t$
- Loss spiral $1 / \left\{ 1 - \frac{\psi_t^{Aa} - \eta_t}{\eta_t} \frac{q'(\eta_t)}{q(\eta_t)/\eta_t} \right\}$ (infinite sum)
- Technological illiquidity $(\kappa, \delta) \Rightarrow$ market illiquidity $q'(\eta)$
 - (dis)investment adjustment cost

2. Technological $(\kappa, \delta) \Rightarrow$ market illiquidity $q'(\eta)$

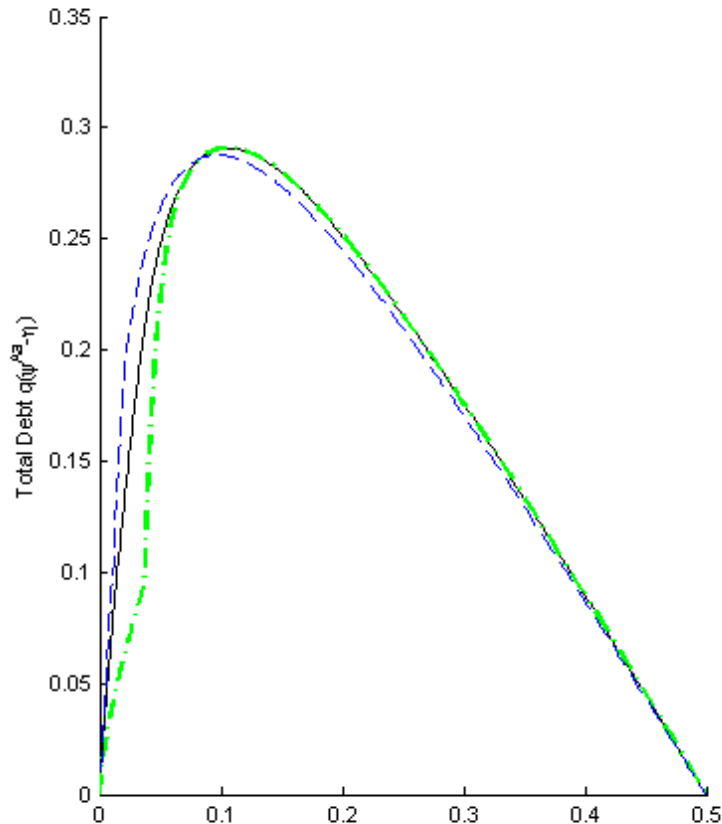
- Quadratic adjustment cost
- Investment rate of $\iota = \Phi + \frac{1}{\kappa} \Phi^2$ generates new capital at rate Φ
- $\Phi(\iota) = \frac{1}{\kappa} (\sqrt{1 + 2\kappa\iota} - 1)$
- Three cases
 - $\kappa = 0 \Rightarrow q = 1$
 - $\kappa = 2$
 - $\kappa_{\iota < 0} = 100$ and $\kappa_{\iota > 0} = 2$



2. Sudden stops: amplification & runs

■ Sudden stop

- Adverse **fundamental triggers** %-decline in debt that exceeds %-decline in net worth; $\frac{\partial(\psi^{Aa}-\eta)}{\partial\eta} \frac{\eta}{\psi^{Aa}-\eta} > 1 \Leftrightarrow \frac{\partial\psi^{Aa}}{\partial\eta} > \frac{\psi^{Aa}}{\eta}$
 \Leftrightarrow pro-cyclical leverage

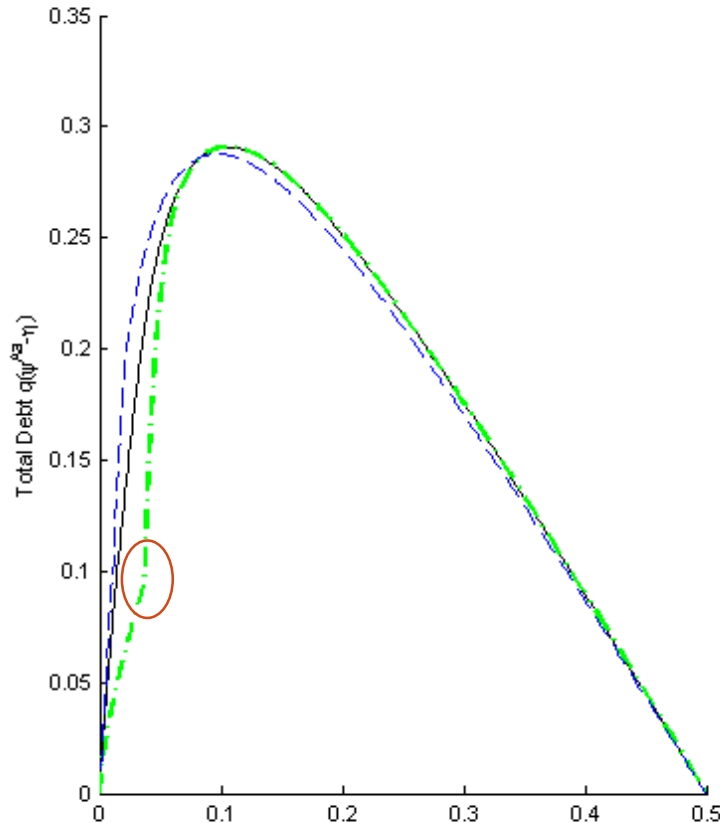


2. Sudden stops: amplification & runs

■ Sudden stop

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Slope of
tangent vs. secant



2. Sudden stops: amplification & runs

■ Sudden stop

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 \Leftrightarrow pro-cyclical leverage

- An unanticipated **sunspot triggers** a sudden capital price drop from q to \tilde{q} , accompanied by a drop in η to $\tilde{\eta}$.

$$\tilde{q}\tilde{\eta} = \max\{\eta q + \psi^{Aa}(\tilde{q} - q), 0\}$$

2. Sudden stops: amplification & runs

■ Sudden stop

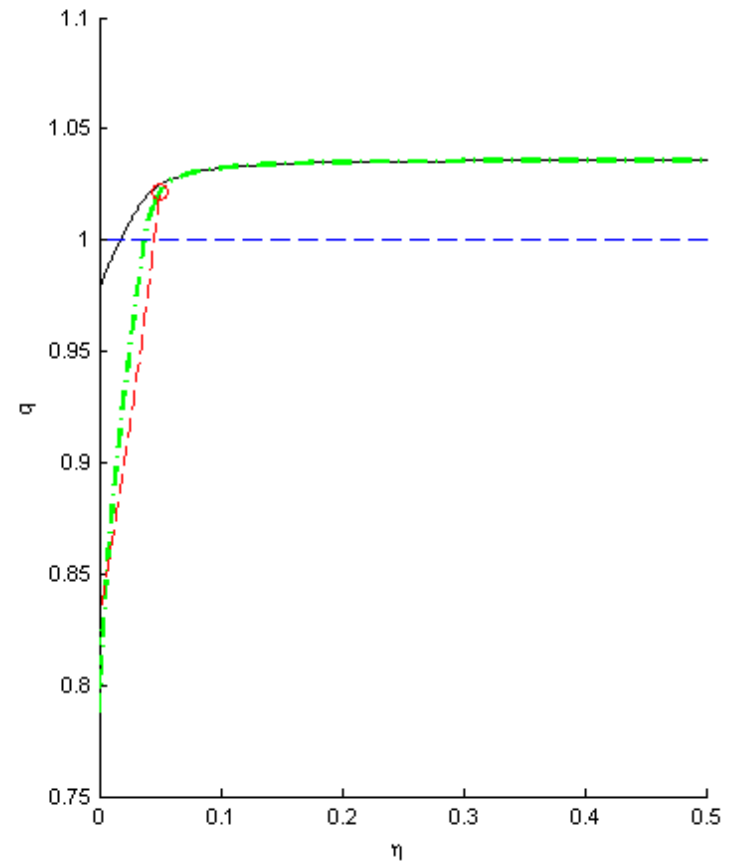
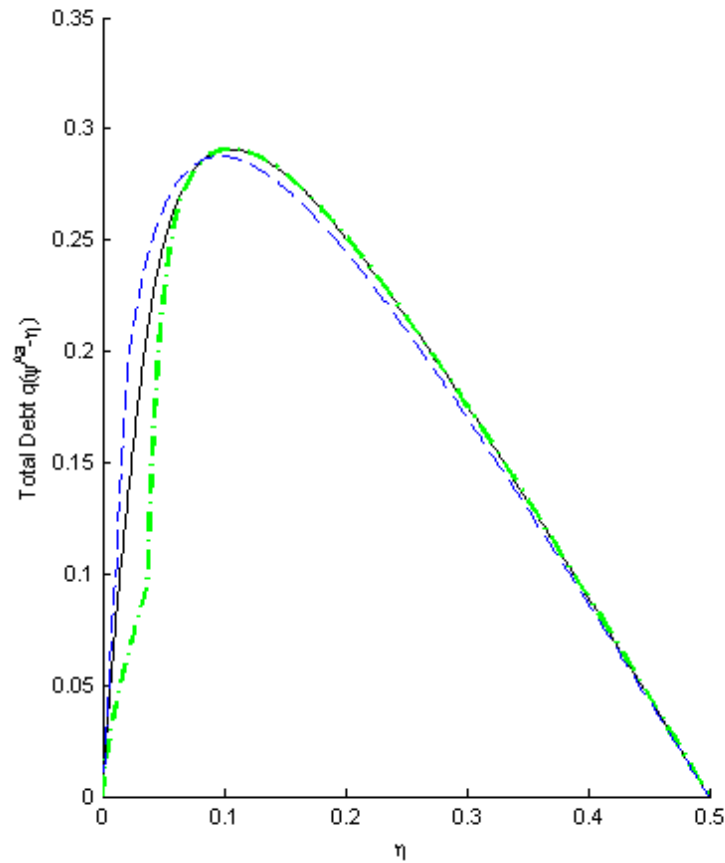
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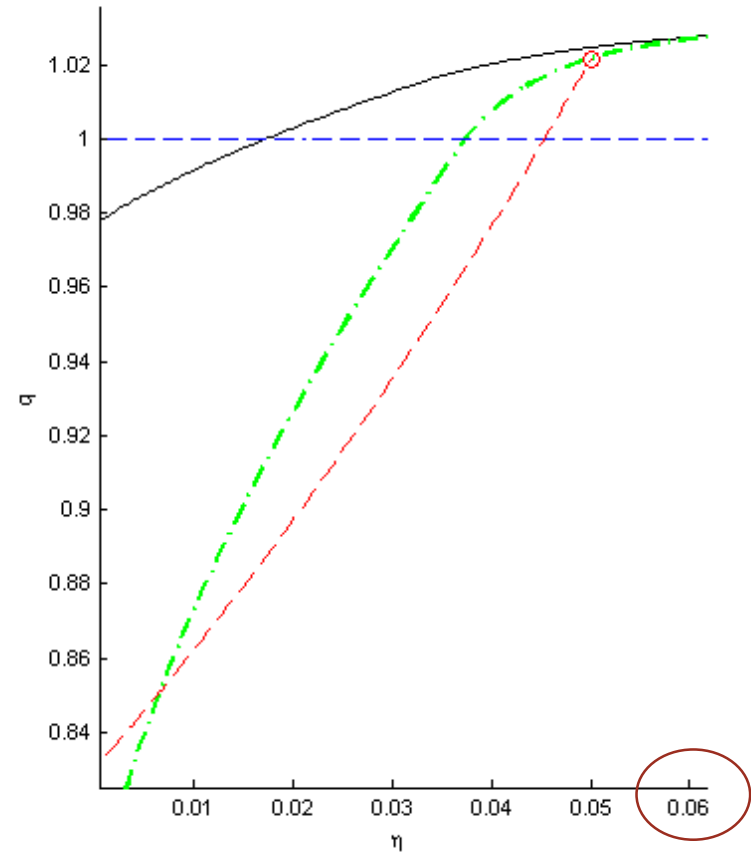
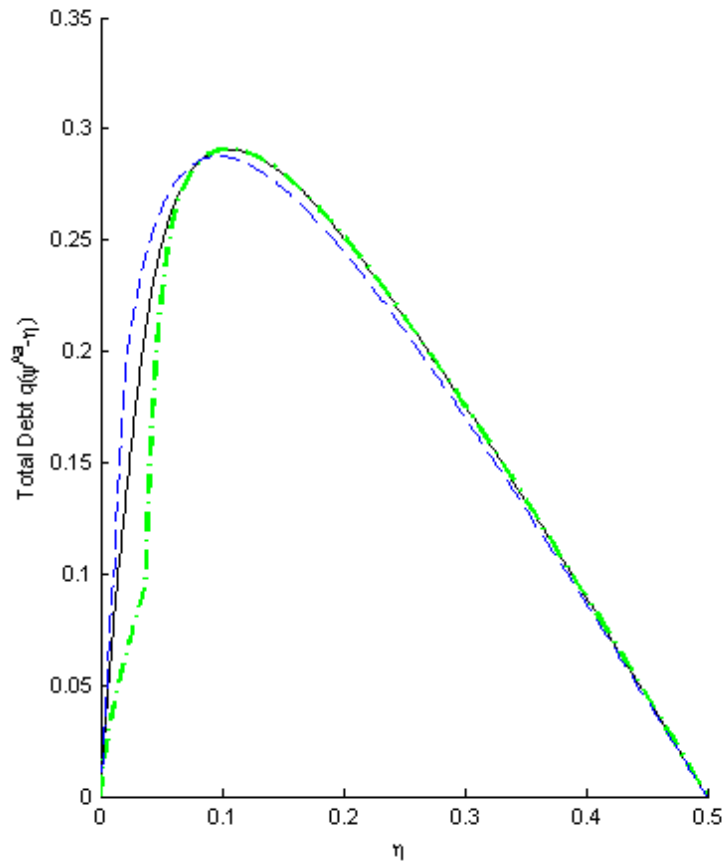
$$\tilde{q} = \frac{\max\{\eta q + \psi^{Aa}(\tilde{q} - q), 0\}}{\tilde{\eta}}$$

hyperbola

2. Sudden stops: amplification & runs



2. Sudden stop due to **run**: Zoomed in



Overview

1. Complete markets \Rightarrow First best
2. Incomplete markets (equity home bias)
 - Levered short-term debt financing
 - Sudden stops: (varying technological illiquidity)
 - Amplification
 - Runs due to sunspots
3. No equity, no debt
 - Closed capital account: capital controls
4. Welfare analysis

Market structures

Trade

Finance

| Markets | Output y^a, y^b | Physical capital K | Debt | Equity |
|---|----------------------|-------------------------|------|--------|
| Complete Markets Full integration/First Best | X | X | X | X |
| Open credit account (equity home bias) | X | X | X | |
| Closed credit account | X | X | | |

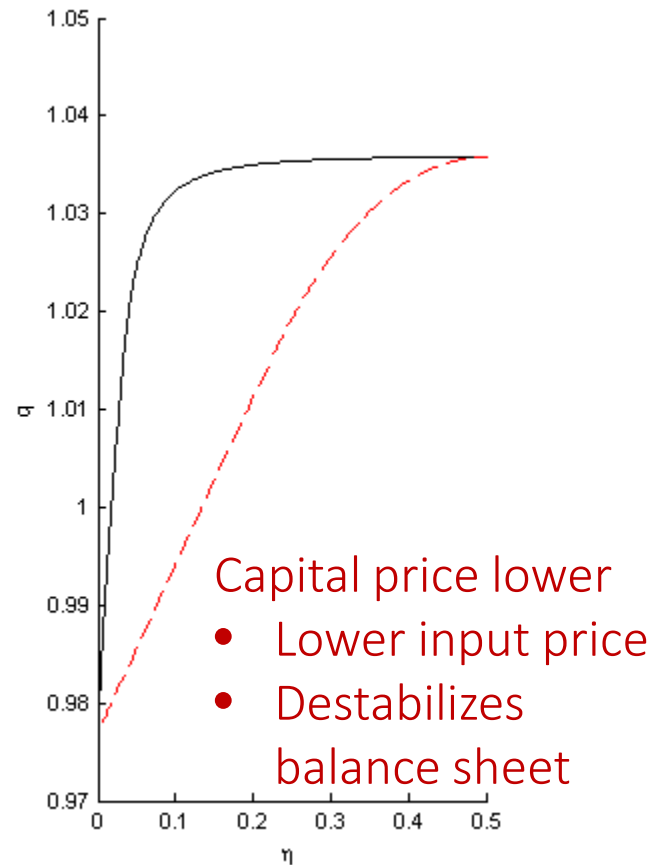
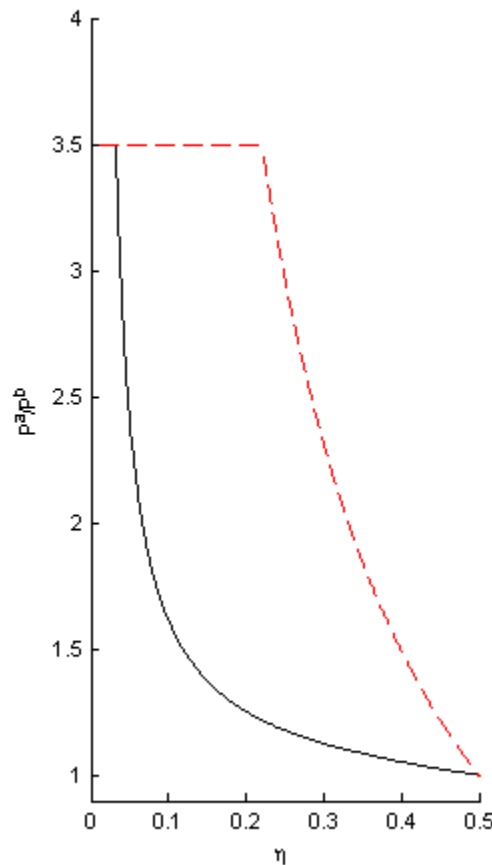
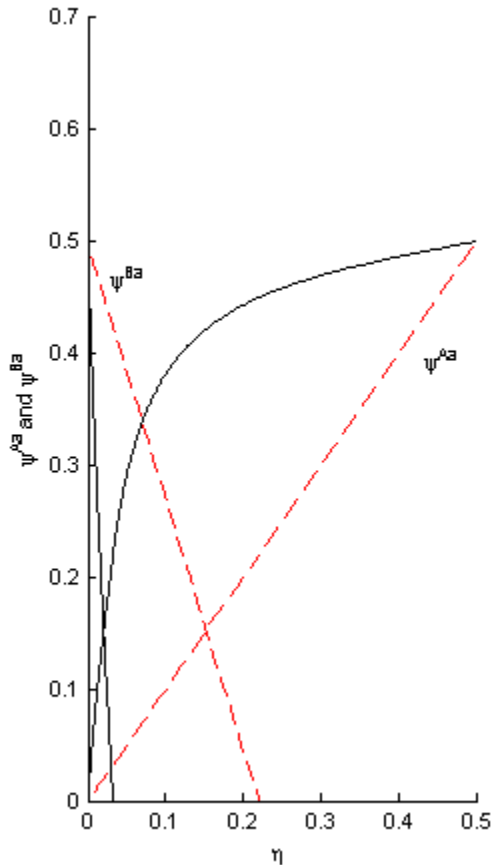
Add taxes/capital controls

intra-temporal

inter-temporal

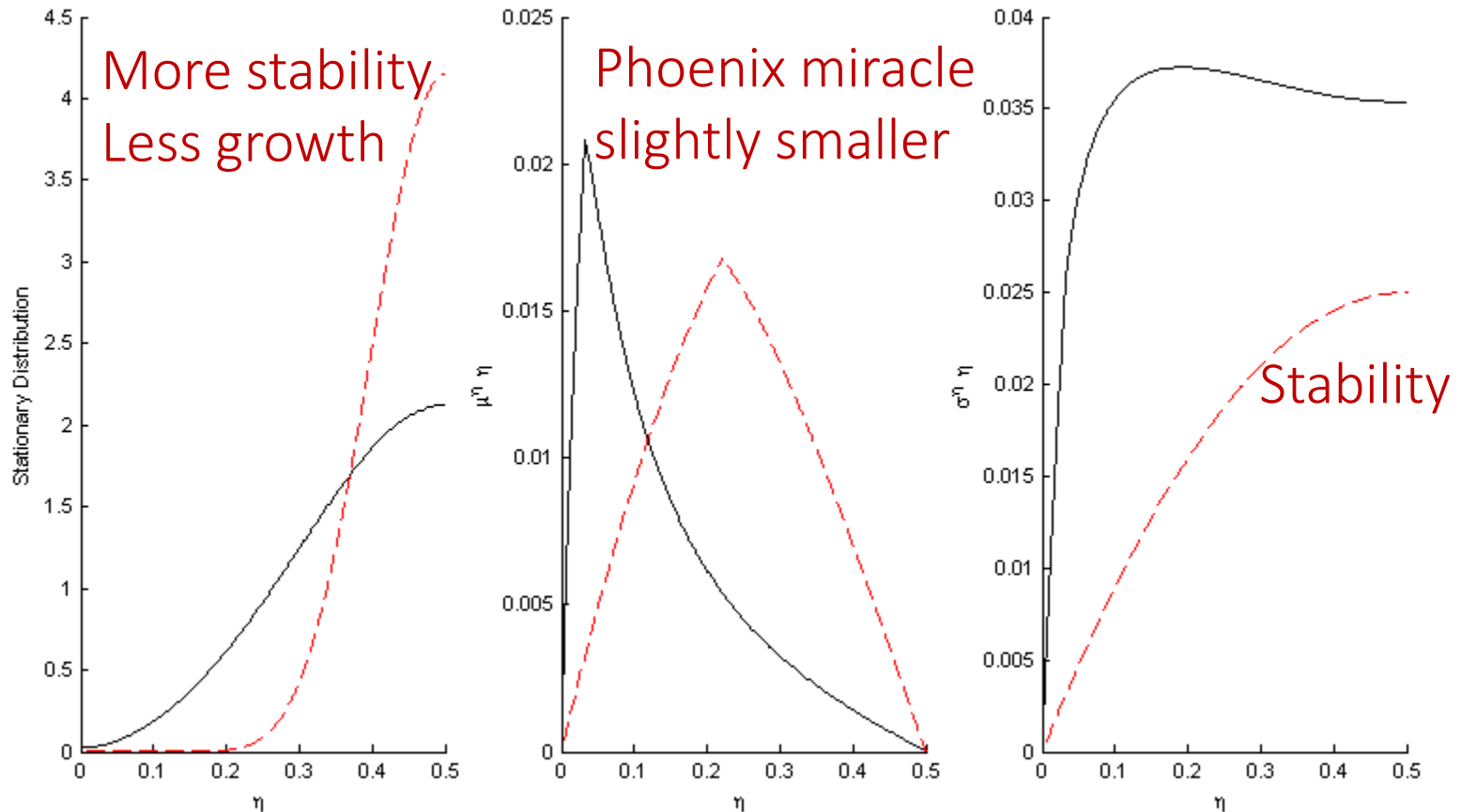
3. Credit account: open vs. closed

- $\rho = 5\%$, $\gamma = 1$, $\bar{a} = 14\%$, $\underline{a} = 4\%$, $\delta = 5\%$, $\kappa = 2$, $\sigma^A = \sigma^B = 10\%$,
 $s = 1$



3. Credit account: open vs. closed

- $\rho = 5\%$, $\bar{a} = 14\%$, $\underline{a} = 4\%$, $\delta = 5\%$, $\kappa = 2$, $\sigma^A = \sigma^B = 10\%$, $s = 1$

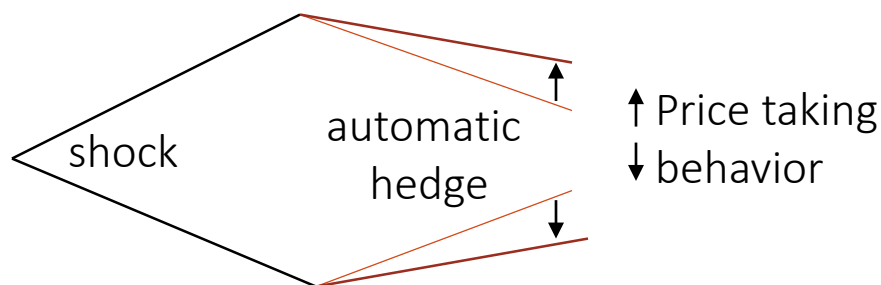


Overview

1. Complete markets \Rightarrow First best
2. Incomplete markets (equity home bias)
3. No equity, no debt: Closed capital account
4. Welfare analysis
 - Pecuniary externalities
 - Welfare calculations + Pareto improving redistributions

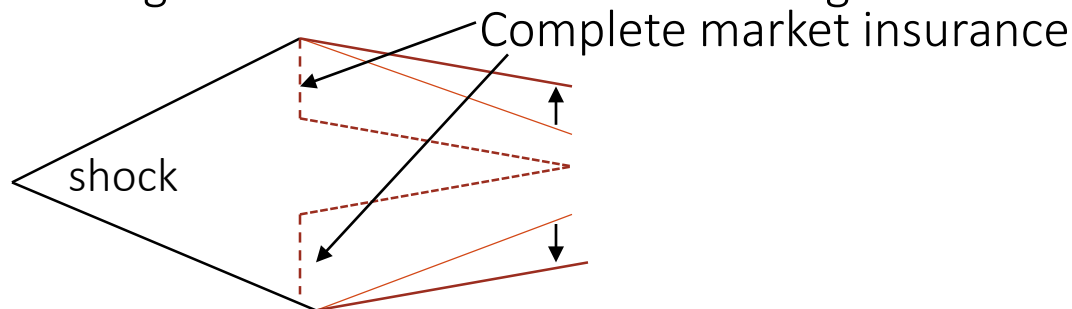
4. When are credit flows excessive?

- Constrained inefficiency (in incomplete market setting) due to pecuniary externalities
 - Price of capital: fire sale externality if leverage is high
 - Price of output good: “terms of trade hedge” restrained competition
 - Price taking behavior undermined this hedge



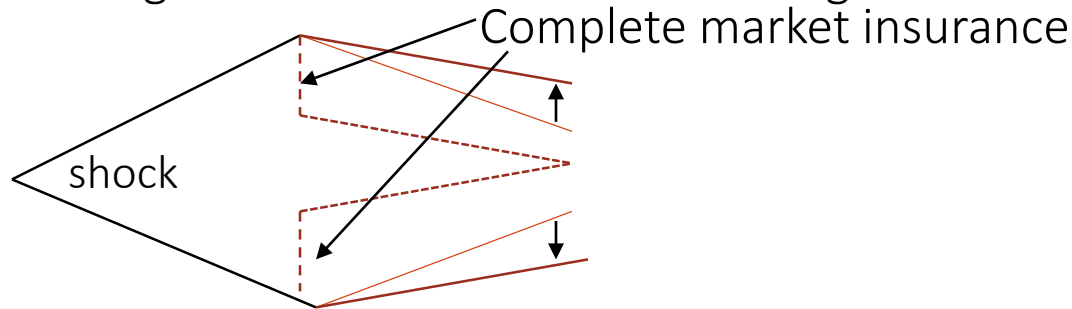
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4. When are credit flows excessive?

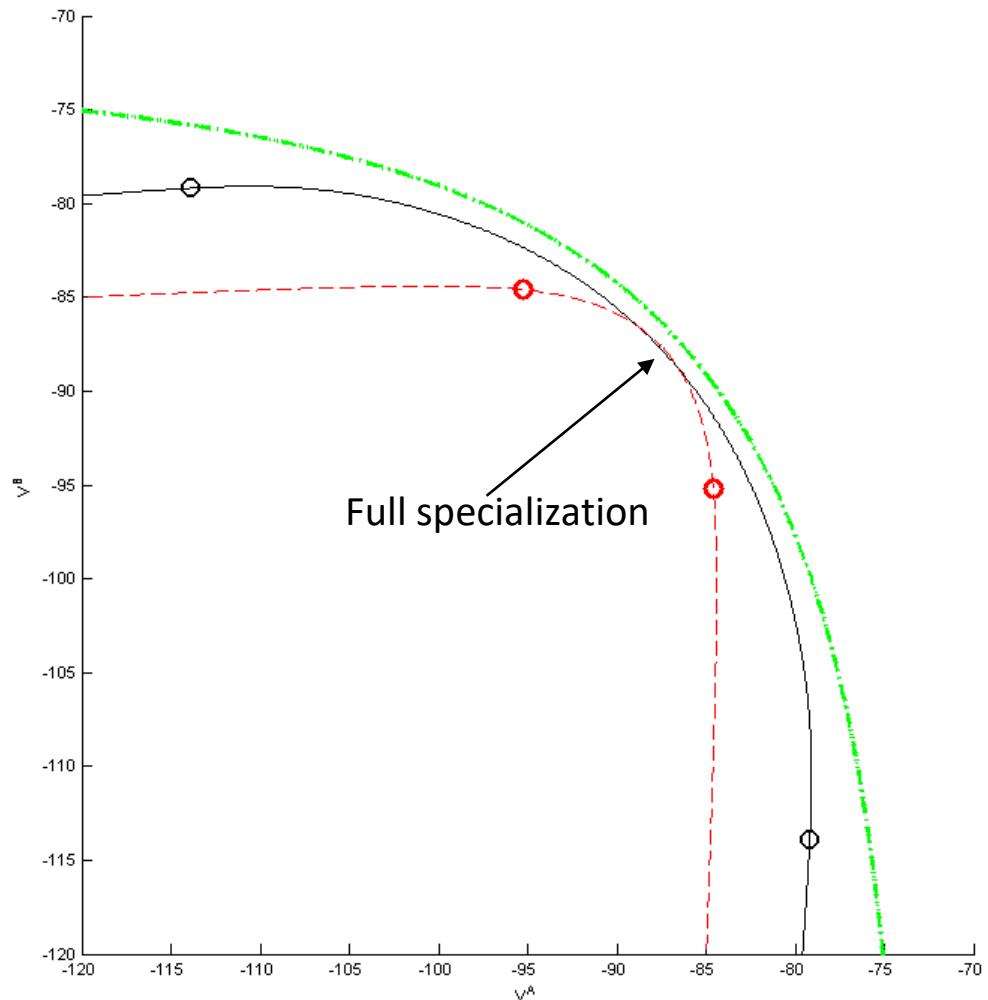
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| Price | Intention | Depends on |
|-----------------------|--|---------------------------------------|
| Capital price (input) | Buy cheaper but capital losses on existing k_t | Adjustment cost, $\Phi(l)$, κ |
| Output price | Sell output more expensive | Elasticity of substitution, s |
| Interest rate | Borrow cheaper | Intertemporal preference |

4. Welfare comparison

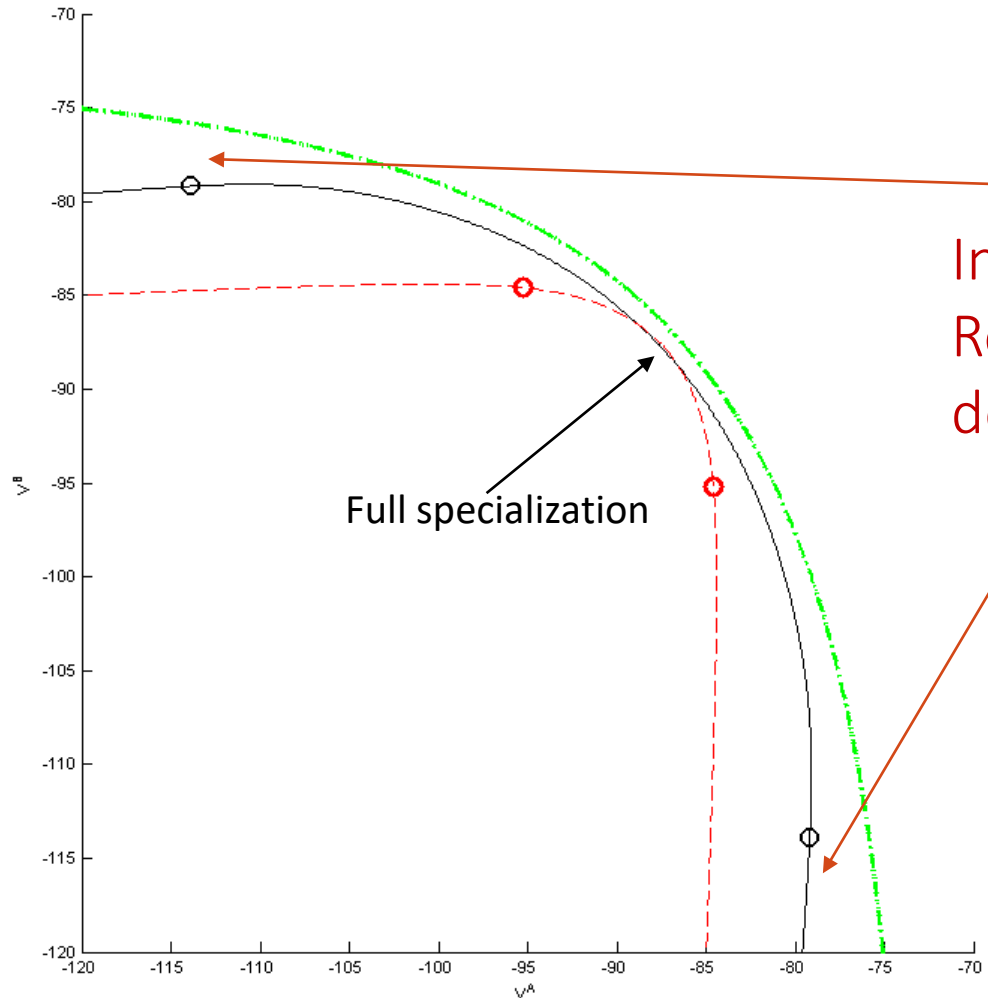
- $\rho = 5\%, \gamma = 1, \bar{a} = 14\%, \underline{a} = 4\%, \delta = 5\%, \kappa = 2, \sigma^A = \sigma^B = 10\%$



- No friction, first best
- No equity
- No equity, no debt

4. Welfare comparison

- $\rho = 5\%$, $\gamma = 1$, $\bar{a} = 14\%$, $\underline{a} = 4\%$, $\delta = 5\%$, $\kappa = 2$, $\sigma^A = \sigma^B = 10\%$



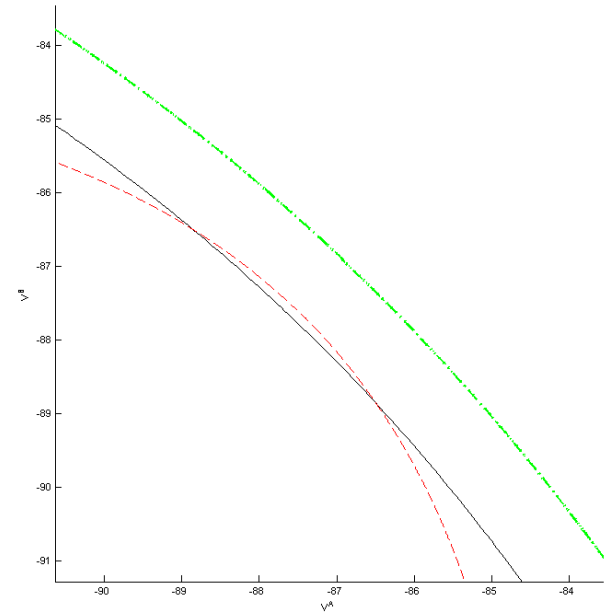
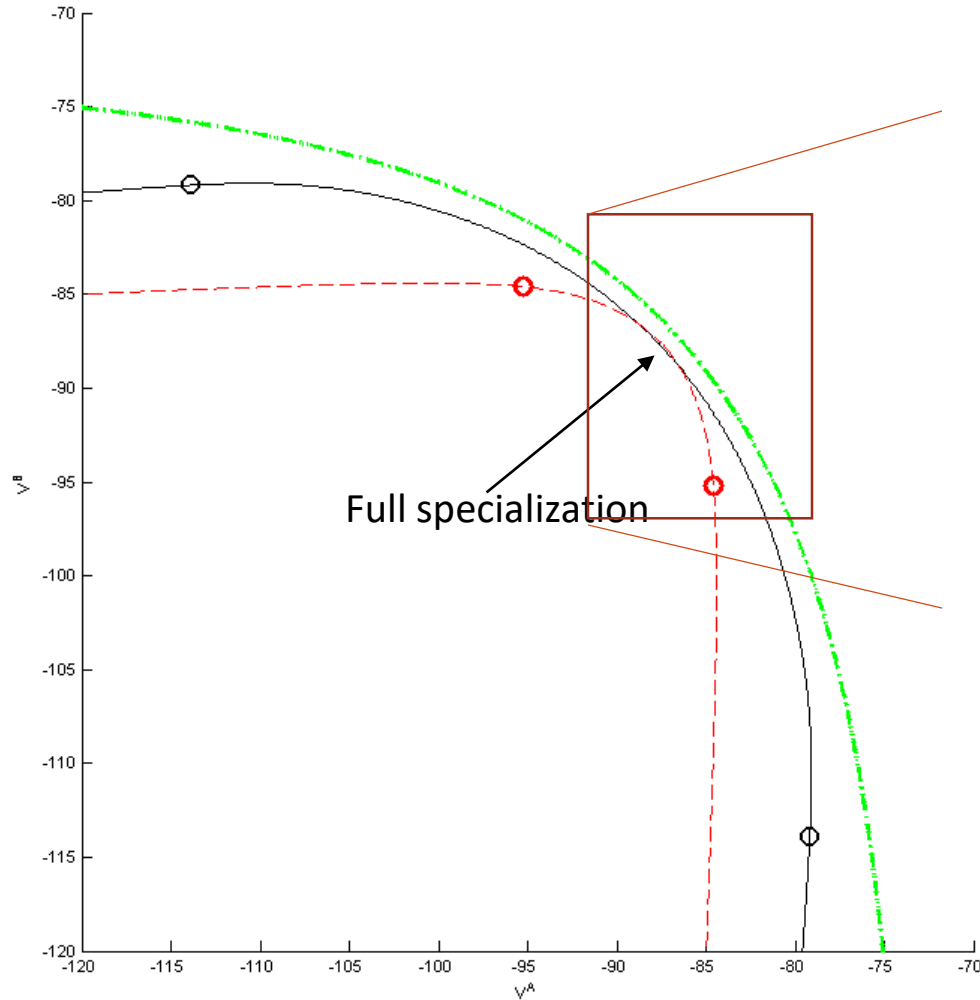
Inefficiency at the extremes:
Role for redistributive Policy
default/bail-out/debt-relief

Pareto improving

Intuition:
Other country's output
price is high

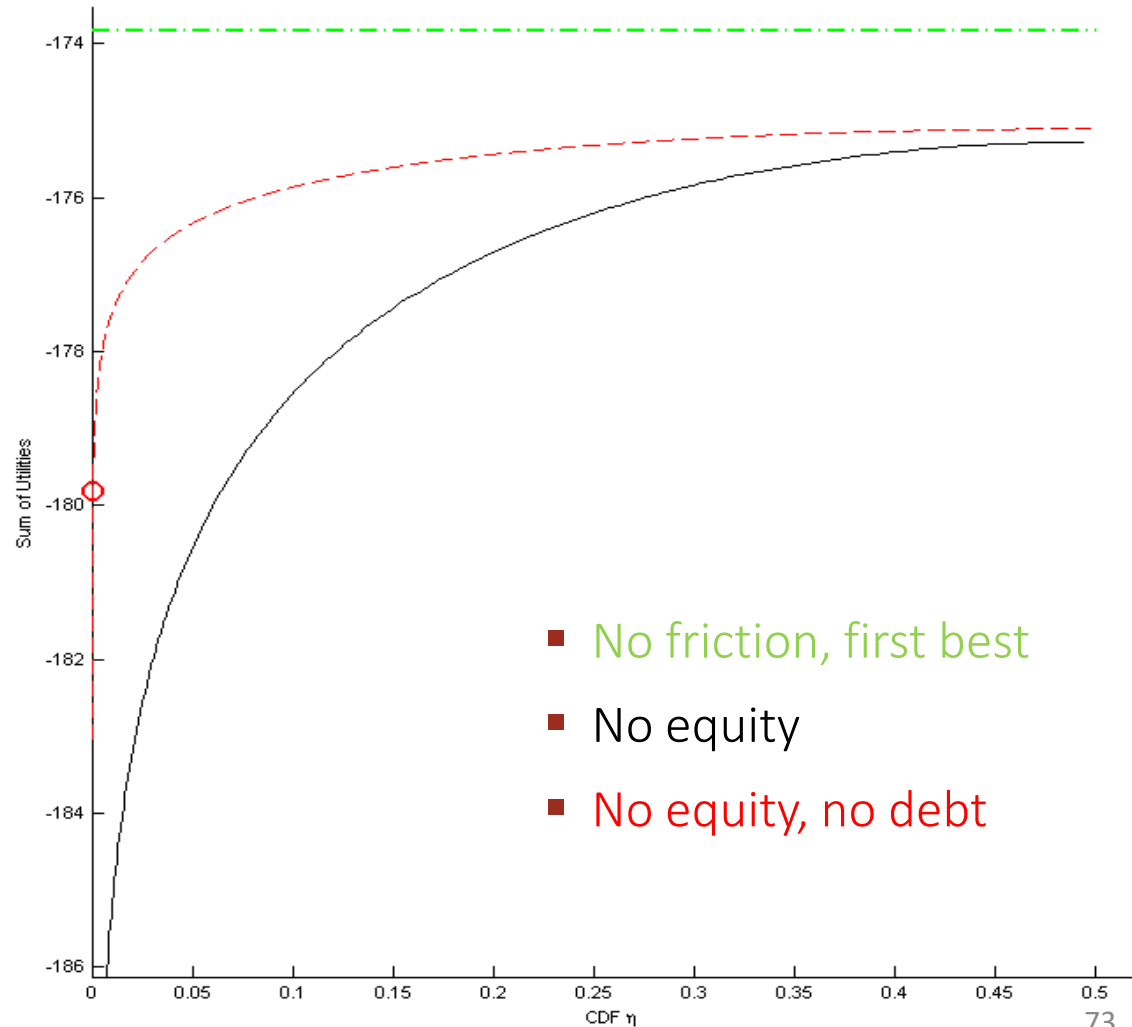
4. Welfare comparison

- $\rho = 5\%, \gamma = 1, \bar{a} = 14\%, \underline{a} = 4\%, \delta = 5\%, \kappa = 2, \sigma^A = \sigma^B = 10\%$



4. Welfare comparison

- Any monotone transformation of η would be equally good state variable
- Normalization:
take CDF of η
 - Uniform stationary distribution!



Conclusion

- Symmetric setup (productivity, discount rate, ...)
 - Derive $A(\psi)$
- Sudden stops
 - Amplification of fundamental shock
 - Runs due to sunspots – vulnerability region
- Phoenix miracle

- Tradeoff between capital allocation & risk sharing
 - “Terms of trade hedge”
- When are short-term credit flows excessive?
 - When can capital controls (financial liberalization) be welfare enhancing (reducing)?
 - Pecuniary externality
 - Price of physical capital fire-sales externality – technological illiquidity
 - Price of output goods: “terms of trade hedge” externality
- Bailout/Restructuring
Redistributive policy can be Pareto improving if one country is sufficiently balance sheet impaired
 - Reduces output good price

Next to do ... Problem Set

- Solve model with CRRA utility functions numerically
 - Follow steps from previous lecture
- Allow for idiosyncratic risk in one country
- Plot fan charts and distribution impulse response functions
- Allow for anticipated jumps
 - Incorporate (compensated) jump process in probability space/proposed processes