International Monetary Theory: Trade, Exchange Rates and Spillovers

by

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Motivation

- Global currency spillovers
  - “Flight to safety” – dollar appreciation when risk rises
  - How large country monetary policy affects capital flows/exchange rates
  - Local currency: good store of value/hedge for idiosyncratic risk
  - Global currency: good hedge for international competitiveness risk

- Monetary policy space: “Nuanced Mundell-Fleming Trilemma”
  - Local and global money have different risk profile (imperfect substitutes) increases policy space
  - Too high inflation: local citizens substitute local currency for global currency limits policy space

- Reserve currency management – irrelevance theorem
# Modeling Framework

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- Risk & Dynamic financial frictions include elements that incorporate risk into the economic models, which can influence the behavior of agents and the overall market equilibrium.
- The open economy models, such as the Mundell-Fleming framework, allow for international trade and capital flows, which can affect exchange rates and interest rates.
- The New Keynesian model extends the classical Keynesian framework by incorporating features like sticky prices and wages, which can lead to more complex dynamics compared to the Hicksian IS-LM framework.
- Obstfeld and Rogoff's model is particularly useful for understanding the implications of trade and capital flows on macroeconomic variables.
**Frictions**

- Incomplete markets
  - Within country
    - only with respect to idiosyncratic risk
    - other risk can be shared within national economy
  - Across countries
    - only currencies can be traded

- Money is a bubble
  - Like in Samuelson, Bewley

- Prices are fully flexible
International setting

- Small Economy
  - Local currency
    - Store of value to insure against idiosyncratic risk
  - Shocks to productivities of tradables
    - export good 1, productivity $b_t$
    - import good 2

- Large Economy*
  - Global currency* $\$
  - Store of value, hedge idiosyncratic risk cross-holdings to hedge “trade risk”

Good 1 productivity $b_t^*$
Good 2 productivity ...
International setting

- **Small Economy**
  - Local currency
    - Store of value to insure against *idiosyncratic* risk
  - Shocks to productivities of tradables
    - Export good 1, productivity $b_t$

- **Large Economy**
  - Global currency* $¹$
    - Store of value, hedge *idiosyncratic* risk
  - Cross-holdings to hedge “trade risk”

$¹$ demand depends on $b_t^*/b_t$ risk (optimal hedge level)
Total (local currency + $¹$) depends on friction / *idiosyncratic* risk
Currencies are imperfect substitutes
Intuition

- Small country: trades global goods
- Hold global money as Net Foreign Asset Position to hedge trade risk

- Value of money
  - Total money demand depends on frictions inside country (idiosyncratic risk)
  - Global money portion depends on optimal hedging of trade risk
  - 2 money can coexist (even though both are “bubbles”)
    - Different return-risk profile
Large country

logarithmic utility

\[
E \left[ \int_0^\infty e^{-\rho t} \log c_t^* \, dt \right]
\]

where \(c_t^*\) is the consumption of the tradable good

- Capital produces \(a_t^*\) units of the (non-tradable) investment good or \(b_t^*\) of the tradable consumption good
- Agents choose fraction \(\alpha_t^*\) of capital to produce the consumption good

\[
\frac{dk_t^*}{k_t^*} = (\Phi(i_t^*) - \delta^*) dt + \tilde{\sigma}^* d\tilde{Z}_t, \quad i_t^* = (1 - \alpha_t^*)a^*
\]

- value of all goods produced is \(a_t^* K_t^*\)
- value of all consumption \((a_t^* - i_t^*) K_t^*\)
- "" "" physical capital \(q_t^* K_t^*\)
- "" "" large country’s money \(p_t^* K_t^*\)

Assume parameters s.t. optimal portfolio weight on small country’s money is 0
Optimality & Market clearing

- Consumption = \( \rho \times \text{net worth} \), so

\[
\rho = 1 - \theta^* \left( \frac{a^*}{q^*} - l^* \right) + \theta^* \cdot 0,
\]

where \( \Phi'(l^*) = \frac{1}{q^*} \).

... money portfolio share, \( \theta^* \), determines \( q^* \) and \( l^* \).

- Portfolio Choice: diff in returns = Cov(diff in risk, wealth risk)
  - capital gain on money/capital = \( \Phi(l^*) - \delta^* \)

\[
\frac{\rho}{1 - \theta^*} = \frac{a^* - l^*}{q^*} = \frac{\tilde{\sigma}^*}{\text{difference in dividend yield}} \cdot (1 - \theta^*) \tilde{\sigma}^*
\]

- \( \Rightarrow \theta^* = 1 - \frac{\sqrt{\rho}}{\tilde{\sigma}^*} \) money demand increases in \( \tilde{\sigma}^* \)
Return on global money ($)

- $a^*$ is constant $\Rightarrow$ money has risk-free return in the investment good

\[ r^* = \Phi(\iota^*) - \delta = \text{growth of large country's capital} \]

- Return in terms of the tradable good is

\[ dr_t^G = r^* dt + db_t^* / b_t^* \]

"tradable good productivity"
Small country

- Same preferences preferences:
  \[ E \left[ \int_0^\infty e^{-\rho t} \log C_t \, dt \right] \]

- \( \hat{\alpha}_t K_t \) devoted to produce tradable consumption good
  ... then traded for tradable basket / global money

- \( C_t = \xi_t b_t K_t \) consumption

- \( (\hat{\alpha}_t - \xi_t) b_t K_t \) trade imbalance (net export)

- \( G_t > 0 \), net foreign asset position (only global money)
  (measured in tradable basket)

\[
\frac{dG_t}{G_t} = r_*^* dt + \frac{db_t^*}{b_t^*} + \frac{(\hat{\alpha}_t - \xi_t) b_t K_t}{G_t} \, dt
\]

\[ d_{rG} \]
State variable

- Net foreign asset position to tradable production potential

\[ \nu_t = \frac{G_t}{(b_t K_t)} \]

- Assume

\[ \frac{d(b_t / b_t^*)}{b_t / b_t^*} = \mu^{PT} dt + \sigma^{PT} dZ_t \]

\[ \frac{dG_t}{G_t} = r^* dt + \frac{db_t^*}{b_t^*} + \frac{\alpha_t - \xi_t}{\nu_t} dt \Rightarrow \]

\[ \frac{d\nu_t}{\nu_t} = \left( r^* - \Phi(t_t) + \delta - \mu^{PT} + (\sigma^{PT})^2 \right) dt + \frac{\hat{\alpha}_t - \xi_t}{\nu_t} dt - \sigma^{PT} dZ_t. \]
Equilibrium in the small country

• Equilibrium in the small country is a map

history of shocks $\{Z_s, Z_s^*, 0 \leq s \leq t\}$  \rightarrow  \text{prices } q_t, p_t, \text{ allocations} \\ \hat{\alpha}_t, \iota_t, \xi_t \& \text{portfolio weights } (1 - \theta_t - \xi_t, \theta_t, \xi_t)$

• such that
  • all agents maximize utility (choose portfolio, consumption, technology) 
  • all markets clear (capital, local and global money)
Equilibrium in the small country is a map

history of shocks \{Z_s, Z_s^*, 0 \leq s \leq t\}

prices \(q_t, p_t\), allocations \(\hat{\alpha}_t, \iota_t, \xi_t\) & portfolio weights \((1 - \theta_t - \xi_t, \theta_t, \xi_t)\)

\[v_t = G_t/(b_t K_t)\]
Portfolio choice & asset pricing (dynamic)

- Portfolio share (processes)
  - Local money
  - Global money
- Returns expressed with country net worth $N_t$, as numeraire
  - Return on individual net worth
  - Return on local money
  - Return on global money ($)
- Asset pricing equation (with log utility)

\[
\begin{align*}
\frac{d\theta_t}{\theta_t} &= \mu_\theta^0 dt + \sigma_\theta^0 dZ_t \\
\frac{d\xi_t}{\xi_t} &= \mu_\xi^t dt + \sigma_\xi^t dZ_t \\
\frac{d\theta_t}{\theta_t} &= \mu_t^\theta dt + \sigma_t^\theta dZ_t \\
\frac{d\xi_t}{\xi_t} &= \mu_t^\xi dt + \sigma_t^\xi dZ_t \\
\frac{d\xi_t}{\xi_t} &= \xi_t - \hat{\alpha}_t dt + \frac{d\xi_t}{\xi_t} \\
\frac{d\xi_t}{\xi_t} &= \frac{\xi_t - \hat{\alpha}_t}{\nu_t} dt + \frac{d\xi_t}{\xi_t} \\
E[dr_t^n - dr_t^{MG}] &= \text{Cov}[dr_t^n - dr_t^{MG}, dr_t^n] \Rightarrow \rho - \frac{\xi_t - \hat{\alpha}_t}{\nu_t} - \mu_t^\xi = (1 - \theta_t - \xi_t)^2 \tilde{\sigma}^2 \\
E[dr_t^n - dr_t^{ML}] &= \text{Cov}[dr_t^n - dr_t^{ML}, dr_t^n] \Rightarrow \rho - \mu_t^\theta = (1 - \theta_t - \xi_t)^2 \tilde{\sigma}^2
\end{align*}
\]
Money valuation

\[ \rho - \mu^\theta_t = (1 - \theta_t - \xi_t)^2 \tilde{\sigma}^2 \Rightarrow \theta_0 = E \left[ \int_0^\infty e^{-\rho t} (1 - \theta_t - \xi_t)^2 \tilde{\sigma}^2 \theta_t \, dt \right] \]

- Constant environment (fixed \( \zeta \)): fixed point where

\[ \frac{(1 - \theta - \zeta)^2 \tilde{\sigma}^2}{\rho} = 1 \]

- \( \theta = 0 \) also a fixed point...

- Global money valuation equation a bit harder to interpret

\[ \rho - \frac{\xi_t - \hat{\alpha}_t}{\nu_t} - \mu^\xi_t = (1 - \theta_t - \xi_t)^2 \tilde{\sigma}^2 \]

but a back-of-the-envelope calculation gives

\[ \xi = \frac{r^* - (\Phi(u) - \delta) - \mu^{PT} + (\sigma^{PT})^2}{(\sigma^{PT})^2} \]
Consumption & Investment (static equations)

- Wealth (in tradable) \( G_t / \xi_t \)
- Value of all capital (in tradable) \( (1 - \xi_t - \theta_t)G_t / \xi_t \)

- Consumption market clearing
  \[
  \rho \underbrace{\frac{G_t}{\xi_t}}_{\text{wealth in tradable basket}} = \xi_t b_t K_t
  \]

- Optimal investment
  \[
  \Phi'((1 - \hat{\alpha}_t)a) a \frac{(1 - \xi_t - \theta_t)G_t}{\xi_t K_t} = b_t \quad \text{if } \hat{\alpha}_t \in (0,1) \\
  \leq b_t \quad \text{if } \hat{\alpha}_t = 1 \\
  \geq b_t \quad \text{if } \hat{\alpha}_t = 0
  \]
Computation

“Time step”

\[ \theta(v,t-\epsilon), \zeta(v,t-\epsilon) \]

portfolio weights \( \theta(v,t), \zeta(v,t) \)

law of motion of \( v \), asset-pricing equations

For details of this method, see survey “Money, Macro and Finance: A Continuous-Time Approach” with Markus Brunnermeier
Computation

convergence
(prices in very
distant future
matter little for
prices today)

terminal
conditions
(-arbitrary)

t - \epsilon t

t

T

time
Numerical Example

\[ \rho = 5\%, \quad \tilde{\sigma} = 0.3, \quad \mu^{PT} = 1\%, \quad \sigma^{PT} = 0.15, \]
\[ a = 0.13, \quad \delta = 0.02, \quad \Phi(t) = \log(\kappa t + 1)/\kappa, \quad \kappa = 2, \quad r^* = 2.2\% \]
(Co-)Existence of Money

- Proposition 1:
  - If $\tilde{\sigma}^2 > \rho$ and $M(0) \leq 0$, then local money has value and $\nu = 0$ (no NFAP) is absorbing state
  - Otherwise, if $\tilde{\sigma}^2 - \rho + M(0) > 0$ then positive amount of global money is held in the small country (and local money may or may not have value)

$$M(\nu_t) = r^* - (\Phi(t) - \delta) - \mu^{PT} + (\sigma^{PT})^2$$
Local and global money demand
Flight to safety (into dollar)

- Unanticipated increase in $\sigma^{PT}$
  - terms of trade or productivity becomes more volatile

- Portfolio share held in dollars increases
  - Dollar valuation is higher

- Transition
  - $\nu_t = G_t/(b_tK_t)$ remains unchanged upon shock, then drifts up
Numerical Example, $\sigma^{PT}$ up from .15 to .16

$\rho = 5\%$, $\tilde{\sigma} = 0.3$, $\mu^{PT} = 1\%$, 

$a = 0.13$, $\delta = 0.02$, $\Phi(\iota) = \log(\kappa\iota + 1) / \kappa$, $\kappa = 2$, $r^* = 2.2\%$
Exchange rate dynamics - UIP

- For $i_t = i_t^*$ ($ = 0$)
  - foreign currency is expected to appreciate relative to local (if held in positive quantity)

- Exchange rate and $\nu$
  - high $\nu$: dollar falls in value relative to local currency (and expected to appreciate faster as $\nu$ moves back to the steady state)
  - locals overhedge trade risk, so demand for dollar falls

$$i_t^* - i_t = E_t[\Delta E] + \psi_t$$

$$\psi_t = -\sigma^\theta \left( \sigma^\zeta - \sigma^\theta \right)$$

from asset pricing equations
Spillover from lowering $r^*$
**Higher inflation $\pi_t$**

- Seigniorage subsidizes capital
- Local money is less attractive $\rho + \pi_t - \mu_t^\theta = (1 - \theta_t - \zeta_t)^2 \bar{\sigma}^2$
- Short run: investment drops, long run: investment rises (slightly)
Mundell-Fleming Trilemma

- Trilemma: Can only pick a 2 desiderata out of 3 – 1 side

- “Dilemma”: Pick only 1
Mundell-Fleming Trilemma

- Trilemma: Can only pick a 2 desiderata out of 3 — 1 side

- “Dilemma”: Pick only 1
Floating Exchange Rate

- With floating exchange rate & open capital account

Still range of monetary policy (local & global money imperfect substitutes)

- Inflation boosts growth, but only possible up to limit \( \bar{\pi}(r^*) \)
- Beyond limit, monetary policy has no bite
  - Citizens of small country hold only global money
- Range is higher with higher inflation in the large country
  - Large country’s monetary policy determines range for small country

- Policy range is larger
  if local money is backed by taxes
Closed capital account

- Range of monetary policy is much larger up to $\bar{\pi} = \bar{\sigma}^2 - \rho$

(physical capital is risky store of value)
Fixed exchange rate regimes & no MoPo

- **Dollarization** = (fully backed) **Currency Board**

- **Exchange rate peg**
  - Requires strong fiscal backing (since no backing by holding global reserves)
    - After a string of adverse shocks, government must tax and use to proceed proceeds to remove some of the local currency in circulation
Foreign Currency Reserves

- Irrelevance Theorem: If central bank holds global money (reserves) citizens in small country will hold accordingly less

Remarks:
- If central banks holds more $-reserves than citizens would like to hold, then agents borrow foreign currency from abroad
- If local money is worthless (without foreign reserves), then the value of local money derives only from the reserves
- If government borrows $, small country citizens will hold accordingly more $
- With fiscal backing of the local money, there are more nuances
Conclusion

- Endogenous value of money in 2 countries
  - Local currency: better hedge for idiosyncratic risk (non-tradeable consumption)
  - Global currency: hedge against ToT + export productivity shocks
  - Exchange rate endogenous

- Spillover effects from foreign monetary policy

- Nuanced Mundell-Fleming Trilemma
  - Local and global money have different risk profile (imperfect substitutes)
    ⇒ increases MoPo space
  - Too high inflation:
    local citizens substitute local currency for global currency
    ⇒ limits MoPo space

- Central Bank’s foreign reserves holding: Irrelevance Result

- Optimal Monetary Policy
  - Idiosyncratic risk – correct pecuniary externality (real interest rate)
  - International savings
Thank you!