The I Theory of Money
&
On the Optimal Inflation Rate

Markus Brunnermeier & Yuliy Sannikov

Princeton Initiative 2018

Princeton, Sept. 8th, 2018

“Money and Banking” (in macro-finance)

- Money → store of value/safe asset

“Money and Banking” (in macro-finance)

- Money → store of value/safe asset
- Banking → “diversifier”
  holds risky assets, issues inside money

“Money and Banking” (in macro-finance)

- Money → store of value/safe asset
- Banking → “diversifier”

  holds risky assets, issues inside money
- Amplification/endogenous risk dynamics
  - Value of capital declines due to fire-sales **Liquidity spiral**
    - Flight to safety
  - Value of money rises **Disinflation spiral** a la Fisher
    - Demand for money rises – less idiosyncratic risk is diversified
    - Supply for inside money declines – less creation by intermediaries
      - Endogenous money multiplier = f(capitalization of critical sector)
  - Paradox of Thrift (in risk terms)

"Money and Banking" (in macro-finance)

- Money — store of value/safe asset
- Banking — "diversifier"
  holds risky assets, issues inside money
- Amplification/endogenous risk dynamics
  - Value of capital declines due to fire-sales — **Liquidity spiral**
    - Flight to safety
  - Value of money rises — **Disinflation spiral** a la Fisher
    - Demand for money rises – less idiosyncratic risk is diversified
    - Supply for inside money declines – less creation by intermediaries
      - Endogenous money multiplier = f(capitalization of critical sector)
  - ~~Paradox of Thrift~~ Paradox of Prudence (in risk terms)

Brunnermeier & Sannikov

"Money and Banking" (in macro-finance)

- Money store of value/safe asset
- Banking "diversifier"
  holds risky assets, issues inside money
- Amplification/endogenous risk dynamics
  - Value of capital declines due to fire-sales **Liquidity spiral**
    - Flight to safety
  - Value of money rises **Disinflation spiral** a la Fisher
    - Demand for money rises – less idiosyncratic risk is diversified
    - Supply for inside money declines – less creation by intermediaries
      - Endogenous money multiplier = f(capitalization of critical sector)
  - ~~Paradox of Thrift~~ Paradox of Prudence (in risk terms)
- Monetary Policy (redistributive)

Some literature

- Roles of money
  - Unit of account
  - Medium of exchange
    (Clower, Lagos & Wright)
  - Store of value
    (Samuelson, Bewley, Aiyagari, Scheinkman & Weiss, Kiyotaki & Moore)
- Models without inside money imply inflation in downturns
  - Less money needed to perform fewer transactions
- "Money view" (Friedman & Schwartz)
  - Downturns → Bank liabilities decrease
- "Credit view"
  - Downturns → equity capital → bank cuts assets/credit
  - BGG, Kiyotaki & Moore, He & Krishnamurthy, BruSan2014,
    Drechsler, Jeanne & Korinek, Savov & Schnabl
- Financial Stability
  - Diamond & Rajan 2010, Curdia & Woodford 2010, Stein 2012

Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

(New) Keynesian Demand Management		I Theory of Money Risk (premium) management
Stimulate aggregate **consumption** Substitution effect		Alleviate **balance sheet constraints** Income/wealth effect
Woodford	Tobin (1982)	BruSan
Price stickiness Perfect capital markets	**Both**	**Financial Frictions** Incomplete markets
Representative Agent	**Heterogeneous Agents**	
Cut i Reduces r due to price stickiness Consumption c rises		- -

Brunnermeier & Sannikov

Monetary Policy Transmission Channel

- Consumption Boost approach to "Bottleneck approach"

(New) Keynesian Demand Management		I Theory of Money Risk (premium) management
Stimulate aggregate **consumption** Substitution effect		Alleviate **balance sheet constraints** Income/wealth effect
Woodford	Tobin (1982)	BruSan
Price stickiness Perfect capital markets	**Both**	**Financial Frictions** Incomplete markets
Representative Agent	**Heterogeneous Agents**	
Cut i Reduces r due to price stickiness Consumption c rises	**Cut i** Changes bond prices Redistributes from low MPC to high MPC consumers	- -

Brunnermeier & Sannikov

Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

(New) Keynesian Demand Management		I Theory of Money Risk (premium) management
Stimulate aggregate **consumption** Substitution effect		Alleviate **balance sheet constraints** Income/wealth effect
Woodford	Tobin (1982)	BruSan
Price stickiness Perfect capital markets	**Both**	**Financial Frictions** Incomplete markets
Representative Agent	**Heterogeneous Agents**	
Cut i Reduces r due to price stickiness Consumption c rises	**Cut i** Changes bond prices Redistributes from low MPC to high MPC consumers	**Cut i** Changes asset prices **Ex-post:** Redistributes to balance sheet impaired sector **QE** - -

Brunnermeier & Sannikov

Literature

- Without intermediaries: Money as store of value = bubble

\Friction	OLG	Incomplete Markets + idiosyncratic risk	
Risk	deterministic	endowment risk borrowing constraint	investment risk
Only money	Samuelson	Bewley	
With capital	Diamond	Aiyagari	Angeletos
		Risk tied up with individual	capital

Literature

- Without intermediaries: Money as store of value = bubble

\Friction	OLG	Incomplete Markets + idiosyncratic risk	
Risk	deterministic	endowment risk borrowing constraint	investment risk
Only money	Samuelson	Bewley	
With capital	Diamond	Aiyagari	Angeletos $q = 1$

depends on
price of capital q

Literature

- Without intermediaries: Money as store of value = bubble

\Friction	OLG	Incomplete Markets + idiosyncratic risk	
Risk	deterministic	endowment risk borrowing constraint	investment risk
Only money	Samuelson	Bewley	
			Basic "I Theory"
With capital	Diamond	Aiyagari	cash flow shock
	$f'(k^*) = r^*$, Dynamic inefficiency $r < r^*, K > K^*$	Inefficiency $r < r^*, K > K^*$	Pecuniary externality Inefficiency $r > r^*, K < K^*$

(money) bubbles if $r < g$

Abel et al. vs. Geerolf

$r^m = g$

Roadmap

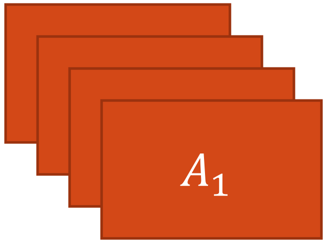
- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - "On the optimal inflation rate" (inflation target)

- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
    - The "Reversal Interest Rate: The Effective Lower Bound"

Model without intermediaries

- Technologies a



- Each household can only operate one firm
  - Physical capital

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma^a dZ_t^a + \tilde{\sigma} d\tilde{Z}_t^a$$

sector; idiosyncratic risk

  - Output

$$y_t = Ak_t$$

- Demand for money

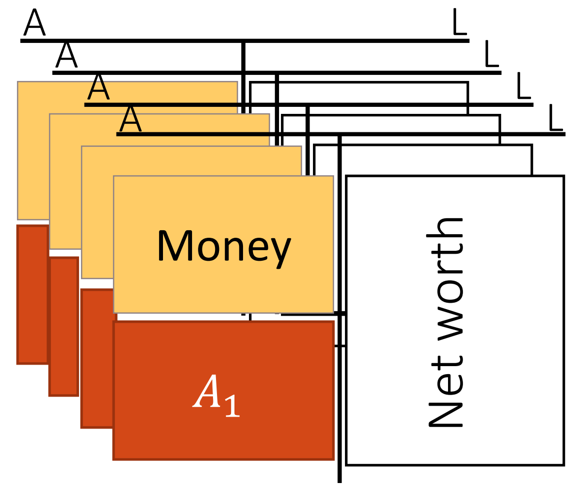
Adding outside money

Outside Money

- Technologies a

- $q_t K_t$ value of physical capital

- $p_t K_t$ value of outside money



- Each household can only operate one firm
  - Physical capital

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma^a dZ_t^a + \tilde{\sigma} d\tilde{Z}_t^a$$

sector idiosyncratic risk

  - Output

$$y_t = A k_t$$

- Demand for money

Solving

1. Postulate
   - Price processes $dp_t/p_t = \mu_t^p dt + \sigma^p dZ_t, \quad dq_t/q_t = \cdots$
   - Portfolio processes dx_t^a/x_t^a

2. Derive return processes
   - $dr^{Ka} = \cdots$
   - $dr^M = \cdots \qquad dt - (\mu^M - \mu^{Mi})dt$

     money supply growth rate that is NOT distributed via interest payment

     Set $\mu^{Mi} = 0$

3. Optimality conditions & Market clearing conditions

4. Solve "undetermined coefficients" $(\mu^x(s_t), \sigma^x(s_t))$
   - Solving ODE with boundary conditions
   - Solve for constants p, q

Solving

1. Postulate
   - Price processes $p_{\not t}$, $q_{\not t} = \cdots$
   - Portfolio processes $x^a_{\not t}$

2. Derive return processes
   - $dr^{Ka} = (\Phi(\iota) - \delta)dt + \sigma^a dZ^a_t + \frac{A-\iota}{q}dt + \tilde{\sigma} d\tilde{Z}_t$
   - $dr^M = (\Phi(\iota) - \delta)dt + \sigma^a dZ^a_t - (\mu^M - \mu^{Mi})dt$

   money supply growth rate that is NOT distributed via interest payment

   Set $\mu^{Mi} = 0$

3. Optimality conditions & Market clearing conditions

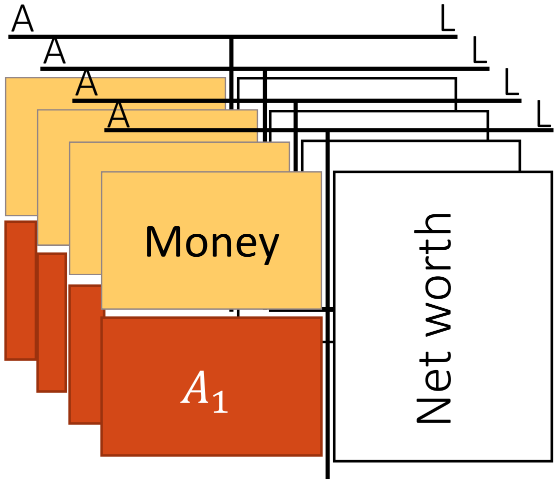
4. Solve "undetermined coefficients" $(\mu^x(s_t), \sigma^x(s_t))$
   - Solving ODE with boundary conditions
   - Solve for constants p, q

Aside: Alternative Shocks

Outside Money

- $q_t K_t$ value of physical capital
- $p_t K_t$ value of outside money

- Technologies a



- Alternative shocks:

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt$$

but

  - Real cash flow shocks

$$\tilde{\sigma} k_t d\tilde{Z}_t^a$$

  - Nominal cash flow shocks

$$p_t \tilde{\sigma} k_t d\tilde{Z}_t^a$$

- Each household can only operate one firm
  - Physical capital shocks

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)dt + \sigma^a dZ_t^a + \tilde{\sigma} d\tilde{Z}_t^a$$

sector — idiosyncratic risk

  - Output

$$y_t = A k_t$$

- Demand for money

Optimality (=) for $E\left[\int_0^\infty e^{-\rho t}\log c_t\, dt\right]$

- Investment rate, ι

- Portfolio choice, x^a

- Consumption, c_t

Optimality (=)

- Investment rate, ι
  - Tobin's q: $\Phi'(\iota) = \frac{1}{q}$ (static problem)
    - For $\Phi(\iota) = \frac{1}{\kappa}\log(\kappa\iota + 1) \Rightarrow \kappa\iota = q - 1$
- Portfolio choice, x^a
- Consumption, c_t

Optimality (=)

- Investment rate, ι
  - Tobin's q: $\Phi'(\iota) = \frac{1}{q}$ (static problem)
    - For $\Phi(\iota) = \frac{1}{\kappa}\log(\kappa\iota + 1) \Rightarrow \kappa\iota = q - 1$
- Portfolio choice, x^a
  - $E[dr^{Ka} - dr^M]/dt = Cov[dr^{Ka} - dr^M, \underbrace{\frac{dn_t}{n_t}}_{dr^M + x^a(dr^{Ka} - dr^M)}] = x^a(\tilde{\sigma})^2$

$$x^a = \frac{E[dr^{Ka} - dr^M]/dt}{(\tilde{\sigma})^2} = \frac{(A-\iota)/q + \mu^M}{(\tilde{\sigma})^2}$$

  - Dividend yield on capital must be ρ
- Consumption, c_t

Optimality (=)

- Investment rate, ι
  - Tobin's q: $\Phi'(\iota) = \frac{1}{q}$ (static problem)
    - For $\Phi(\iota) = \frac{1}{\kappa}\log(\kappa\iota + 1) \Rightarrow \kappa\iota = q - 1$
- Portfolio choice, x^a
  - $E[dr^{Ka} - dr^M]/dt = Cov[dr^{Ka} - dr^M, \underbrace{\frac{dn_t}{n_t}}_{dr^M + x^a(dr^K - dr^M)}] = x^a(\tilde{\sigma})^2$

$$x^a = \frac{E[dr^{Ka} - dr^M]/dt}{(\tilde{\sigma})^2} = \frac{(A-\iota)/q + \mu^M}{(\tilde{\sigma})^2}$$

  - Dividend yield on capital must be ρ
- Consumption, c_t
  - Demand $\rho N_t = \rho(q + p)K_t$

Optimality (=) & market clearing (=)

- Investment rate, ι
  - Tobin's q: $\Phi'(\iota) = \frac{1}{q}$ (static problem)
    - For $\Phi(\iota) = \frac{1}{\kappa}\log(\kappa\iota + 1) \Rightarrow \kappa\iota = q - 1$
- Portfolio choice, x^a
  - $E[dr^{Ka} - dr^M]/dt = Cov[dr^{Ka} - dr^M, \underbrace{\frac{dn_t}{n_t}}_{dr^M + x^a(dr^K - dr^M)}] = x^a(\tilde{\sigma})^2$

$$x^a = \frac{E[dr^{Ka} - dr^M]/dt}{(\tilde{\sigma})^2} = \frac{(A-\iota)/q + \mu^M}{(\tilde{\sigma})^2} = \frac{q}{q+p}$$

Capital market clearing

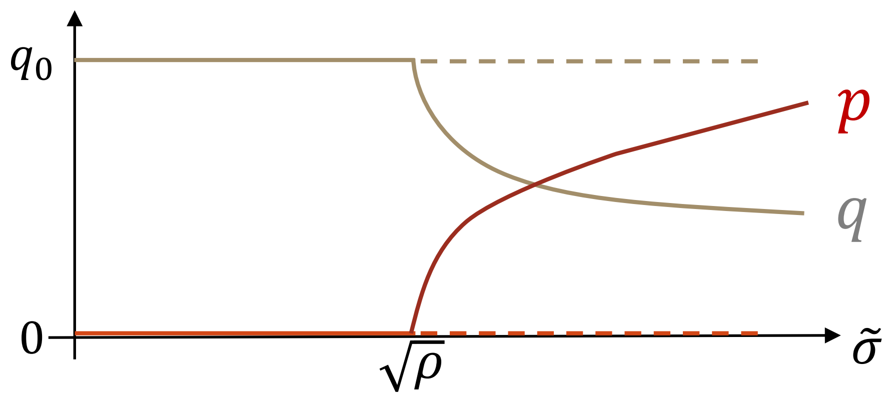
  - Dividend yield on capital must be ρ
- Consumption, c_t
  - Demand $\rho N_t = \rho(q+p)K_t = (A-\iota)K_t$ Supply

Output market clearing

$$q = \underbrace{\left(\frac{q}{q+p}\right)}_{=x^a}(A-\iota)/\rho$$

Equilibrium

Moneyless equilibrium		Money equilibrium
$p_0 = 0$		$p = \frac{\tilde{\sigma} - \sqrt{\rho}}{\sqrt{\rho}} q$
$q_0 = \frac{\kappa A + 1}{\kappa \rho + 1}$	>	$q = \frac{\kappa A + 1}{\kappa \sqrt{\rho} \tilde{\sigma} + 1}$



Welfare analysis

Moneyless equilibrium		Money equilibrium
$p_0 = 0$		$p = \frac{\tilde{\sigma} - \sqrt{\rho}}{\sqrt{\rho}} q$
$q_0 = \frac{\kappa A + 1}{\kappa \rho + 1}$	>	$q = \frac{\kappa A + 1}{\kappa \sqrt{\rho} \tilde{\sigma} + 1}$
g_0	>	g
welfare$_0$	<	welfare

Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - "On the optimal inflation rate" (inflation target)

- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
    - The "Reversal Interest Rate: The Effective Lower Bound"

Steady state MoPo – no intermediaries

- Shock structure: real cash flow shock
  - See paper "On the Optimal Inflation Rate" (AER P&P 2016)

- Policy variable: Money growth rate μ
- Portfolio choice: $x^{k*} = \frac{q(A-\iota^*)}{\tilde{\sigma}^2} + \frac{q^2\mu}{\tilde{\sigma}^2}$

- Capital markets clearing: $\frac{1}{p+q} = \frac{A-\iota^*}{\tilde{\sigma}^2} + \frac{q\mu}{\tilde{\sigma}^2}$

Equilibrium

- Collecting the three equations:

$$q = 1 + \kappa\iota^*$$
$$\rho(p+q) = A - \iota^*$$
$$\frac{\sigma^2}{q+p} = A - \iota^* + q\mu$$

- Equilibrium solved in terms of $\hat{\mu} := x^k\mu$ (monotone transformation)

$$p = \frac{\sigma(1+\kappa\rho)}{\sqrt{\rho+\hat{\mu}}} - (1+\kappa A)$$
$$q = 1 + \kappa A - \frac{\kappa\rho\sigma}{\sqrt{\rho+\hat{\mu}}}$$
$$\iota^* = A - \rho\frac{\sigma}{\sqrt{\rho+\hat{\mu}}}$$

Closed form!

Welfare

- Plug in FOC in value function
- Plug in equilibrium
- All households start symmetrically

- Expected Utility of an individual household

$$V = V_0 + \frac{\frac{1}{\kappa}\log\left(1 + \kappa A - \frac{\kappa\rho\sigma}{\sqrt{\rho+\hat{\mu}}}\right) - \delta + \rho - \frac{1}{2}(\rho+\hat{\mu})}{\rho^2} + \frac{\log\left(\frac{\sigma}{\sqrt{\rho+\hat{\mu}}}\right)}{\rho}.$$

Closed form!

Optimal inflation rate

- Money growth μ affects (steady state) inflation in two ways

$$\pi = \mu^M - \underbrace{\left(\Phi\left(\iota^*(\mu^M)\right) - \delta\right)}_{g}$$

- Proposition:
  - If $\frac{\sigma}{\sqrt{\rho}} > \frac{2(A\kappa+1)}{1+2\kappa\rho}$, welfare maximizing money growth rate $\mu^* > 0$.
    - Market outcome is not even constrained Pareto efficient
    - Economic growth rate, $g > r^m$, is also higher
  - Growth maximizing $\mu^{g*} \geq \mu^{M*}$, s.t. $p^{g*} = 0$, Tobin (1965)

$$\iota^* = A - \rho\frac{\sigma}{\sqrt{\rho + \hat{\mu}}}$$ increasing in $\hat{\mu}$

- Corollary: No super-neutrality of money
  - Nominal money growth rate affects real economy
    - No price/wage rigidity, no monopolistic competition

Optimal inflation rate: Emerging markets

- Proposition: (comparative static)
  - μ^{M*} does not depend on depreciation rate δ, but inflation does
  - μ^{M*} is strictly increasing in idiosyncratic risk σ
    "Emerging markets should have higher inflation target"

Conclusion: our 3 initial questions

- What should the (long-run) optimal inflation rate be?
  - Competitive market outcome is constrained Pareto inefficient.
  - Inflation is Pigouvian & internalizes pecuniary externality!
    - HH take real interest rate as given, but
    - Portfolio choice affects economic growth and real interest rate

- What role do financial frictions play?
  - incomplete markets ⇒ no superneutrality of money
    - No price/wage rigidity needed

- Emerging markets, with less developed financial markets, should have higher inflation rate/target
  - Higher idiosyncratic risk ⇒ higher pecuniary externality

Main results

- HH portfolio choice
  - Physical capital: w/ idiosyncratic risk + dividend
  - Money: w/o idiosyncratic risk + no dividend (bubble)
    - Tilted inefficiently towards money

- Money supply growth ⇒ inflation ⇒ "tax on money"
- ⇒ lowers real interest rate ⇒ tilts portfolio choice
- ⇒ boosts physical investment ⇒ higher economic growth
- ⇒ raises real interest rate (partially undoes inflation tax)
- Pecuniary externality:
  - individual households do not take this GE effect into account.
  - Planner who can print money and distribute seignorage can improve growth + Pareto welfare.
- Derive optimal money growth rate/inflation rate

Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - "On the optimal inflation rate" (inflation target)

- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy

Outline of two sector model

- Technologies b
- Technologies a

- Households have to
  - Specialize in one subsector for one period → sector specific + idiosyncratic risk

$$\frac{dk_t}{k_t} = \cdots dt + \sigma^b dZ_t^b + \tilde{\sigma} d\tilde{Z}_t^b$$

$$\frac{dk_t}{k_t} = \cdots dt + \sigma^a dZ_t^a + \tilde{\sigma} d\tilde{Z}_t^a$$

  - Demand for money

Add outside money

Outside Money

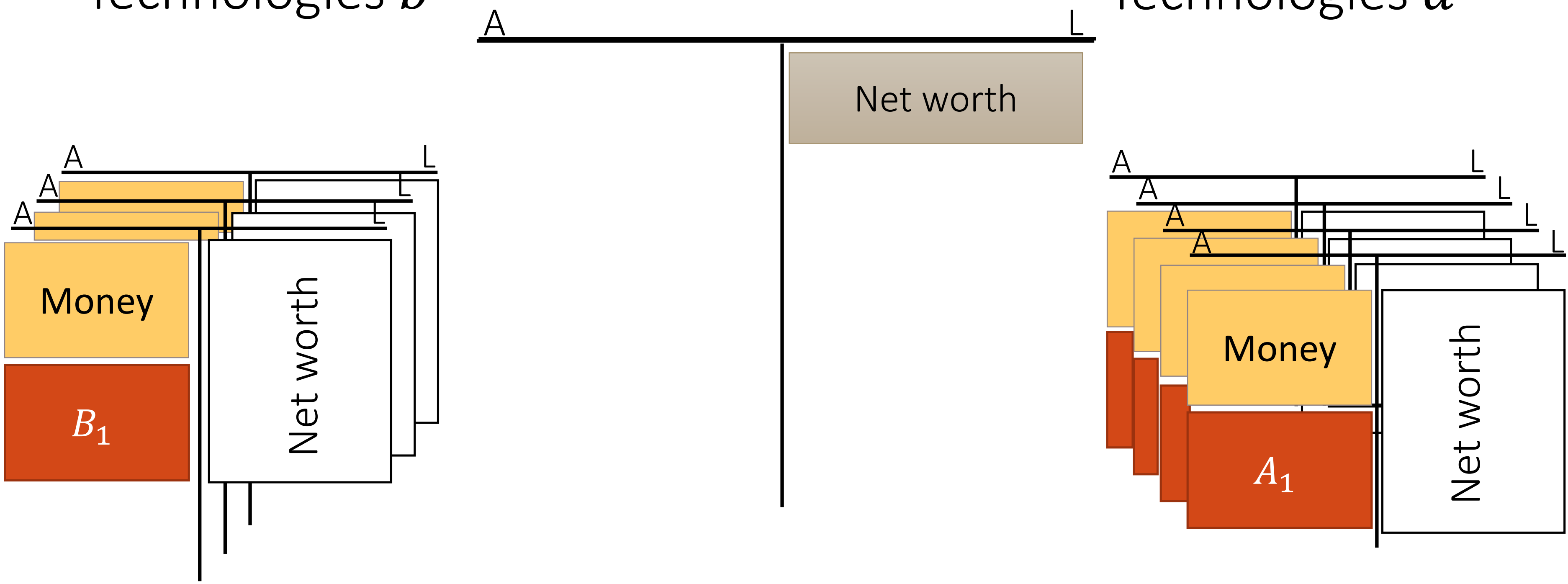
- Technologies b
- Technologies a

- Households have to
  - Specialize in one subsector
    for one period
  - Demand for money

Add intermediaries

Outside Money

- Technologies b
- Technologies a



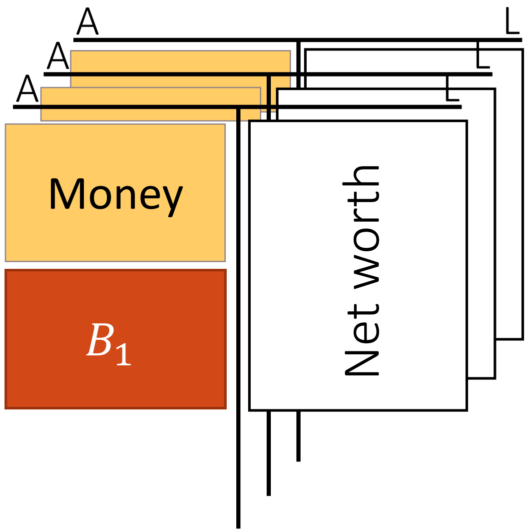
- Risk can be <u>partially sold off</u> to intermediaries

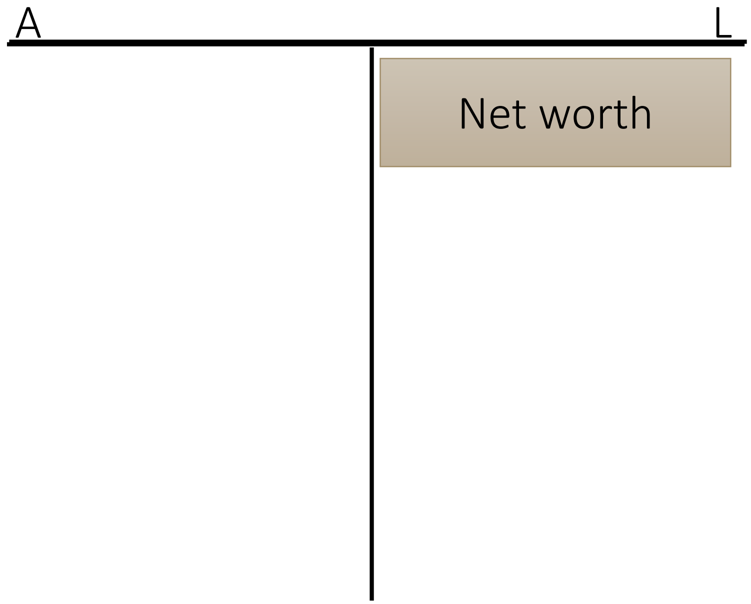
- Risk is <u>not contractable</u> (Plagued with moral hazard problems)

Add intermediaries

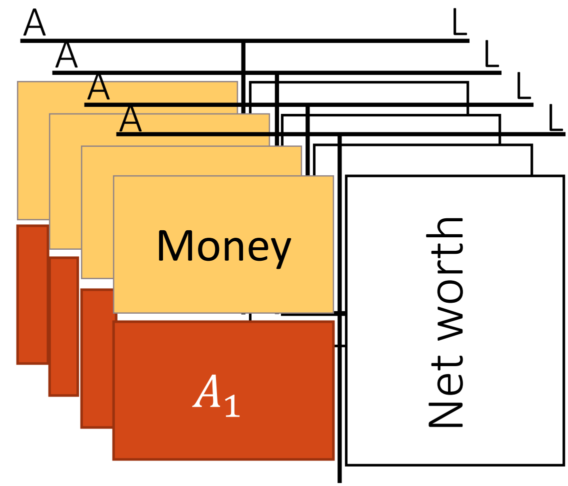
Outside Money

- Technologies b





- Technologies a

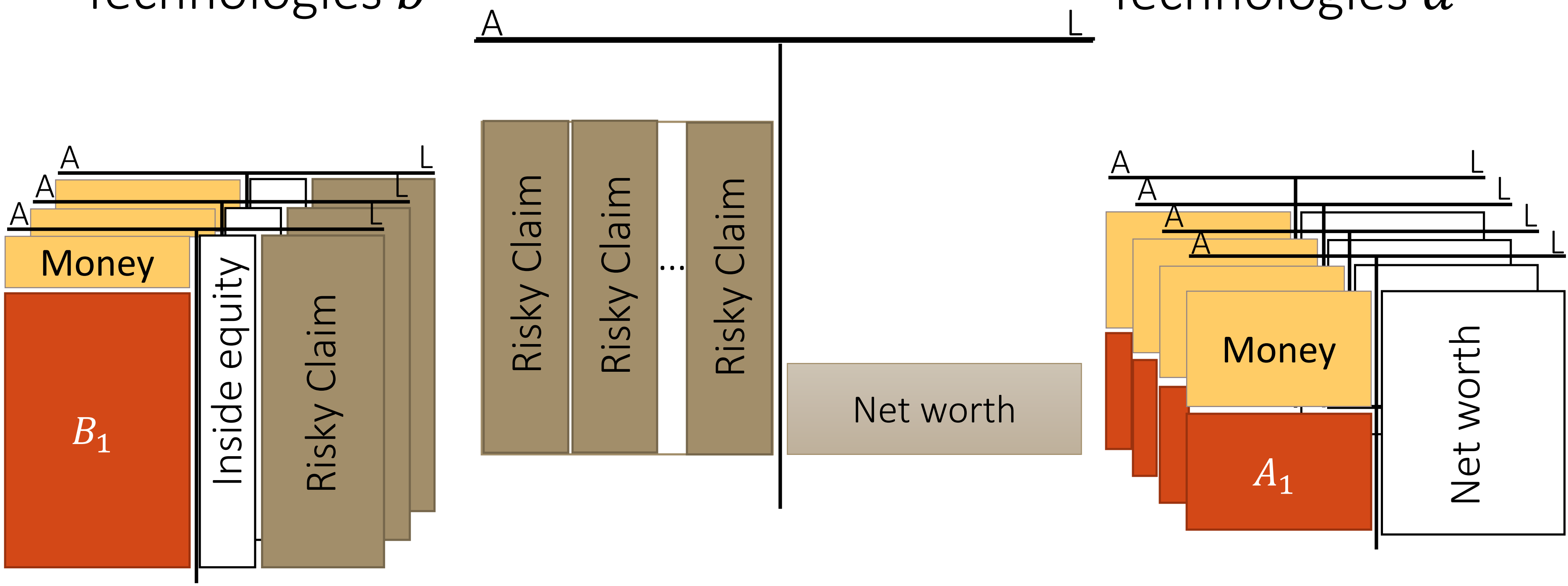


- Intermediaries
  - Can hold outside equity & diversify within sector b
  - Monitoring

Add intermediaries

Outside Money

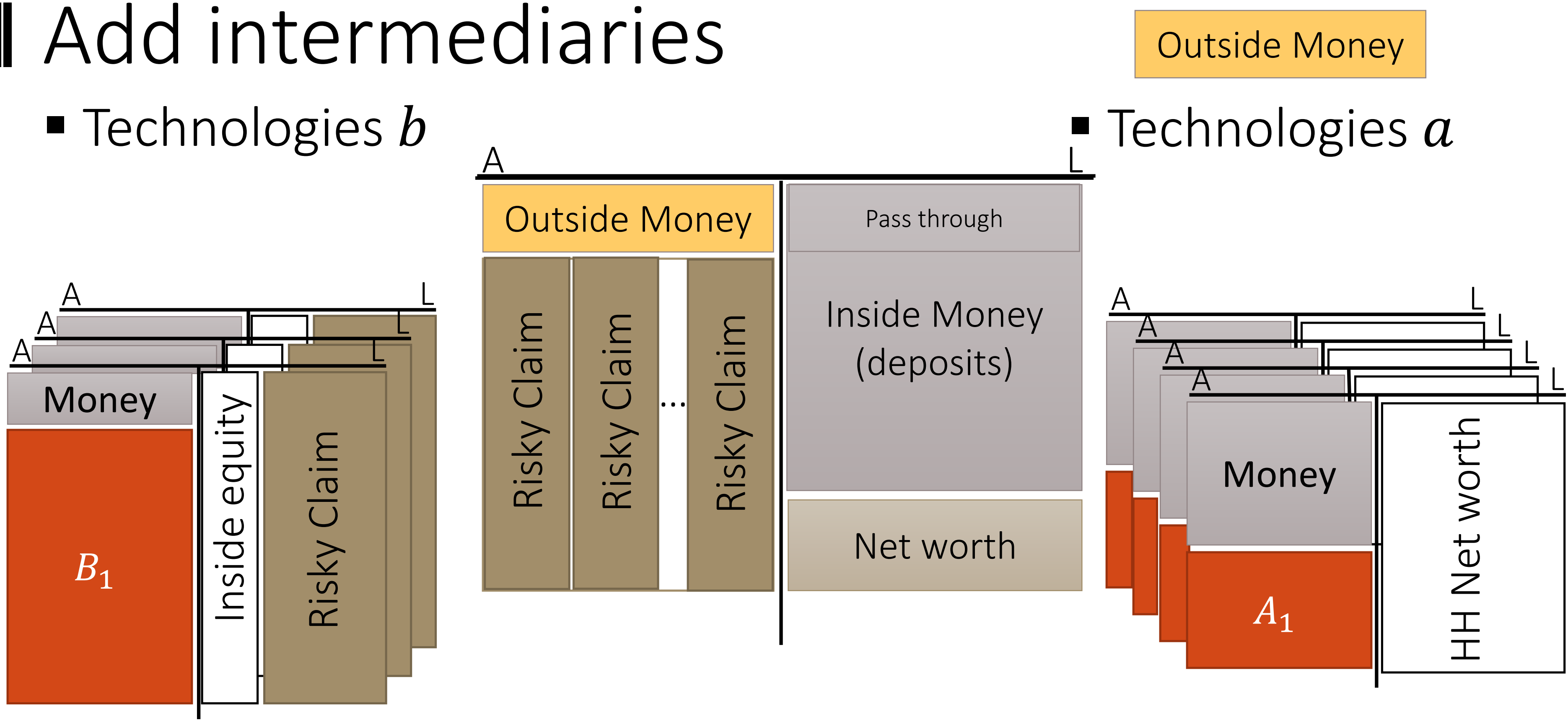
- Technologies b
- Technologies a



- Intermediaries
  - Can hold outside equity & diversify within sector b
  - Monitoring

Add intermediaries

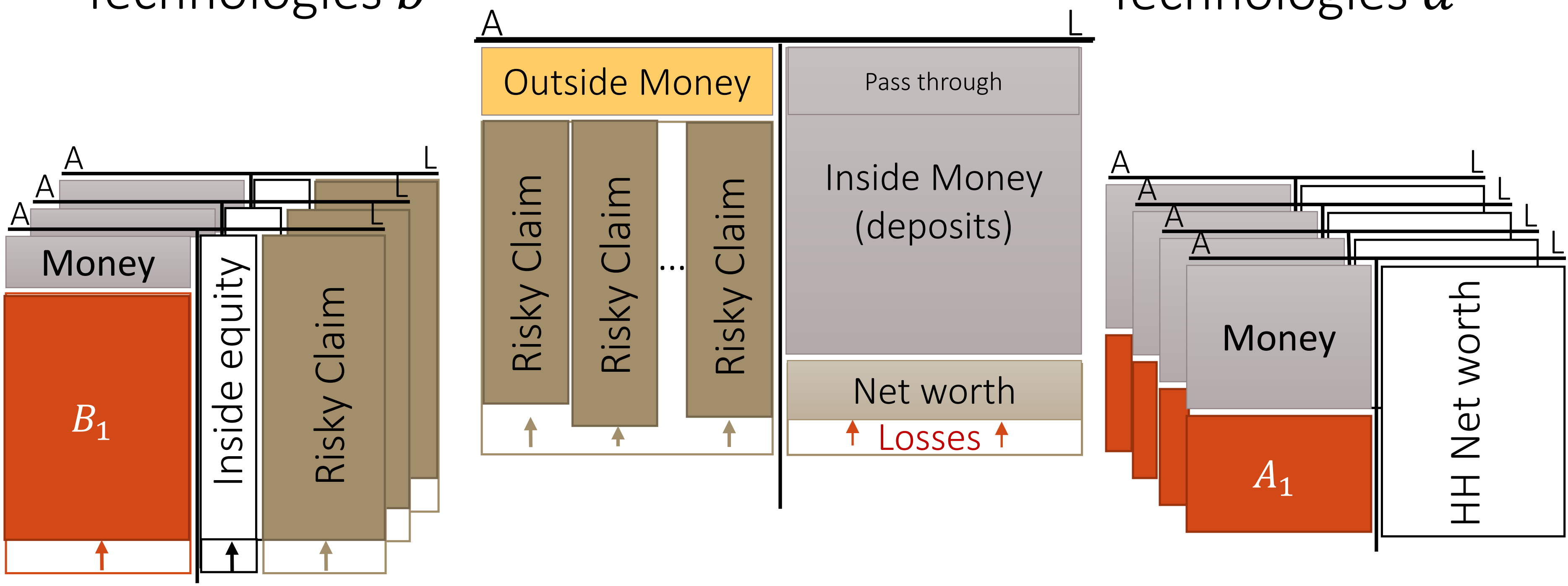
- Technologies b
- Technologies a



- Intermediaries
  - Can hold outside equity & diversify within sector b
  - Monitoring
  - Create inside money
  - Maturity/liquidity transformation

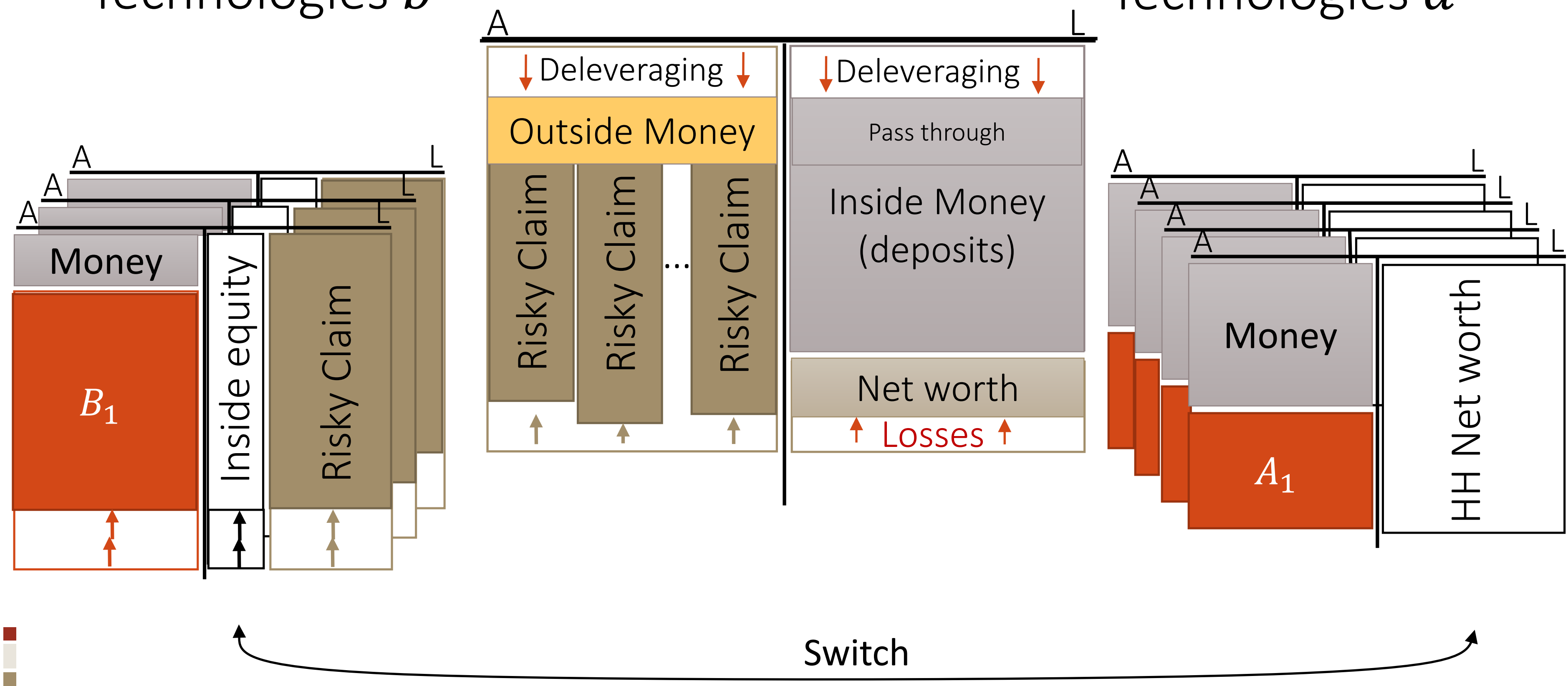
Shock impairs assets: 1st of 4 steps

- Technologies b
- Technologies a



Shrink balance sheet: 2nd of 4 steps

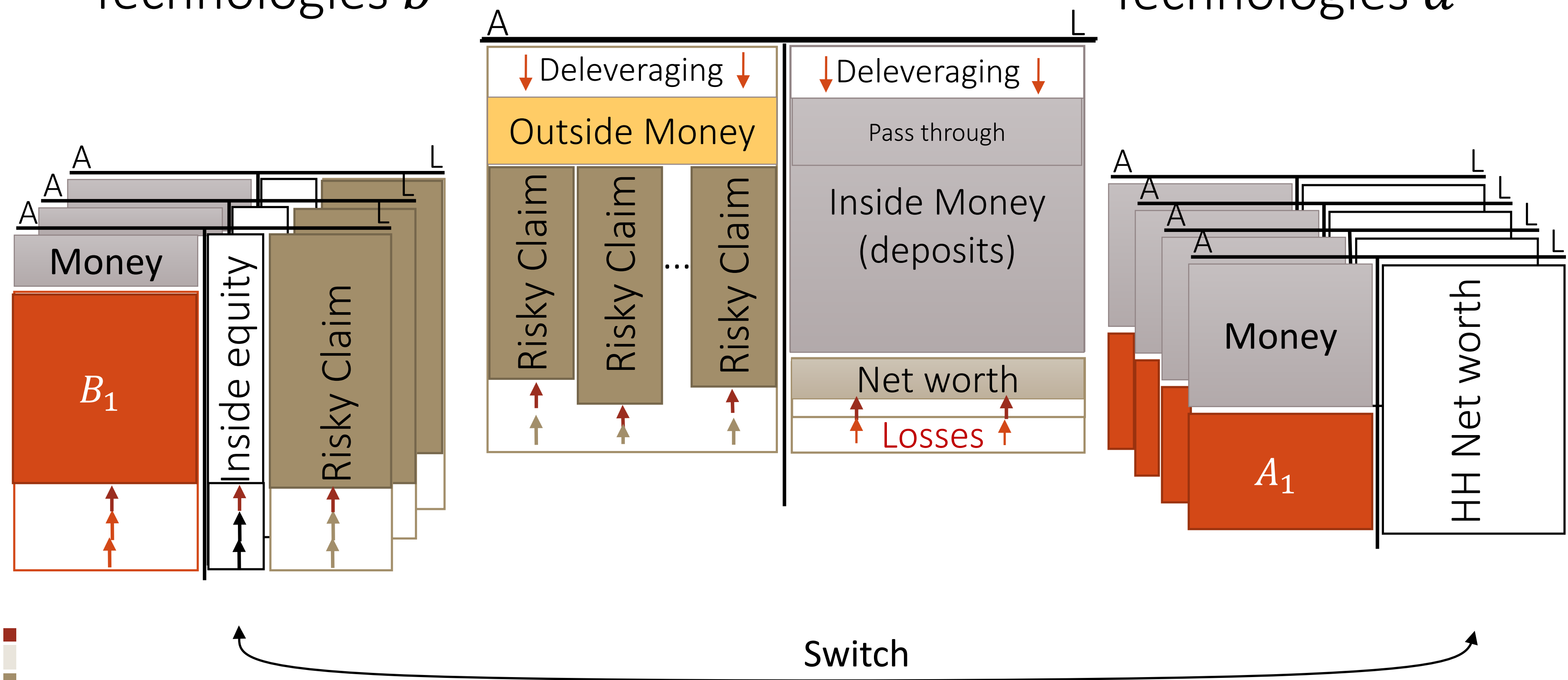
- Technologies b
- Technologies a



"Paradox of Prudence"

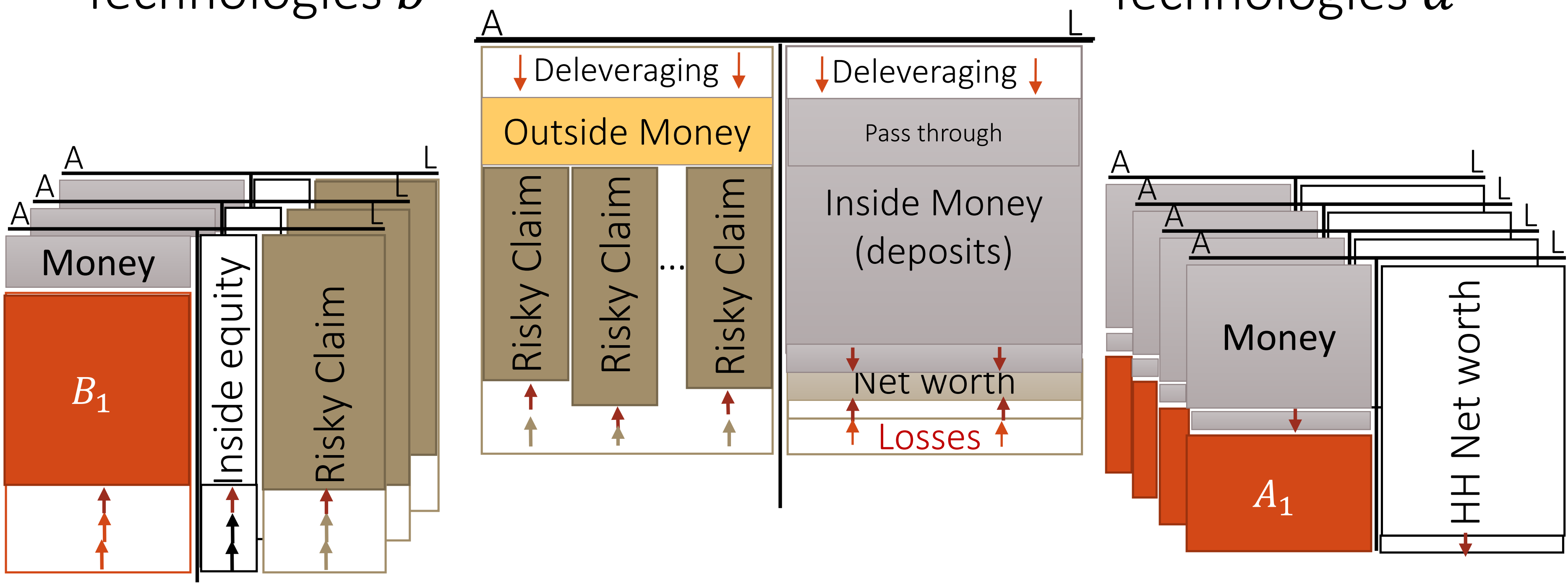
Liquidity spiral: asset price drop: 3rd of 4

- Technologies b
- Technologies a



Disinflationary spiral: 4th of 4 steps

- Technologies b
- Technologies a



… after an adverse shock

- Intermediaries are hit and shrink their balance sheets inducing
  - Asset side liquidity spiral financial stability
  - Liability side disinflation spiral price stability

- Response of intermediaries to adverse shock leads to endogenous risk
  - Amplification
  - Persistence

- Other sectors can also be undercapitalized
  - Japan 1980: corporate sector
  - US 2000s: household sector

Formal model: capital & output

Technologies	b		a
Physical capital K_t - Capital share	ψ_t		$1-\psi_t$
Output goods	$Y_t^b = Ak_t^b$	Imperfect substitutes	$Y_t^a = Ak_t^a$

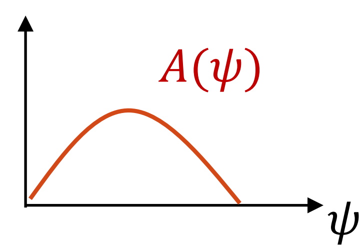
Aggregate good (CES)
- Consumed or invested
- numeraire

$$Y_t = \left(\frac{1}{2}\left(Y_t^b\right)^{(s-1)/s} + \frac{1}{2}\left(Y_t^a\right)^{(s-1)/s}\right)^{s/(s-1)}$$

Price of goods

$$P_t^b = \frac{1}{2}\left(\frac{Y_t}{Y_t^b}\right)^{1/s} \qquad P_t^a = \frac{1}{2}\left(\frac{Y_t}{Y_t^a}\right)^{1/s}$$

- Model setup in paper is more general: $Y_t = A(\psi_t)K_t$



Brunnermeier & Sannikov

Formal model: risk

- When k_t is employed in sector a by agent j

$$dk_t = (\Phi(\iota_t) - \delta)k_t dt + \sigma^a k_t dZ_t^a + \sigma^j k_t d\tilde{Z}_t^a$$

Investment rate (per unit of k_t)

sectorial idiosyncratic
independent Brownian motions
(fundamental cash flow risk)

  - $\Phi(\iota_t)$ is increasing and concave, e.g. $\log[(\kappa\iota_t + 1)/\kappa]$
  - All dZ are independent of each other

- Intermediaries can diversify within sector b
  - Face no idiosyncratic risk
- Households cannot become intermediaries or vice versa

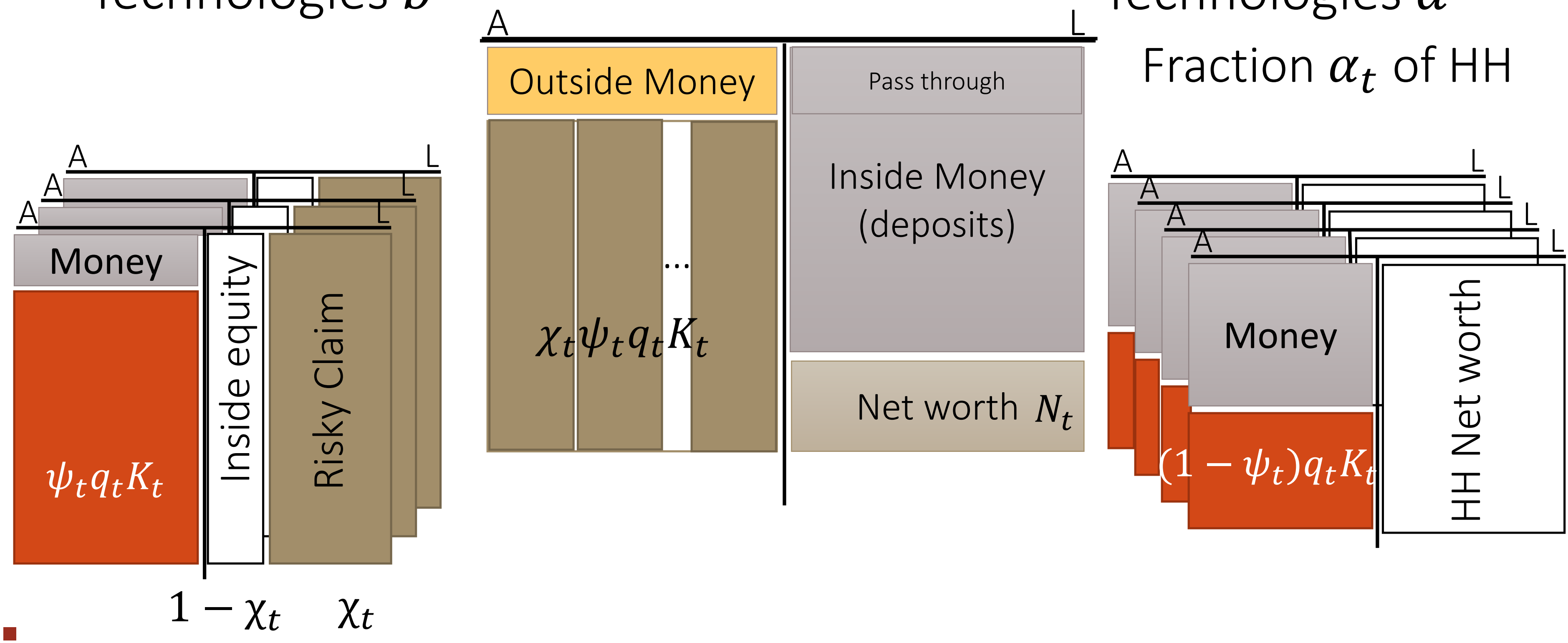
Financing constraints

Technologies	b	a
Equity issuance - Special case	Inside equity $\chi_t \geq \underline{\chi}$ $\underline{\chi} = 0\%$ (no inside equity)	Inside equity only
Households' risk	dZ^b & $d\tilde{Z}^b$ sector & idiosyncratic	dZ^a & $d\tilde{Z}^a$ sector & idiosyncratic
Intermediaries' risk	dZ^b can diversify idiosyncratic risk	

Capital/risk shares

- Technologies b
- Technologies a

Fraction α_t of HH



Formal model: preferences

- All agents have logarithmic utility with discount rate ρ

$$E\left[\int_0^\infty e^{-\rho t}\log c_t\,dt\right]$$

- Implies
  - Consumption = ρ * net worth
  - Equilibrium Sharpe ratio $\propto$ Covariance with net worth

Solution steps

1. Postulate endogenous processes
   - $dq_t/q_t = \mu_t^q dt + \sigma_t^{q,a} dZ_t^a + \sigma_t^{q,b} dZ_t^b$
     - Returns from holding capital
   - $dp_t/p_t = \mu_t^p dt + \sigma_t^{p,a} dZ_t^a + \sigma_t^{p,b} dZ_t^b$
2. Equilibrium conditions
   - Agents' optimization
     - Internal investment (new capital formation)
     - Optimal portfolio choice — Sharpe ratio $\propto$ Cov. with net worth
     - Optimal consumption — $\rho *$ networth
   - Market clearing conditions
3. Law of motion of state variable
   - wealth (share) distribution η_t
4. Express in ODEs of state variable

Brunnermeier & Sannikov

Asset returns on technology b

- Physical capital: (in technology b) also earns dividend yield

Vector dZ_t^a, dZ_t^b

  - If $dq_t/q_t = \mu_t^q dt + (\boldsymbol{\sigma_t^q})^T d\boldsymbol{Z_t}$,
  - $dk_t/k_t = (\Phi(\iota_t) - \delta)dt + \sigma^b dZ_t^b + \tilde{\sigma}^j dZ_t^{b,j}$

Brunnermeier & Sannikov

Asset returns on technology b

- Physical capital: (in technology b) also earns dividend yield
  - If $dq_t/q_t = \mu_t^q dt + (\boldsymbol{\sigma_t^q})^T d\boldsymbol{Z_t}$,
  - $dk_t/k_t = (\Phi(\iota_t) - \delta)dt + \sigma^b dZ_t^b + \tilde{\sigma}^j dZ_t^{b,j}$
  - $dr_t^b = \underbrace{\frac{AP_t^b - \iota_t}{q_t}}_{\text{Dividend yield}} dt + \underbrace{\left(\Phi(\iota_t) - \delta + \mu_t^q + (\boldsymbol{\sigma}_t^q)^T \sigma^i \mathbf{1}^b\right)}_{\text{Expected capital gains}} dt + \left(\boldsymbol{\sigma_t^q} + \sigma^a \mathbf{1}^{\boldsymbol{b}}\right)^T d\boldsymbol{Z_t} + \tilde{\sigma}^j dZ_t^{b,j}$

Asset returns on technology b

- Physical capital: (in technology b) also earns dividend yield
  - If $dq_t/q_t = \mu_t^q dt + (\boldsymbol{\sigma}_t^q)^T d\boldsymbol{Z}_t$,
  - $dk_t/k_t = (\Phi(\iota_t) - \delta)dt + \sigma^b dZ_t^b + \tilde{\sigma}^j dZ_t^{b,j}$
  - $dr_t^b = \frac{AP_t^b - \iota_t}{q_t} dt + (\Phi(\iota_t) - \delta + \mu_t^q + (\boldsymbol{\sigma}_t^q)^T \boldsymbol{\sigma} \mathbf{1}^b)dt + (\boldsymbol{\sigma}_t^q + \boldsymbol{\sigma}\mathbf{1}^{\boldsymbol{b}})^T d\boldsymbol{Z}_t + \tilde{\sigma}^j dZ_t^{b,j}$
  - $dr_t^a =$… (analogous)

$\chi_t dr_t^\chi + (1 - \chi_t) dr_t^I = dr_t^b$

- Return on outside equity held by intermediaries
  - $dr_t^I = dr_t^b - \underbrace{\lambda_t dt}_{\text{risk premium}}$

- Return on inside equity (fraction χ_t) held by b-HH
  - $dr_t^\chi = dr_t^b + \frac{1-\chi_t}{\chi_t} \lambda_t dt$

Asset returns on money

- Money: fixed supply in baseline model, total value $p_t K_t$
  - Return = capital gains (no dividend/interest in baseline model)
  - If $dp_t/p_t = \mu_t^p dt + \boldsymbol{\sigma}_t^{\boldsymbol{p}} d\boldsymbol{Z}_t$,
  - $dK_t/K_t = (\Phi(\iota_t) - \delta)dt + \underbrace{(1-\psi_t)\sigma^a dZ_t^a + \psi_t \sigma^b dZ_t^b}_{(\boldsymbol{\sigma}_t^{\boldsymbol{K}})^T d\boldsymbol{Z}_t}$

$$dr_t^M = \left(\Phi(\iota_t) - \delta + \mu_t^p + \left(\boldsymbol{\sigma}_t^{\boldsymbol{p}}\right)^T \boldsymbol{\sigma}_t^{\boldsymbol{K}}\right) dt + \left(\boldsymbol{\sigma}_t^{\boldsymbol{p}} + \boldsymbol{\sigma}_t^{\boldsymbol{K}}\right) d\boldsymbol{Z}_t$$

- $\vartheta_t = \frac{p_t}{q_t + p_t}$ fraction of wealth in form of money

Allocation

- Equilibrium is a map

Histories of shocks - - - - - - - - - - → prices q_t, p_t, λ_t, allocation

$\{Z_\tau, 0 \leq \tau \leq t\}$ → wealth distribution → α_t, χ_t & portfolio weights (x_t, x_t^a, x_t^b)

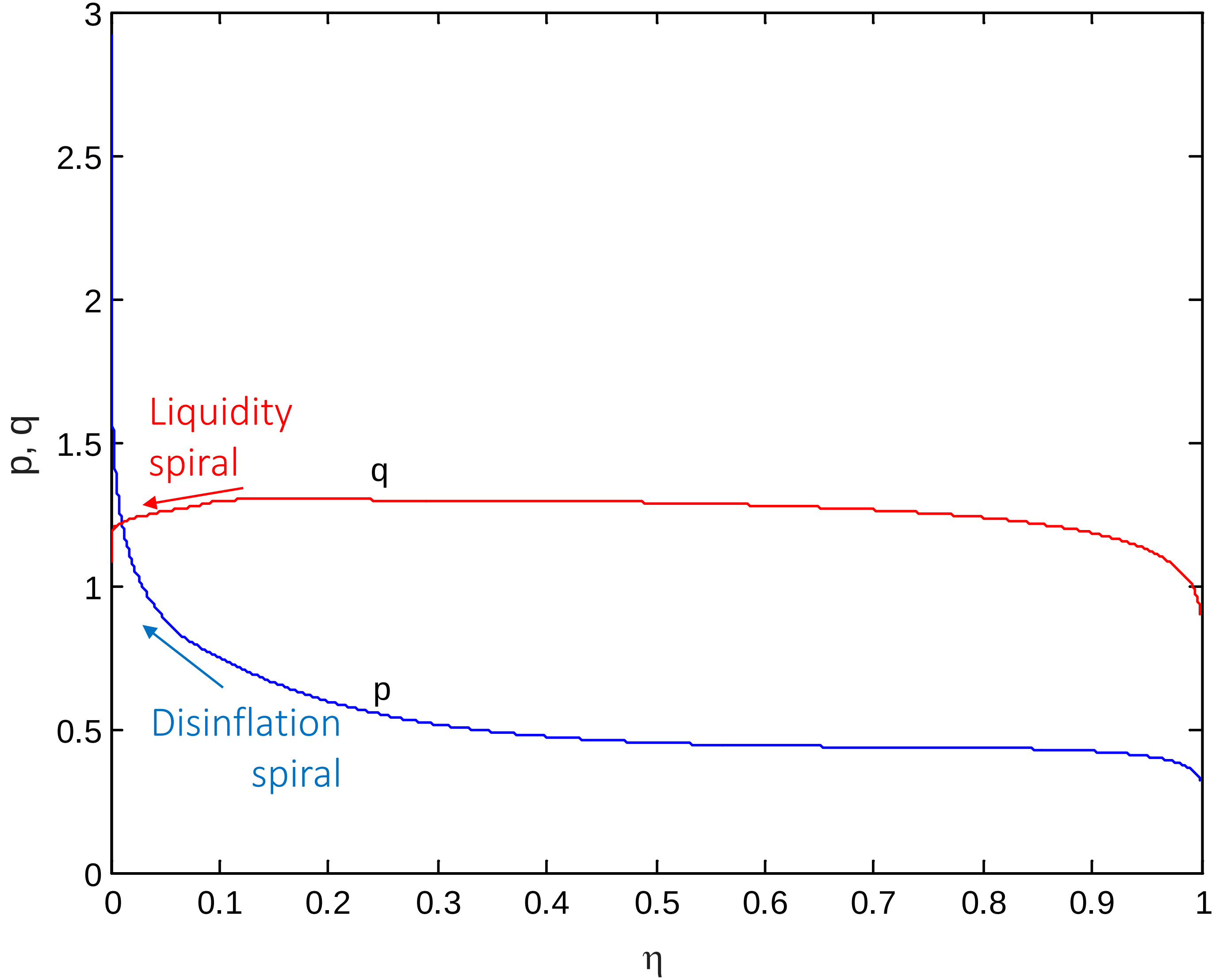
wealth distribution

$$\eta_t = \frac{N_t}{(p_t + q_t)K_t} \in (0,1)$$

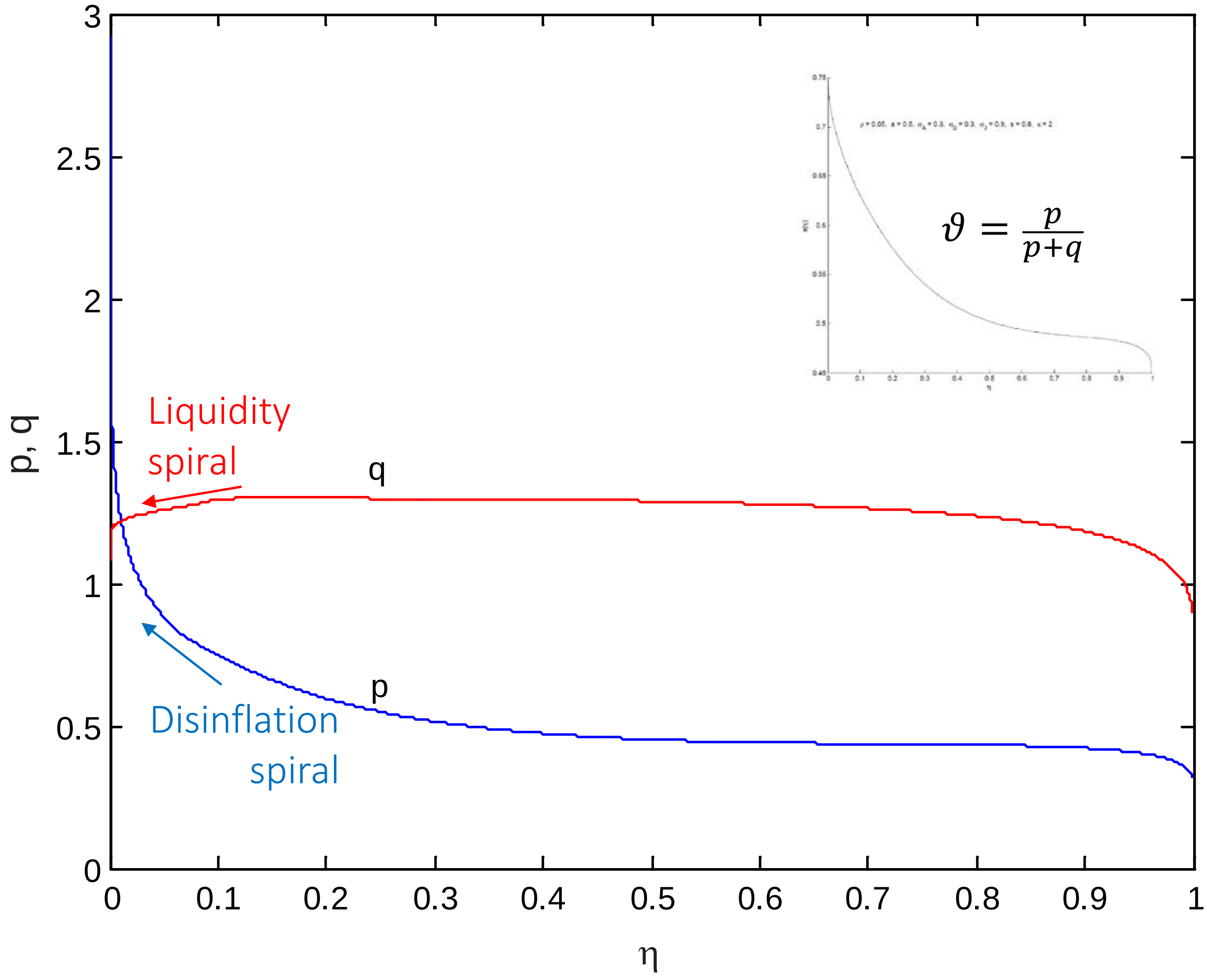
intermediaries' wealth share

- All agents maximize utility
  - Choose: portfolio, consumption, technology
- All markets clear
  - Consumption, capital, money, outside equity of b

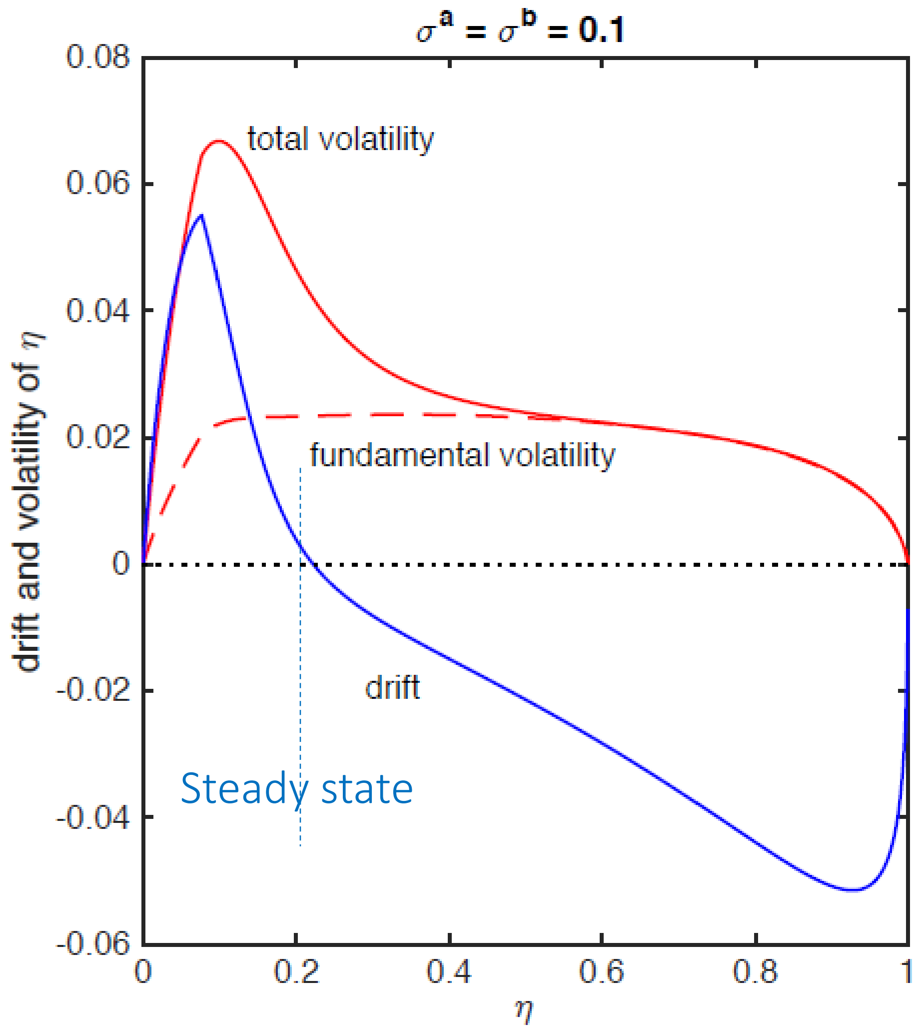
Numerical example: prices



Numerical example: prices



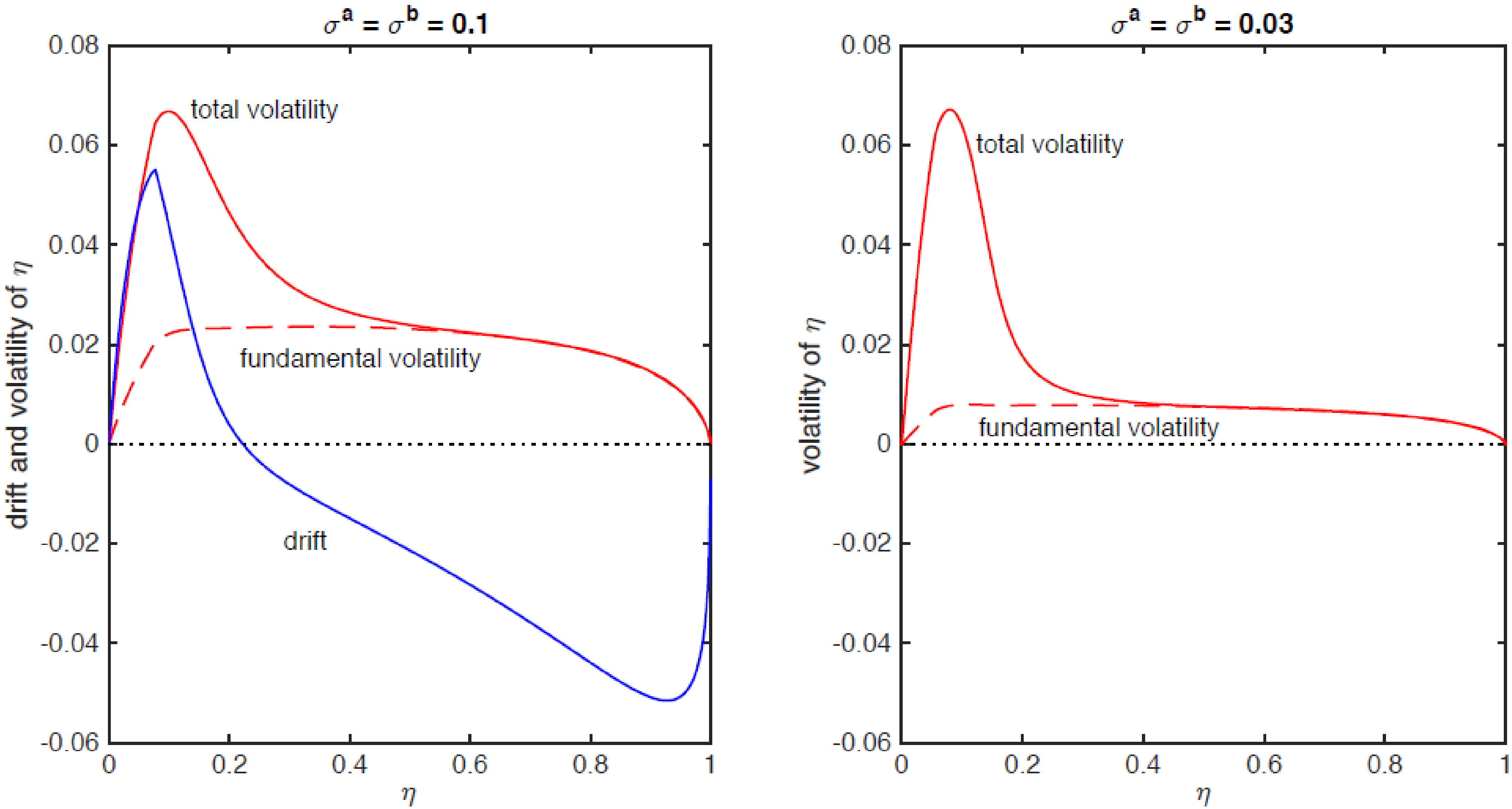
Numerical example: dynamics of η



$$\sigma_t^\eta = \frac{\overbrace{x_t\,(\sigma^b 1^b - \sigma_t^K)}^{\text{fundamental volatility}}}{1 - \underbrace{\underbrace{\left(\frac{x_t}{1-\vartheta_t} - 1\right)}_{\text{leverage}} \underbrace{\frac{-\vartheta'(\eta_t)}{\vartheta/\eta_t}}_{\text{elasticity}}}_{\text{amplification}}}$$

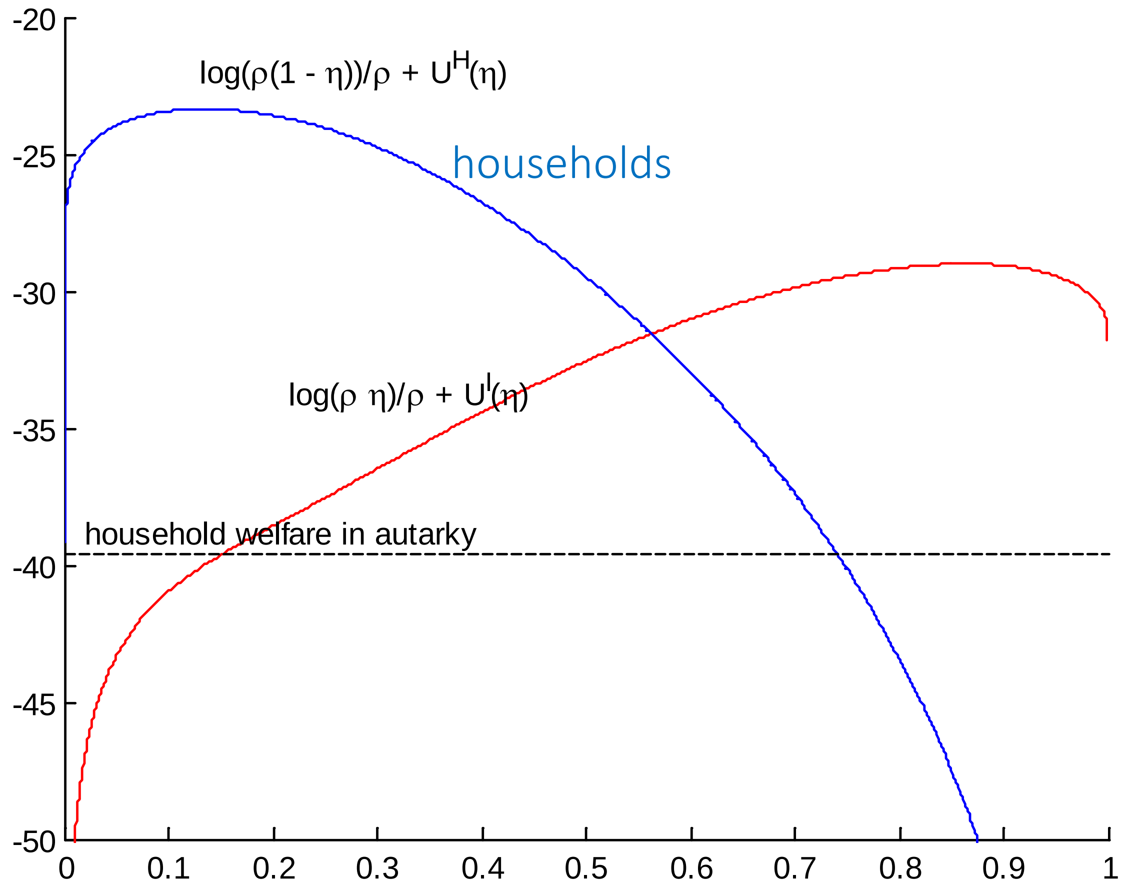
Brunnermeier & Sannikov

Numerical example: dynamics of η



Welfare analysis

- Challenge: Heterogeneous agents with idiosyncratic risks
- Inefficiencies in
  - Production
  - Investment
  - Risk sharing



Brunnermeier & Sannikov

Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - "On the optimal inflation rate" (inflation target)

- Model with intermediaries
  - Fixed outside money supply - Amplification/endogenous risk
    - Liquidity spiral asset side of intermediaries' balance sheet
    - Disinflationary spiral liability side

  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
    - The "Reversal Interest Rate: The Effective Lower Bound"

Monetary Policy: Ex-post perspective

- Money view — Friedman-Schwartz
  - Restore money supply
    - Replace missing inside money with outside money
  - Aim: Reduce deflationary spiral
    - … but banks extent less credit & diversify less idiosyncratic risk away
    - … as households have to hold more idiosyncratic risk, money demand rises
    - Undershoots inflation target

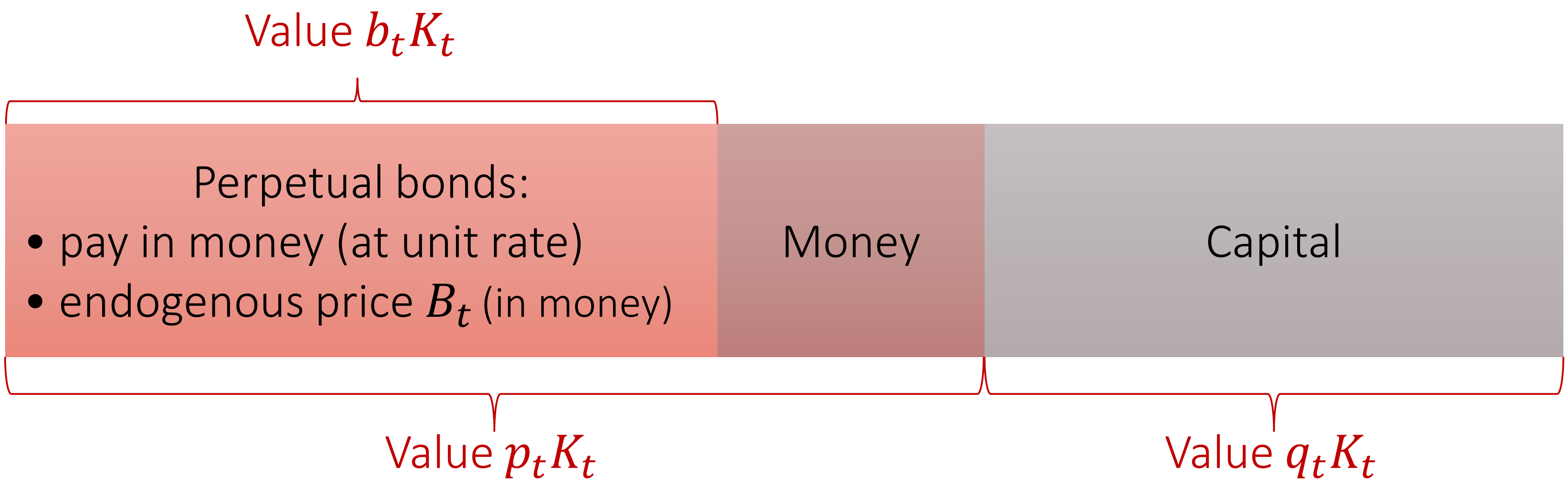
- Credit view — Tobin
  - Restore credit
  - Aim: Switch off deflationary spiral & liquidity spiral

Policy

- Monetary Policy
  - Introduce long-term bond
  - Central bank's actions change money supply/transfer risk
    - Interest rate cuts in downturns raise the value of long-term bonds
    - Change relative price between long-term bond and short-term money
    - Risk transfer (ex-post redistribution)
- Macro-prudential policy
  1. Leverage upper bounds
  2. Affect agents portfolio choice directly

Introducing Long-term Gov. Bond

- Introduce long-term (perpetual) bond
  - No default ... held by intermediaries in equilibrium

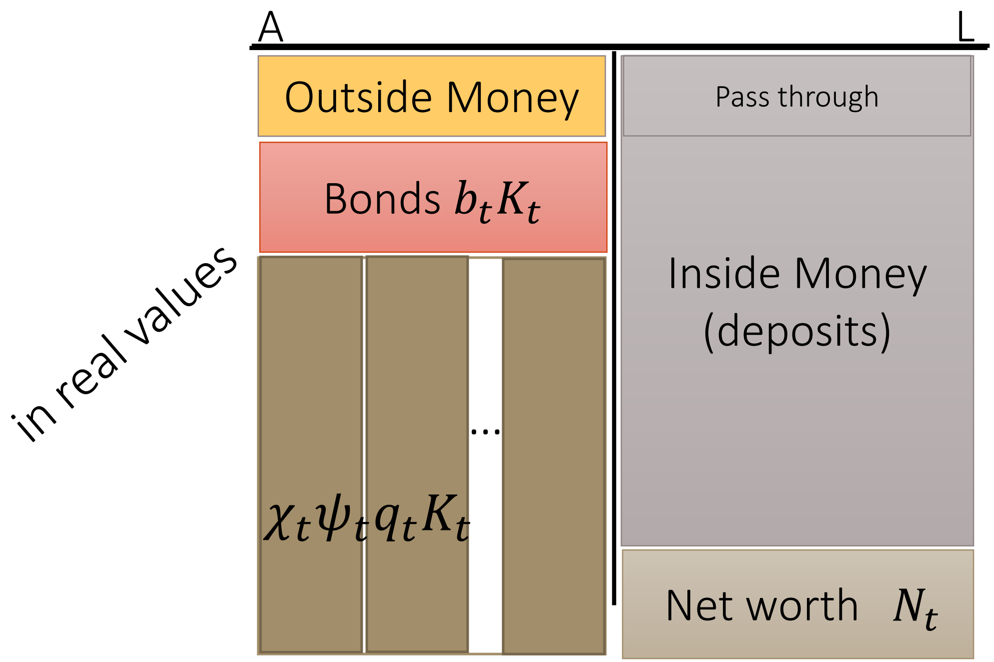


  - Value of long-term bond is endogenous

$$dB_t/B_t = \mu_t^B dt + \boldsymbol{\sigma}_t^{\boldsymbol{B}} d\boldsymbol{Z}_t$$

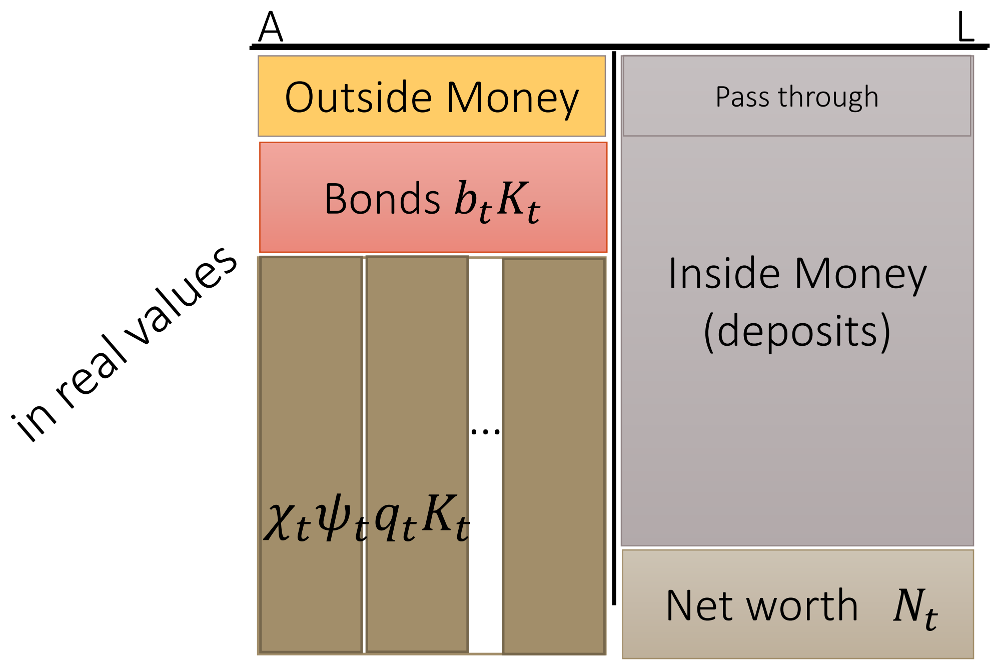
Brunnermeier & Sannikov

Redistributive MoPo: Ex-post perspective



- Adverse shock → value of risky claims drops
- Monetary policy
  - Interest rate cut ⇒ long-term bond price ↑
  - Asset purchase ⇒ asset price ↑
  - ⇒ "stealth recapitalization" - redistributive
  - ⇒ risk premia ↓
- Liquidity & Deflationary Spirals are mitigated

Redistributive MoPo: Ex-post perspective



- Adverse shock $\rightarrow$ value of risky claims drops
- Monetary policy
  - Interest rate cut $\Rightarrow$ long-term bond price ↑
  - Asset purchase $\Rightarrow$ asset price ↑
  - $\Rightarrow$ "stealth recapitalization" - redistributive
  - $\Rightarrow$ risk premia ↓
- Liquidity & Deflationary Spirals are mitigated

"stealth recapitalization"

LTRO, QE

Monetary policy and endogenous risk

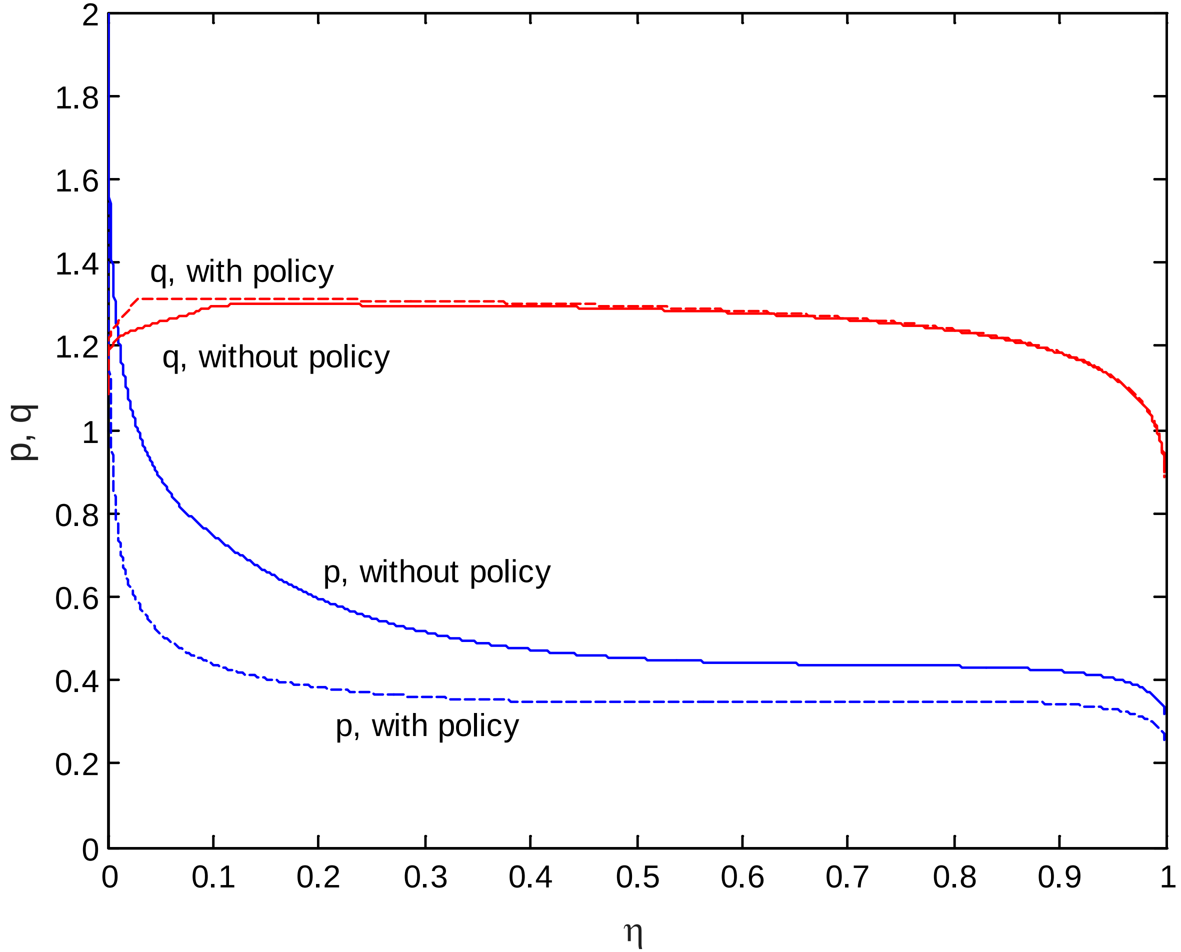
- Intermediaries' risk (borrow to scale up)

$$\sigma_t^\eta = \frac{\overbrace{x_t\,(1^b\sigma^b - \sigma_t^K)}^{\text{fundamental risk}}}{1 + \underbrace{\left(\frac{\chi_t\,\psi_t - \eta}{\eta_t}\right)\frac{\vartheta'(\eta_t)}{\vartheta/\eta_t}}_{\text{amplification}} - \underbrace{\left(x_t + \vartheta_t\frac{1-\eta_t}{\eta_t}\right)\frac{b_t}{p_t}\frac{B'(\eta_t)}{B(\eta_t)/\eta_t}}_{\text{mitigation}}}$$

- MoPo works through $\frac{B'(\eta_t)}{B(\eta_t)/\eta_t}$
  - with right monetary policy bond price $B(\eta)$ rises as η drops "stealth recapitalization"
  - Switch off liquidity and disinflationary spiral
- Example:
  Remove amplification s.t. $\sigma_t^\eta = x_t(1^b\sigma^b - \sigma_t^K)$

Numerical example with monetary policy

- Prices

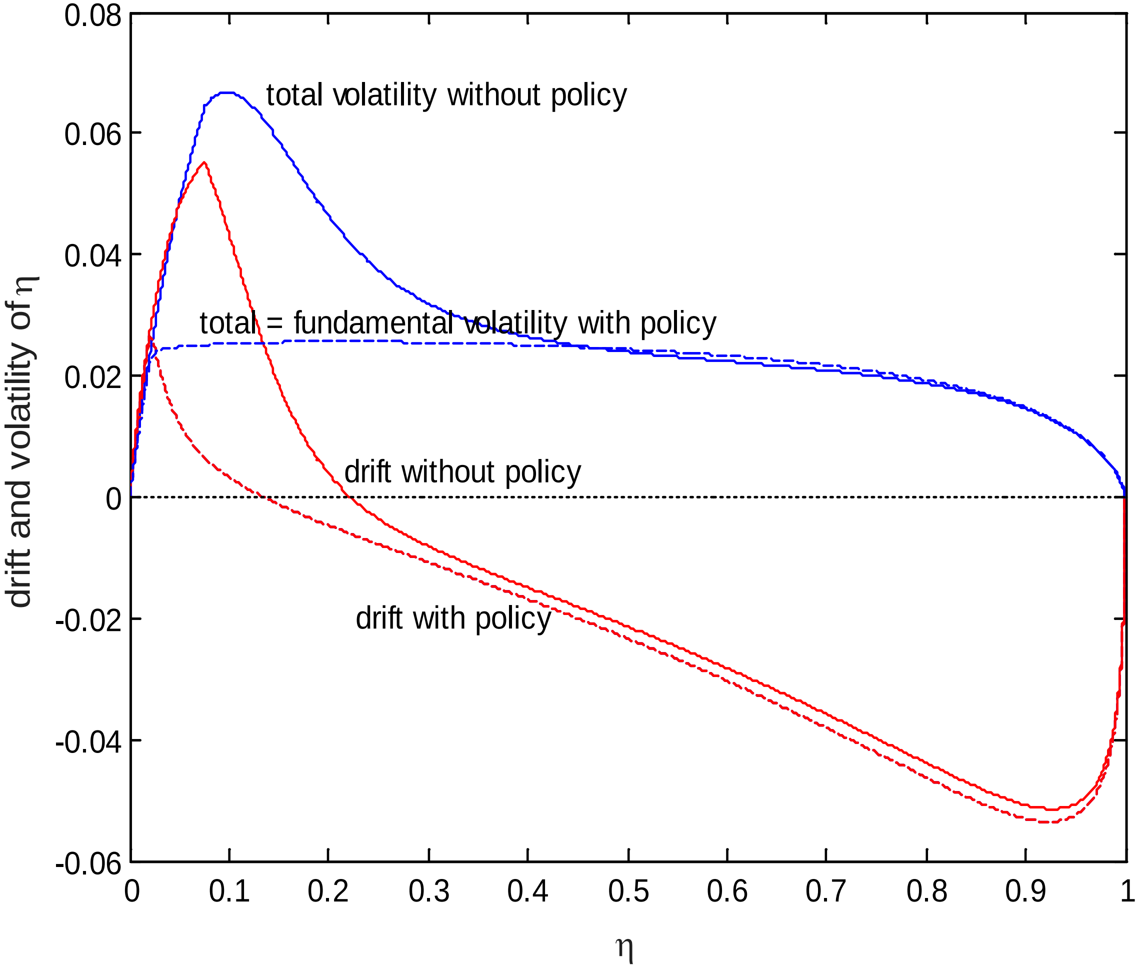


q is more stable

p less disinflation

Numerical example with monetary policy

- Drift and volatility of η



Observations

- As interest rate are cut in downturns, bonds held by intermediaries appreciate, this
  - protects intermediaries against shocks
  - increases the supply of asset that can be used as storage (weakens disinflation)
- Ex-post stabilization
  - Liquidity spiral
  - Disinflationary spiral
- Ex-ante
  - Higher leverage
  - (shift in steady state)

Monetary policy … in the limit

- full risk sharing of all aggregate risk

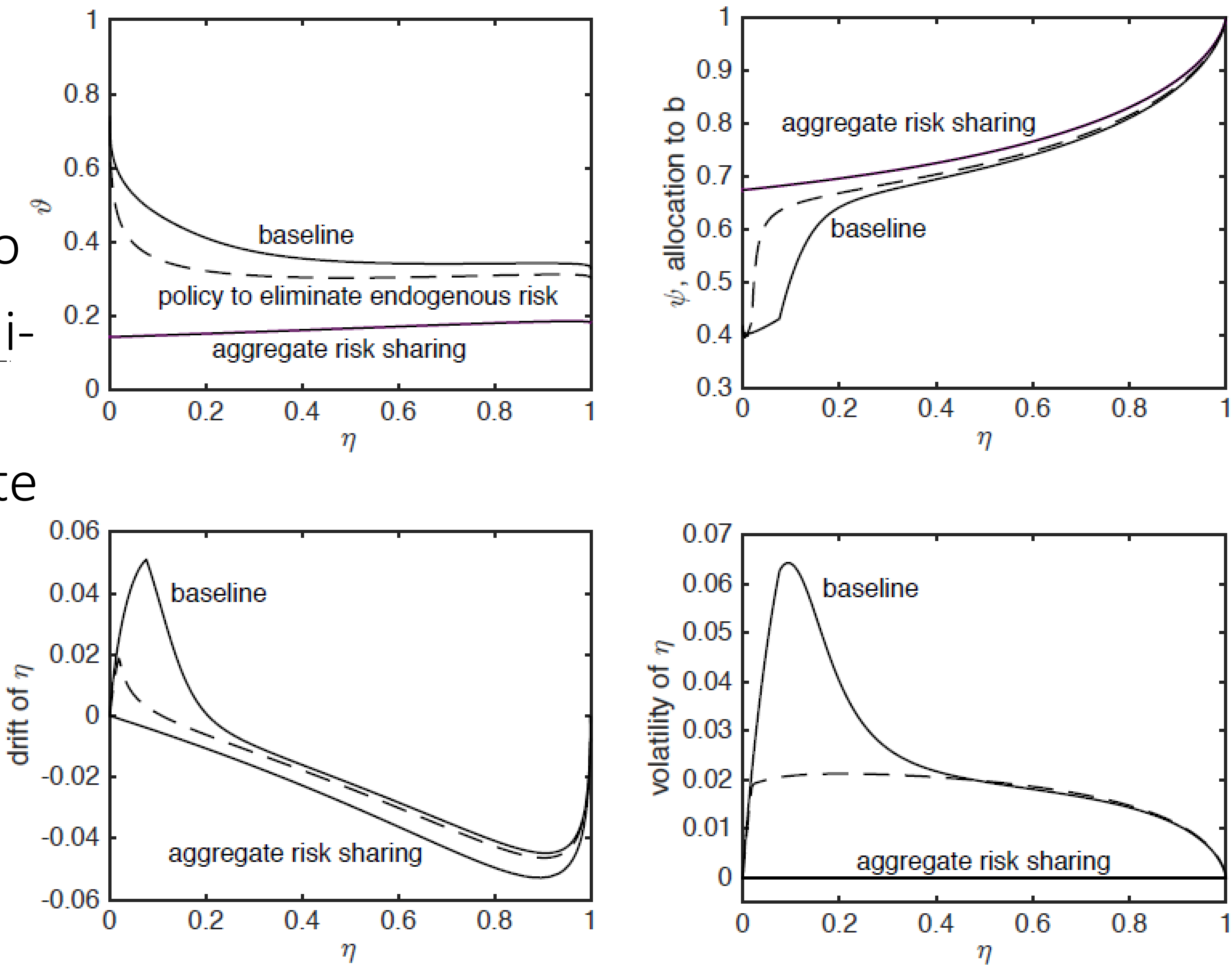
- $\sigma_t^\eta = \dfrac{x_t(1^b\sigma^b - \sigma_t^K)}{1 - \left(\frac{\chi\psi-\eta}{\eta}\right)\frac{-\vartheta'(\eta)}{\vartheta/\eta} + \left((1-\vartheta)\frac{\psi\chi-\eta}{\eta} + \vartheta\frac{1-\eta}{\eta}\right)\frac{b_t - B'(\eta)}{p_t B(\eta)/\eta}}$

  $\longrightarrow -\infty$

- η is deterministic and converges over time towards $\mathbf{0}$

Monetary policy: 3 versions

- No MoPo
- No Amplification
- Aggregate risk sharing



Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
<tr><th colspan="2">(New) Keynesian
Demand Management</th><th>I Theory of Money
Risk (premium) management</th></tr>
<tr><td colspan="2">Stimulate aggregate consumption
Substitution effect</td><td>Alleviate balance sheet constraints
Income/wealth effect</td></tr>
<tr><td>Woodford</td><td>Tobin (1982)</td><td>BruSan</td></tr>
<tr><td>Price stickiness
Perfect capital markets</td><td>Both</td><td>Financial Frictions
Incomplete markets</td></tr>
<tr><td>Representative Agent</td><td colspan="2">Heterogeneous Agents</td></tr>
<tr><td>Cut i
Reduces r due to price stickiness
Consumption c rises</td><td></td><td>- -</td></tr>
</table>

Brunnermeier & Sannikov

Monetary Policy Transmission Channel

- Consumption Boost approach to "Bottleneck approach"

(New) Keynesian Demand Management		I Theory of Money Risk (premium) management
Stimulate aggregate **consumption** Substitution effect		Alleviate **balance sheet constraints** Income/wealth effect
Woodford	Tobin (1982)	BruSan
Price stickiness Perfect capital markets	**Both**	**Financial Frictions** Incomplete markets
Representative Agent	Heterogeneous Agents	
Cut i Reduces r due to price stickiness Consumption c rises	Cut i Changes bond prices Redistributes from low MPC to high MPC consumers	- -

Brunnermeier & Sannikov

Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

(New) Keynesian Demand Management		I Theory of Money Risk (premium) management
Stimulate aggregate **consumption** Substitution effect		Alleviate **balance sheet constraints** Income/wealth effect
Woodford	Tobin (1982)	BruSan
Price stickiness Perfect capital markets	**Both**	**Financial Frictions** Incomplete markets
Representative Agent	**Heterogeneous Agents**	
Cut i Reduces r due to price stickiness Consumption c rises	**Cut i** Changes bond prices Redistributes from low MPC to high MPC consumers	**Cut i** Changes asset prices **Ex-post:** Redistributes to balance sheet impaired sector **QE** - **-**

Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
<tr><th colspan="2">(New) Keynesian
Demand Management</th><th>I Theory of Money
Risk (premium) management</th></tr>
<tr><td colspan="2">Stimulate aggregate consumption
Substitution effect</td><td>Alleviate balance sheet constraints
Income/wealth effect</td></tr>
<tr><td>Woodford</td><td>Tobin (1982)</td><td>BruSan</td></tr>
<tr><td>Price stickiness
Perfect capital markets</td><td>Both</td><td>Financial Frictions
Incomplete markets</td></tr>
<tr><td>Representative Agent</td><td colspan="2">Heterogeneous Agents</td></tr>
<tr><td>Cut i
Reduces r due to price stickiness
Consumption c rises</td><td>Cut i
Changes bond prices
Redistributes from low MPC to high MPC consumers</td><td>Cut i
Changes asset prices
Ex-post: Redistributes to balance sheet impaired sector

QE
- US: QE1 & QE3: MBS
- Japan 1990: corporate bonds</td></tr>
</table>

Overview

- No monetary economics
  - Fixed outside money supply
  - Amplification/endogenous risk through
    - Liquidity spiral — asset side of intermediaries' balance sheet
    - Disinflationary spiral — liability side
- Monetary policy
  - Wealth shifts by affecting relative price between
    - Long-term bond
    - Short-term money
  - Risk transfers – reduce endogenous aggregate risk
- Macroprudential policy
  - Directly affect portfolio positions

MacroPru

- MacroPru complements MoPo
  - Not substitutes
- Good MacroPru enables more aggressive MoPo
  - More redistribution ex-post
  - More risk-transfers/insurance ex-ante
  - Lower q
    - reduces cost to repurchase capital after shock
    - Lowers importance of idiosyncratic shocks

MacroPru policy

- Regulator can control
  - Portfolio choice ψs, xs

cannot control

- investment decision $\iota(q)$
- consumption decision c

of intermediaries and households

MacroPru policy

- Regulator can control
  - Portfolio choice ψs, xs

of intermediaries and households

  - De-facto controls q and p within some range
  - De-factor controls wealth share η
    - Force agents to hold certain assets and generate capital gains

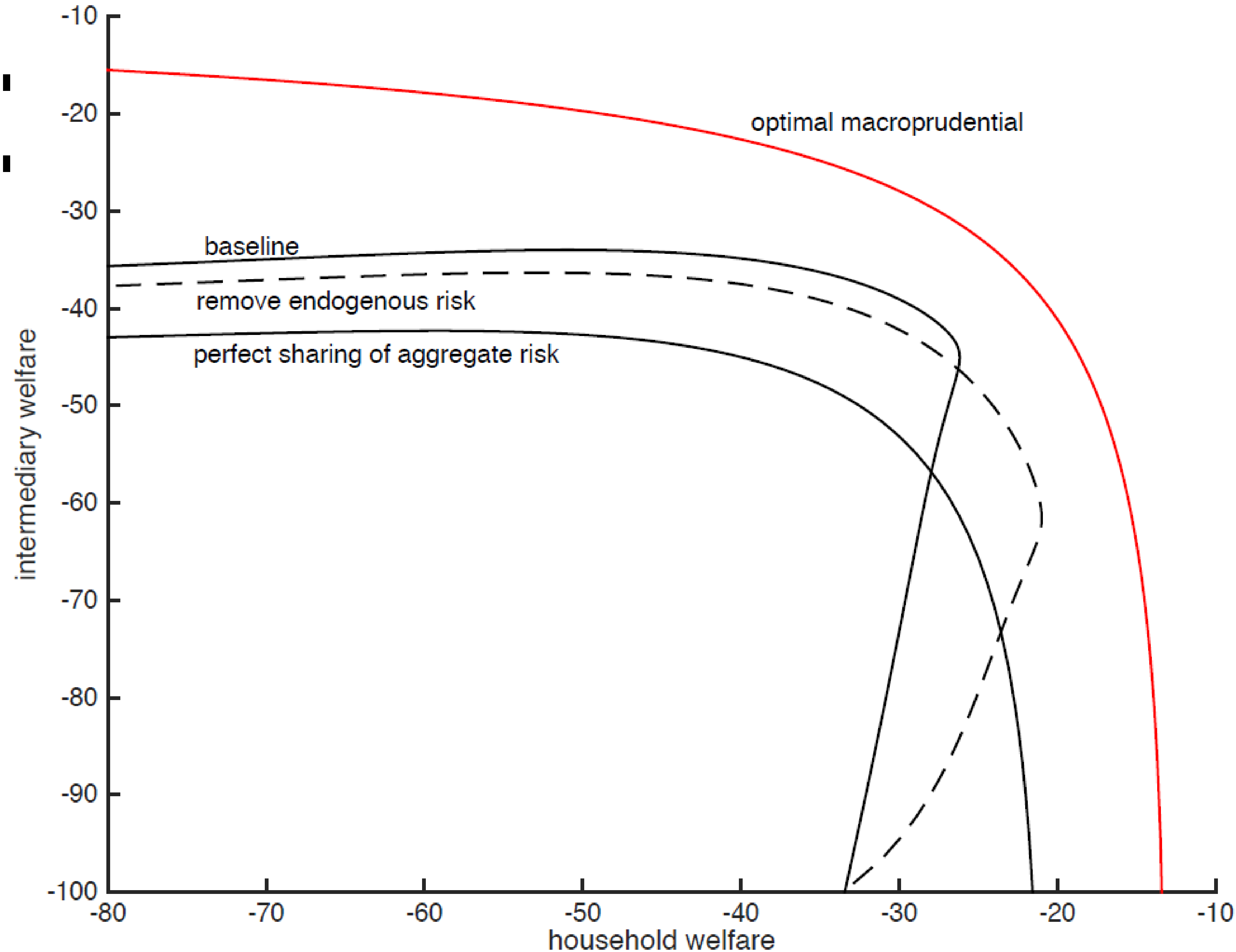
cannot control

- investment decision $\iota(q)$
- consumption decision c

distorts

- In sum,
  regulator maximizes sum of agents value function
  - Choosing ψ^b, q, η

MacroPru policy: Welfare frontier



Brunnermeier & Sannikov

Conclusion

- Unified macro "Money and Banking" model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral

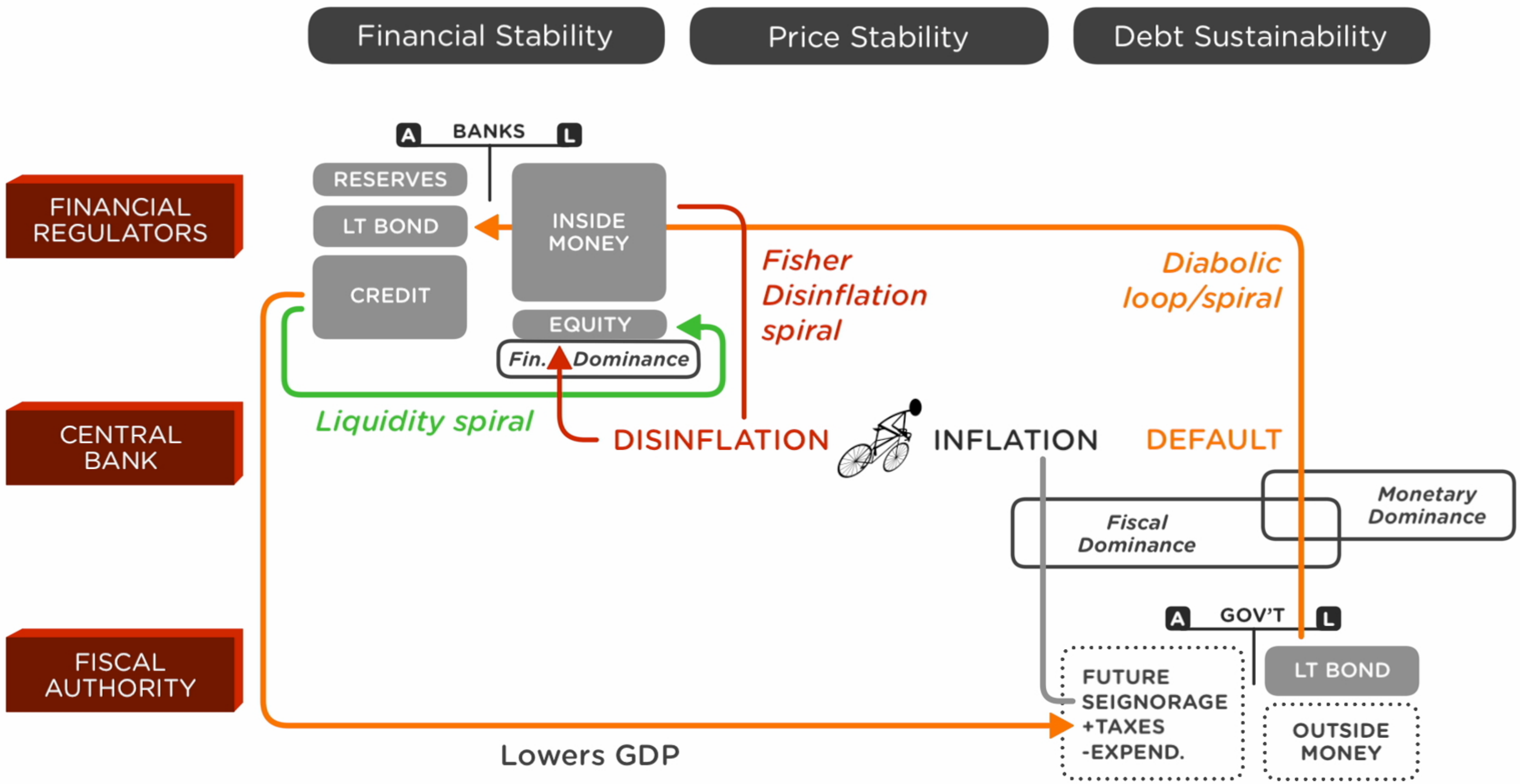
- Exogenous risk &
  - Sector specific
  - idiosyncratic
- Endogenous risk
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable

"Paradox of Prudence"

- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking
- Macro-prudential policies
  - MacroPru + MoPo to achieve superior welfare optimum

Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes
YouTube channel: Markus.economicus



Brunnermeier & Sannikov