The I Theory of Money &
On the Optimal Inflation Rate
Markus Brunnermeier & Yuliy Sannikov
“Money and Banking” (in macro-finance)

- Money → store of value/safe asset
“Money and Banking” (in macro-finance)

- Money  ➔ store of value/safe asset
- Banking  ➔ “diversifier”
  holds risky assets, issues inside money
“Money and Banking” (in macro-finance)

- Money → store of value/safe asset
- Banking → “diversifier”
  holds risky assets, issues inside money

Amplification/endogenous risk dynamics
- Value of capital declines due to fire-sales  **Liquidity spiral**
  - Flight to safety
- Value of money rises  **Disinflation spiral** a la Fisher
  - Demand for money rises – less idiosyncratic risk is diversified
  - Supply for inside money declines – less creation by intermediaries
    - Endogenous money multiplier = f(capitalization of critical sector)

- Paradox of Thrift  (in risk terms)
“Money and Banking” (in macro-finance)

- Money store of value/safe asset
- Banking “diversifier”
  - holds risky assets, issues inside money

Amplification/endogenous risk dynamics
- Value of capital declines due to fire-sales Liquidity spiral
  - Flight to safety
- Value of money rises Disinflation spiral a la Fisher
  - Demand for money rises – less idiosyncratic risk is diversified
  - Supply for inside money declines – less creation by intermediaries
    - Endogenous money multiplier = f(capitalization of critical sector)

Paradox of Prudence
- Paradox of Thrift (in risk terms)
“Money and Banking” (in macro-finance)

- Money store of value/safe asset
- Banking “diversifier” holds risky assets, issues inside money

Amplification/endogenous risk dynamics
- Value of capital declines due to fire-sales Liquidity spiral
  - Flight to safety
- Value of money rises Disinflation spiral a la Fisher
  - Demand for money rises – less idiosyncratic risk is diversified
  - Supply for inside money declines – less creation by intermediaries
    - Endogenous money multiplier = f(capitalization of critical sector)

- Paradox of Prudence
- Paradox of Thrift (in risk terms)

- Monetary Policy (redistributive)
Some literature

- Roles of money
  - Unit of account
  - Medium of exchange (Clower, Lagos & Wright)
  - Store of value (Samuelson, Bewley, Aiyagari, Scheinkman & Weiss, Kiyotaki & Moore)

- Models without inside money imply inflation in downturns
  - Less money needed to perform fewer transactions

- “Money view” (Friedman & Schwartz)
  - Downturns → Bank liabilities decrease

- “Credit view”
  - Downturns → equity capital → bank cuts assets/credit
  - BGG, Kiyotaki & Moore, He & Krishnamurthy, BruSan2014, Drechsler, Jeanne & Korinek, Savov & Schnabl

- Financial Stability
  - Diamond & Rajan 2010, Curdia & Woodford 2010, Stein 2012
### Monetary Policy Transmission Channel

- **Consumption Boost approach to “Bottleneck approach”**

<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th>I Theory of Money Risk (premium) management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate <strong>consumption</strong> Substitution effect</td>
<td>Alleviate balance sheet constraints Income/wealth effect</td>
</tr>
<tr>
<td>Woodford</td>
<td>Tobin (1982)</td>
</tr>
<tr>
<td>Price stickiness Perfect capital markets</td>
<td>Both</td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Heterogeneous Agents</td>
</tr>
<tr>
<td>Cut $i$ Reduces $r$ due to price stickiness Consumption $c$ rises</td>
<td>-</td>
</tr>
</tbody>
</table>
Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th>Theory of Money Risk (premium) management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate consumption</td>
<td>Alleviate balance sheet constraints</td>
</tr>
<tr>
<td>Substitution effect</td>
<td>Income/wealth effect</td>
</tr>
<tr>
<td>Woodford</td>
<td>Tobin (1982)</td>
</tr>
<tr>
<td></td>
<td>BruSan</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>Both</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td>Financial Frictions</td>
</tr>
<tr>
<td></td>
<td>Incomplete markets</td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Heterogeneous Agents</td>
</tr>
<tr>
<td>Cut $i$</td>
<td>Cut $i$</td>
</tr>
<tr>
<td>Reduces $r$ due to price</td>
<td>Changes bond prices</td>
</tr>
<tr>
<td>stickiness</td>
<td>Redistributions from</td>
</tr>
<tr>
<td>Consumption $c$ rises</td>
<td>low MPC to high MPC consumers</td>
</tr>
<tr>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>
## Monetary Policy Transmission Channel

### Consumption Boost approach to “Bottleneck approach”

<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th>I Theory of Money Risk (premium) management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate \textbf{consumption} Substitution effect</td>
<td>Alleviate balance sheet constraints Income/wealth effect</td>
</tr>
<tr>
<td>Woodford</td>
<td>Tobin (1982)</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>Both</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td></td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Heterogeneous Agents</td>
</tr>
<tr>
<td>Cut $i$ Reduces $r$ due to price stickiness Consumption $c$ rises</td>
<td>Cut $i$ Changes bond prices Redistributes from low MPC to high MPC consumers</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Woodford**
- **Tobin (1982)**
- **BruSan**

- **Price stickiness**
- **Perfect capital markets**
- **Both**

- **Financial Frictions**
- **Incomplete markets**

- **Representative Agent**
- **Heterogeneous Agents**
- **Cut $i$**
- **Reduces $r$ due to price stickiness**
- **Consumption $c$ rises**
- **Cut $i$**
- **Changes bond prices**
- **Redistributes from low MPC to high MPC consumers**
- **Cut $i$**
- **Changes asset prices**
- **Ex-post: Redistributes**
- **QE**
Literature

- Without intermediaries: Money as store of value = bubble

<table>
<thead>
<tr>
<th>Friction</th>
<th>OLG</th>
<th>Incomplete Markets + idiosyncratic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>deterministic</td>
<td>endowment risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>borrowing constraint</td>
</tr>
<tr>
<td></td>
<td></td>
<td>investment risk</td>
</tr>
</tbody>
</table>

- Only money
  - Samuelson
  - Bewley

- With capital
  - Diamond
  - Aiyagari
  - Angeletos

Risk tied up with individual capital
## Literature

- Without intermediaries: Money as store of value = bubble

<table>
<thead>
<tr>
<th>Friction</th>
<th>OLG</th>
<th>Incomplete Markets + idiosyncratic risk</th>
</tr>
</thead>
</table>
| Risk     | deterministic | endowment risk  
|          |            | investment risk  
|          |            | borrowing constraint                                   |

<table>
<thead>
<tr>
<th>Only money</th>
<th>Samuelson</th>
<th>Bewley</th>
</tr>
</thead>
</table>
| With capital | Diamond | Aiyagari  
|              |            | Angeletos $q = 1$          |

depends on price of capital $q$
## Literature

- Without intermediaries: Money as store of value = bubble

<table>
<thead>
<tr>
<th>Friction</th>
<th>OLG</th>
<th>Incomplete Markets + idiosyncratic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>deterministic</td>
<td>endowment risk borrowing constraint</td>
</tr>
<tr>
<td></td>
<td></td>
<td>investment risk</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Only money</th>
<th>Samuelson</th>
<th>Bewley</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Basic “I Theory”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>With capital</th>
<th>Diamond</th>
<th>Aiyagari</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>cash flow shock</td>
</tr>
</tbody>
</table>

\[
f'(k^*) = r^*, \]

Dynamic inefficiency

\[
r < r^*, K > K^* \]

Inefficiency

\[
r < r^*, K > K^* \]

Pecuniary externality

\[
r > r^*, K < K^* \]

(money) bubbles if \( r < g \)

Abel et al. vs. Geerolf

\[
r^m = g \]
Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - “On the optimal inflation rate” (inflation target)

- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
  - The “Reversal Interest Rate: The Effective Lower Bound”
Model without intermediaries

- Technologies $\alpha$

- Each household can only operate one firm
  - Physical capital
    \[
    \frac{dk_t}{k_t} = (\Phi(t_t) - \delta)dt + \sigma^a dZ^a_t + \tilde{\sigma} d\tilde{Z}^a_t
    \]
  - Output
    \[
    y_t = A k_t
    \]
- Demand for money
  - Sector idiosyncratic risk
  - Demand for money
Adding outside money

- $q_t K_t$ value of physical capital
- $p_t K_t$ value of outside money

- Each household can only operate one firm
  - Physical capital
    \[
    \frac{dk_t}{k_t} = (\Phi(t_t) - \delta)dt + \sigma^a dZ_t^a + \tilde{\sigma} d\tilde{Z}_t^a
    \]
  - Output
    \[
    y_t = A k_t
    \]

- Demand for money

- Technologies $\alpha$

Outside Money

Technologies $\alpha$

Each household can only operate one firm

- Physical capital
  \[
  \frac{dk_t}{k_t} = (\Phi(t_t) - \delta)dt + \sigma^a dZ_t^a + \tilde{\sigma} d\tilde{Z}_t^a
  \]

- Output
  \[
  y_t = A k_t
  \]
Solving

1. Postulate
   • Price processes \( \frac{dp_t}{p_t} = \mu_t^p \, dt + \sigma^p \, dZ_t, \quad \frac{dq_t}{q_t} = \cdots \)
   • Portfolio processes \( dx_t^a / x_t^a \)

2. Derive return processes
   • \( dr^{Ka} = \cdots \)
   • \( dr^M = \cdots \)

   \[ dt - (\mu^M - \mu^{Mi}) dt \]

   money supply growth rate that is NOT distributed via interest payment
   Set \( \mu^{Mi} = 0 \)

3. Optimality conditions & Market clearing conditions

4. Solve “undetermined coefficients” \( (\mu^x(s_t), \sigma^x(s_t)) \)
   • Solving ODE with boundary conditions
   • Solve for constants \( p, q \)
Solving

1. Postulate
   • Price processes \( \frac{d p_t}{p_t} = \mu_t \sigma_d Z_t \), \( q_t = \ldots \)
   • Portfolio processes \( x^a_t \)

2. Derive return processes
   • \( dr^{Ka} = (\Phi(\ell) - \delta)dt + \sigma^a dZ^a_t + \frac{A-t}{q}dt + \tilde{\sigma} d\tilde{Z}_t \)
   • \( dr^M = (\Phi(\ell) - \delta)dt + \sigma^a dZ^a_t - (\mu^M - \mu^{Mi}) dt \)
     money supply growth rate that is NOT distributed via interest payment
     Set \( \mu^{Mi} = 0 \)

3. Optimality conditions & Market clearing conditions

4. Solve “undetermined coefficients” \( (\mu^x(s_t), \sigma^x(s_t)) \)
   • Solving ODE with boundary conditions
   • Solve for constants \( p, q \)
Aside: Alternative Shocks

- $q_t K_t$ value of physical capital
- $p_t K_t$ value of outside money

Alternative shocks:
\[
\frac{dk_t}{k_t} = (\Phi(t) - \delta)dt
\]
but
- Real cash flow shocks
  \[
  \tilde{\sigma} k_t d\tilde{Z}_t^a
  \]
- Nominal cash flow shocks
  \[
  p_t \tilde{\sigma} k_t d\tilde{Z}_t^a
  \]

Each household can only operate one firm
- Physical capital shocks
  \[
  \frac{dk_t}{k_t} = (\Phi(t) - \delta)dt + \sigma^a dz_t^a + \tilde{\sigma} d\tilde{Z}_t^a
  \]
- Output
  \[
  y_t = Ak_t
  \]
- Demand for money
Optimality (\(=\)) for \(E\left[\int_0^\infty e^{-\rho t} \log c_t \, dt\right]\)

- Investment rate, \(\ell\)
- Portfolio choice, \(x^a\)
- Consumption, \(c_t\)
Optimality (≡)

- **Investment rate, $\iota$**
  - Tobin’s q: $\Phi'(\iota) = \frac{1}{q}$ (static problem)
    - For $\Phi(\iota) = \frac{1}{\kappa} \log(\kappa \iota + 1)$ $\Rightarrow \kappa \iota = q - 1$

- **Portfolio choice, $x^a$**

- **Consumption, $c_t$**
Investment rate, $i$
• Tobin’s q: $\Phi'(i) = \frac{1}{q}$ (static problem)
  - For $\Phi(i) = \frac{1}{\kappa} \log(\kappa i + 1) \Rightarrow \kappa = q - 1$

Portfolio choice, $x^a$
• $E[dr^{Ka} - dr^M]/dt = Cov[dr^{Ka} - dr^M, \frac{dn_t}{n_t}] = x^a(\sigma)^2$
  \[
x^a = \frac{E[dr^{Ka} - dr^M]/dt}{(\sigma)^2} = \frac{dr^M + x^a(dr^{Ka} - dr^M)}{(A - i)/q + \mu^M (\sigma)^2}
\]
• Dividend yield on capital must be $\rho$

Consumption, $c_t$
Optimality (=)

- **Investment rate, \( \ell \)**
  - Tobin’s q: \( \Phi'(\ell) = \frac{1}{q} \) (static problem)
    - For \( \Phi(\ell) = \frac{1}{\kappa} \log(\kappa \ell + 1) \Rightarrow \kappa = q - 1 \)

- **Portfolio choice, \( x^a \)**
  - \( E[dr^Ka - dr^M]/dt = Cov[dr^Ka - dr^M, \frac{dn_t}{n_t}] = x^a(\bar{\sigma})^2 \)
  - \( x^a = E[dr^Ka - dr^M]/dt = \frac{dr^M + x^a(dr^K - dr^M)}{(A-\ell)/q + \mu^M} \)
    - Dividend yield on capital must be \( \rho \)

- **Consumption, \( c_t \)**
  - Demand \( \rho N_t = \rho(q + p)K_t \)
Optimality (=) & market clearing (=)

- **Investment rate, \( l \)**
  - Tobin’s q: \( \Phi'(l) = \frac{1}{q} \) (static problem)
    - For \( \Phi(l) = \frac{1}{k} \log(\kappa l + 1) \Rightarrow \kappa l = q - 1 \)

- **Portfolio choice, \( x^a \)**
  - \( E[dr^{Ka} - dr^M]/dt = \text{Cov}[dr^{Ka} - dr^M, \frac{dnt}{nt}] = x^a(\bar{\sigma})^2 \)
    \[
    x^a = \frac{E[dr^{Ka} - dr^M]/dt}{(\bar{\sigma})^2} = \frac{(A - \iota)/q + \mu^M}{(\bar{\sigma})^2} = \frac{q}{q + p} \]
  - Dividend yield on capital must be \( \rho \)

- **Consumption, \( c_t \)**
  - Demand \( \rho N_t = \rho(q + p)K_t \Rightarrow (A - \iota)K_t \) Supply
    \[
    q = \frac{q}{q + p} \frac{(A - \iota)/\rho}{x^a} \]

Capital market clearing

Output market clearing
### Equilibrium

<table>
<thead>
<tr>
<th>Moneyleess equilibrium</th>
<th>Money equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_0 = 0 )</td>
<td>( p = \frac{\bar{\sigma} - \sqrt{\rho}}{\sqrt{\rho}} q )</td>
</tr>
<tr>
<td>( q_0 = \frac{\kappa A + 1}{\kappa \rho + 1} )</td>
<td>( q = \frac{\kappa A + 1}{\kappa \sqrt{\rho} \bar{\sigma} + 1} )</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between \( q_0, q, p \) and \( \sqrt{\rho}, \bar{\sigma} \).](image)
# Welfare analysis

<table>
<thead>
<tr>
<th>Moneyless equilibrium</th>
<th>Money equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_0 = 0 )</td>
<td>( p = \frac{\bar{\sigma} - \sqrt{\rho}}{\sqrt{\rho}} q )</td>
</tr>
<tr>
<td>( q_0 = \frac{\kappa A + 1}{\kappa \rho + 1} )</td>
<td>( q = \frac{\kappa A + 1}{\kappa \sqrt{\rho \bar{\sigma}} + 1} )</td>
</tr>
<tr>
<td>( g_0 )</td>
<td>( g )</td>
</tr>
<tr>
<td>welfare_0</td>
<td>welfare</td>
</tr>
</tbody>
</table>
Roadmap

- Model **without intermediaries**
  - Fixed (outside) money supply
  - Optimal **money growth rate**
    - “On the optimal inflation rate” (inflation target)

- Model **with intermediaries**
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
  - The “Reversal Interest Rate: The Effective Lower Bound”
Steady state MoPo – no intermediaries

- **Shock structure:** real cash flow shock
  - See paper “On the Optimal Inflation Rate” (AER P&P 2016)

- **Policy variable:** Money growth rate $\mu$

- **Portfolio choice:**
  $$x^{k^*} = \frac{q(A-t^*)}{\sigma^2} + \frac{q^2 \mu}{\sigma^2}$$

- **Capital markets clearing:**
  $$\frac{1}{p+q} = \frac{A-t^*}{\sigma^2} + \frac{q \mu}{\sigma^2}$$
Equilibrium

- Collecting the three equations:

\[ q = 1 + \kappa \iota^* \]
\[ \rho(p + q) = A - \iota^* \]
\[ \frac{\sigma^2}{q + p} = A - \iota^* + q\mu \]

- Equilibrium solved in terms of \( \hat{\mu} := x^\kappa \mu \) (monotone transformation)

\[ p = \frac{\sigma(1 + \kappa\rho)}{\sqrt{\rho + \hat{\mu}}} - (1 + \kappa A) \]
\[ q = 1 + \kappa A - \frac{\kappa\rho\sigma}{\sqrt{\rho + \hat{\mu}}} \]
\[ \iota^* = A - \rho\frac{\sigma}{\sqrt{\rho + \hat{\mu}}} \]

Closed form!
Welfare

- Plug in FOC in value function
- Plug in equilibrium
- All households start symmetrically

- Expected Utility of an individual household

\[ V = V_0 + \frac{1}{\kappa} \log \left( 1 + \kappa A - \frac{\kappa \rho \sigma}{\sqrt{\rho + \mu}} \right) - \delta + \rho - \frac{1}{2} (\rho + \hat{\mu}) + \frac{\log \left( \frac{\sigma}{\sqrt{\rho + \mu}} \right)}{\rho}. \]
Optimal inflation rate

- Money growth $\mu$ affects (steady state) inflation in two ways
  \[ \pi = \mu^M - \left( \Phi\left( i^*(\mu^M) \right) - \delta \right) g \]

- Proposition:
  - If $\frac{\sigma}{\sqrt{\rho}} > \frac{2(A\kappa + 1)}{1+2\kappa \rho}$, welfare maximizing money growth rate $\mu^* > 0$.
    - Market outcome is not even constrained Pareto efficient
    - Economic growth rate, $g > r^m$, is also higher
  - Growth maximizing $\mu^{g*} \geq \mu^M^*$, s.t. $p^{g*} = 0$, Tobin (1965)
    \[ i^* = A - \rho \frac{\sigma}{\sqrt{\rho} + \hat{\mu}} \text{ increasing in } \hat{\mu} \]

- Corollary: No super-neutrality of money
  - Nominal money growth rate affects real economy
    - No price/wage rigidity, no monopolistic competition
Proposition: (comparative static)
- $\mu^{M*}$ does not depend on depreciation rate $\delta$, but inflation does
- $\mu^{M*}$ is strictly increasing in idiosyncratic risk $\sigma$
  "Emerging markets should have higher inflation target"
Conclusion: our 3 initial questions

- What should the (long-run) optimal inflation rate be?
  - Competitive market outcome is constrained Pareto inefficient.
  - Inflation is Pigouvian & internalizes pecuniary externality!
    - HH take real interest rate as given, but
    - Portfolio choice affects economic growth and real interest rate

- What role do financial frictions play?
  - incomplete markets ⇒ no superneutrality of money
    - No price/wage rigidity needed

- Emerging markets, with less developed financial markets, should have higher inflation rate/target
  - Higher idiosyncratic risk ⇒ higher pecuniary externality
Main results

- **HH portfolio choice**
  - Physical capital: with idiosyncratic risk + dividend
  - Money: without idiosyncratic risk + no dividend (bubble)
    - Tilted inefficiently towards money

- Money supply growth $\Rightarrow$ inflation $\Rightarrow$ “tax on money”
- $\Rightarrow$ lowers real interest rate $\Rightarrow$ tilts portfolio choice
- $\Rightarrow$ boosts physical investment $\Rightarrow$ higher economic growth
- $\Rightarrow$ raises real interest rate (partially undoes inflation tax)

- **Pecuniary externality:**
  - Individual households do not take this GE effect into account.
  - Planner who can print money and distribute seignorage can improve growth + Pareto welfare.

- Derive optimal money growth rate/inflation rate
Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - “On the optimal inflation rate” (inflation target)

- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy
Outline of two sector model

- Technologies $b$
  - Households have to
    - Specialize in one subsector for one period
    - Demand for money

- Technologies $a$
  - Demand for money

\[
\frac{dk_t}{k_t} = \cdots dt + \sigma^b dZ_t^b + \tilde{\sigma} d\tilde{Z}_t^b
\]

\[
\frac{dk_t}{k_t} = \cdots dt + \sigma^a dZ_t^a + \tilde{\sigma} d\tilde{Z}_t^a
\]
Add outside money

- Technologies $b$

- Technologies $a$

Households have to
  - Specialize in one subsector for one period
  - Demand for money

Switch technology
Add intermediaries

- Technologies $b$
  - Risk can be partially sold off to intermediaries

- Technologies $a$
  - Risk is not contractable (Plagued with moral hazard problems)
Add intermediaries

- Technologies \(b\)

- Intermediaries
  - Can hold outside equity & diversify within sector \(b\)
  - Monitoring

Outside Money

Technologies \(a\)
Add intermediaries

- Technologies $b$
- Technologies $a$

- Intermediaries
  - Can hold outside equity & diversify within sector $b$
  - Monitoring

Outside Money

Net worth

Money

Inside equity

Risky Claim

Risky Claim

Risky Claim

Add intermediaries

$B_1$

$A_1$
Add intermediaries

- Technologies $b$
  - Intermediaries
    - Can hold outside equity & diversify within sector $b$
    - Monitoring
    - Create inside money
    - Maturity/liquidity transformation

- Technologies $a$
  - Outside Money
    - Inside Money (deposits)
    - Net worth
Shock impairs assets: 1st of 4 steps

- Technologies $b$

- Technologies $a$

```
A  
\[ \text{Technologies } b \]
  
\[ \text{Shock impairs assets: 1st of 4 steps} \]
  
\[ \text{Technologies } a \]
```

```
\[ \begin{align*}
\text{Inside Money (deposits)} & \quad \text{Pass through} \\
\text{Risky Claim} & \quad \text{Risky Claim} \\
\text{Net worth} & \quad \text{Losses}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Risky Claim} & \quad \text{Losses} \\
\text{Net worth} & \quad \text{Losses}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Inside equity} & \quad \text{Risky Claim} \\
\text{Money} & \quad \text{Money}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Inside equity} & \quad \text{Risky Claim} \\
\text{Money} & \quad \text{Money}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Inside equity} & \quad \text{Risky Claim} \\
\text{Money} & \quad \text{Money}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Inside equity} & \quad \text{Risky Claim} \\
\text{Money} & \quad \text{Money}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Inside equity} & \quad \text{Risky Claim} \\
\text{Money} & \quad \text{Money}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Inside equity} & \quad \text{Risky Claim} \\
\text{Money} & \quad \text{Money}
\end{align*} \]
```

```
\[ \begin{align*}
\text{Inside equity} & \quad \text{Risky Claim} \\
\text{Money} & \quad \text{Money}
\end{align*} \]
```
Shrink balance sheet: 2nd of 4 steps

- Technologies \( b \)
- Technologies \( a \)

```
Inside Money (deposits)  
Outside Money  
Risky Claim  
Risky Claim  
Risky Claim  
Risky Claim  

\( A_1 \)
```

```
Money  
Risky Claim  
Risky Claim  
Risky Claim  
Risky Claim  

\( B_1 \)
```

“Paradox of Prudence”
Liquidity spiral: asset price drop: 3rd of 4

- Technologies $b$

- Technologies $a$

Switch
Disinflationary spiral: 4th of 4 steps

- Technologies $b$

- Technologies $a$
... after an adverse shock

- Intermediaries are hit and shrink their balance sheets inducing
  - Asset side liquidity spiral financial stability
  - Liability side disinflation spiral price stability

- Response of intermediaries to adverse shock leads to endogenous risk
  - Amplification
  - Persistence

- Other sectors can also be undercapitalized
  - Japan 1980: corporate sector
  - US 2000s: household sector
Formal model: capital & output

Technologies

Physical capital $K_t$
- Capital share

Output goods

Aggregate good (CES)
- Consumed or invested
- Numeraire

Price of goods

- Model setup in paper is more general: $Y_t = A(\psi_t)K_t$

\[
\begin{align*}
\text{Technologies} & \\
\text{Physical capital } K_t & \\
& - \text{Capital share} \\
\text{Output goods} & \\
\text{Aggregate good (CES)} & - \text{Consumed or invested} \\
& - \text{Numeraire} \\
\text{Price of goods} & \\
\end{align*}
\]

\[
\begin{align*}
& b & a \\
& \psi_t & 1 - \psi_t \\
Y_t^b &= Ak_t^b & \text{Imperfect substitutes} & Y_t^a &= Ak_t^a \\
Y_t &= \left(\frac{1}{2}(Y_t^b)^{(s-1)/s} + \frac{1}{2}(Y_t^a)^{(s-1)/s}\right)^{s/(s-1)} \\
P_t^b &= \frac{1}{2}\left(\frac{Y_t}{Y_t^b}\right)^{1/s} & P_t^a &= \frac{1}{2}\left(\frac{Y_t}{Y_t^a}\right)^{1/s}
\end{align*}
\]
Formal model: risk

- When $k_t$ is employed in sector $a$ by agent $j$

  $$dk_t = (\Phi(\iota_t) - \delta)k_t dt + \sigma^a k_t dZ^a_t + \sigma^j k_t d\tilde{Z}^a_t$$

  - Investment rate (per unit of $k_t$)
  - $\Phi(\iota_t)$ is increasing and concave, e.g. $\log[(\kappa \iota_t + 1)/\kappa]$ 
  - All $dZ$ are independent of each other

- Intermediaries can diversify within sector $b$
  - Face no idiosyncratic risk

- Households cannot become intermediaries or vice versa
Financing constraints

Technologies

Equity issuance
- Special case

Inside equity $\chi_t \geq \chi$
$\chi = 0\%$ (no inside equity)

Households’ risk

Intermediaries’ risk

$dZ^b$ & $d\tilde{Z}^b$
sector & idiosyncratic
$dZ^a$ & $d\tilde{Z}^a$
sector & idiosyncratic

$dZ^b$
can diversify
idiosyncratic risk
**Capital/risk shares**

- Technologies $b$
  - Money
  - Inside equity
  - Risky Claim
  - Risky Claim

- Technologies $a$
  - Inside Money (deposits)
  - Net worth $N_t$
  - Fraction $\alpha_t$ of HH

\[
\begin{align*}
1 - \chi_t & \quad \chi_t \\
\psi_t q_t K_t & \\
\chi_t \psi_t q_t K_t & \\
(1 - \psi_t) q_t K_t & \\
\end{align*}
\]
Formal model: preferences

- All agents have logarithmic utility with discount rate $\rho$

$$ E \left[ \int_0^\infty e^{-\rho t} \log c_t \, dt \right] $$

- Implies
  - Consumption = $\rho \times$ net worth
  - Equilibrium Sharpe ratio $\propto$ Covariance with net worth
### Solution steps

1. **Postulate endogenous processes**
   - \[ \frac{dq_t}{q_t} = \mu_t^q \, dt + \sigma_t^{q,a} \, dZ_t^a + \sigma_t^{q,b} \, dZ_t^b \]
     - Returns from holding capital
   - \[ \frac{dp_t}{p_t} = \mu_t^p \, dt + \sigma_t^{p,a} \, dZ_t^a + \sigma_t^{p,b} \, dZ_t^b \]

2. **Equilibrium conditions**
   - **Agents’ optimization**
     - Internal investment (new capital formation)
     - Optimal portfolio choice: Sharpe ratio \( \propto \) Cov. with net worth
     - Optimal consumption: \( \rho \) * networth
   - **Market clearing conditions**

3. **Law of motion of state variable**
   - wealth (share) distribution \( \eta_t \)

4. **Express in ODEs of state variable**
Asset returns on technology $b$

- Physical capital: (in technology $b$) also earns dividend yield

  \[
  \begin{align*}
  dq_t/q_t &= \mu^q_t \, dt + (\sigma^q_t)^T \, dZ_t, \\
  dk_t/k_t &= (\Phi(\nu_t) - \delta) \, dt + \sigma^b \, dZ^b_t + \tilde{\sigma}^j \, dZ^{b,j}_t
  \end{align*}
  \]

  Vector $dZ^a_t, dZ^b_t$
Asset returns on technology $b$

- Physical capital: (in technology $b$) also earns dividend yield
  
  \[ \frac{dq_t}{q_t} = \mu_t^q dt + (\sigma_t^q)^T dZ_t, \]
  
  \[ \frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta) dt + \sigma^b dZ_t^b + \tilde{\sigma}^j dZ_t^{b,j}, \]
  
  \[ dr_t^b = \frac{AP_t^b - \iota_t}{q_t} dt + (\Phi(\iota_t) - \delta + \mu_t^q + (\sigma_t^q)^T \sigma^i 1^b) dt + (\sigma_t^q + \sigma^a 1^b)^T dZ_t + \tilde{\sigma}^j dZ_t^{b,j} \]

  Dividend yield  Expected capital gains
Asset returns on technology $b$

- Physical **capital**: (in technology $b$) also earns dividend yield

  - If $dq_t/q_t = \mu^q_t \, dt + (\sigma^q_t)^T \, dZ_t$, 
  - $dk_t/k_t = (\Phi(\iota_t) - \delta) \, dt + \sigma^b \, dZ^b_t + \tilde{\sigma}^j \, dZ^b_{t,j}$

  - $dr^b_t = \frac{AP^b_t - \iota_t}{q_t} \, dt + (\Phi(\iota_t) - \delta + \mu^q_t + (\sigma^q_t)^T \sigma 1^b) \, dt + (\sigma^q_t + \sigma 1^b)^T \, dZ_t + \tilde{\sigma}^j \, dZ^b_{t,j}$
  - $dr^a_t = \ldots$ (analogous)

  $\chi_t dr^\chi_t + (1 - \chi_t)dr^l_t = dr^b_t$

- Return on **outside equity** held by intermediaries

  - $dr^l_t = dr^b_t - \lambda_t \, dt$

  risk premium

- Return on **inside equity** (fraction $\chi_t$) held by $b$-HH

  - $dr^\chi_t = dr^b_t + \frac{1 - \chi_t}{\chi_t} \lambda_t \, dt$
Asset returns on money

- **Money**: fixed supply in baseline model, total value $p_tK_t$
  - Return = capital gains (no dividend/interest in baseline model)
  - If $dp_t/p_t = \mu^p_t dt + \sigma^p_t dZ_t$,
  - $dK_t/K_t = (\Phi(\iota_t) - \delta)dt + (1 - \psi_t)\sigma^a_t dZ^a_t + \psi_t \sigma^b_t dZ^b_t$

$$dr^M_t = \left(\Phi(\iota_t) - \delta + \mu^p_t + (\sigma^p_t)^T \sigma^K_t\right) dt + (\sigma^p_t + \sigma^K_t) dZ_t$$

- $\vartheta_t = \frac{p_t}{q_t + p_t}$ fraction of wealth in form of money
Allocation

- Equilibrium is a map

Histories of shocks $\{Z_\tau, 0 \leq \tau \leq t\}$

$\rightarrow$ prices $q_t, p_t, \lambda_t$, allocation

$\alpha_t, \chi_t$ & portfolio weights $(x_t, x_t^a, x_t^b)$

wealth distribution

\[ \eta_t = \frac{N_t}{(p_t+q_t)K_t} \in (0,1) \]

intermediaries’ wealth share

- All agents maximize utility
  - Choose: portfolio, consumption, technology

- All markets clear
  - Consumption, capital, money, outside equity of $b$
Numerical example: prices

Disinflation spiral

Liquidity spiral
Numerical example: prices

Disinflation spiral

Liquidity spiral

\[ \vartheta = \frac{p}{p+q} \]
Numerical example: dynamics of $\eta$

$$\sigma^\eta_t = \frac{x_t \left( \sigma^b 1^b - \sigma^K_t \right)}{1 - \left( \frac{x_t}{1 + \vartheta_t} \right) \frac{-\vartheta'(\eta_t)}{\vartheta/\eta_t}}$$

fundamental volatility

leverage

elasticity

amplification
Numerical example: dynamics of $\eta$

\[ \sigma^a = \sigma^b = 0.1 \]

\[ \sigma^a = \sigma^b = 0.03 \]
Welfare analysis

- Challenge: Heterogeneous agents with idiosyncratic risks
- Inefficiencies in
  - Production
  - Investment
  - Risk sharing
Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - “On the optimal inflation rate” (inflation target)

- Model with intermediaries
  - Fixed outside money supply - Amplification/endogenous risk
    - Liquidity spiral asset side of intermediaries’ balance sheet
    - Disinflationary spiral liability side

  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
  - The “Reversal Interest Rate: The Effective Lower Bound”
Monetary Policy: Ex-post perspective

**Money view**  
Friedman-Schwartz

- Restore money supply
  - Replace missing inside money with outside money
- Aim: Reduce deflationary spiral
  - ... but banks extent less credit & diversify less idiosyncratic risk away
  - ... as households have to hold more idiosyncratic risk, money demand rises
  - Undershoots inflation target

**Credit view**  
Tobin

- Restore credit
- Aim: Switch off deflationary spiral & liquidity spiral
Policy

- Monetary Policy
  - Introduce long-term bond
  - Central bank’s actions change money supply/transfer risk
    - Interest rate cuts in downturns raise the value of long-term bonds
    - Change relative price between long-term bond and short-term money
    - Risk transfer (ex-post redistribution)

- Macro-prudential policy
  1. Leverage upper bounds
  2. Affect agents portfolio choice directly
Introducing Long-term Gov. Bond

- Introduce long-term (perpetual) bond
  - No default ... held by intermediaries in equilibrium

\[ \frac{d B_t}{B_t} = \mu_t^B dt + \sigma_t^B dZ_t \]
Redistributive MoPo: Ex-post perspective

- Adverse shock → value of risky claims drops
- Monetary policy
  - Interest rate cut ⇒ long-term bond price
  - Asset purchase ⇒ asset price
  - ⇒ “stealth recapitalization” - redistributive
  - ⇒ risk premia
- Liquidity & Deflationary Spirals are mitigated
Redistributive MoPo: Ex-post perspective

- Adverse shock → value of risky claims drops
- Monetary policy
  - Interest rate cut ⇒ long-term bond price
  - Asset purchase ⇒ asset price
  - ⇒ “stealth recapitalization” - redistributive
  - ⇒ risk premia
- Liquidity & Deflationary Spirals are mitigated

\[ N_t \]
\[ b_t K_t \]
\[ \chi_t \psi_t q_t K_t \]
\[ \text{Net worth} \]
\[ \text{Inside Money (deposits)} \]
\[ \text{Outside Money} \]
\[ \text{Pass through} \]
Monetary policy and endogenous risk

- Intermediaries’ risk (borrow to scale up)

\[
\sigma_t^\eta = \frac{x_t \left(1^b \sigma^b - \sigma^K_t\right)}{1 + \left(\frac{\chi_t \psi_t - \eta}{\eta_t}\right) \frac{\vartheta'(\eta_t)}{\vartheta/\eta_t} - \left(x_t + \vartheta_t \frac{1 - \eta_t}{\eta_t}\right) \frac{b_t}{p_t} \frac{B'(\eta_t)}{B(\eta_t)/\eta_t}}
\]

- MoPo works through \(\frac{B'(\eta_t)}{B(\eta_t)/\eta_t}\)
  - with right monetary policy bond price \(B(\eta)\) rises as \(\eta\) drops “stealth recapitalization”
  - Switch off liquidity and disinflationary spiral

- Example:
  Remove amplification s.t. \(\sigma_t^\eta = x_t \left(1^b \sigma^b - \sigma^K_t\right)\)
Numerical example with monetary policy

- Prices

$q, \text{ without policy}$

$q, \text{ with policy}$

$p, \text{ without policy}$

$p, \text{ with policy}$

$q$ is more stable

$p$ less disinflation
Numerical example with monetary policy

- Drift and volatility of $\eta$

![Graph showing drift and volatility of $\eta$]
Observations

- As interest rates are cut in downturns, bonds held by intermediaries appreciate, this
  - protects intermediaries against shocks
  - increases the supply of asset that can be used as storage (weakens disinflation)

- Ex-post stabilization
  - Liquidity spiral
  - Disinflationary spiral

- Ex-ante
  - Higher leverage
  - (shift in steady state)
Monetary policy ... in the limit

- full risk sharing of all aggregate risk

\[ \sigma_t^\eta = \frac{x_t(1^b \sigma^b - \sigma^K)}{1 - \left(\frac{\chi \psi - \eta}{\eta}\right) - \varphi'(\eta) + \left(1 - \varphi\right)\psi \frac{\chi - \eta}{\eta} + \varphi \frac{1 - \eta}{\eta}} \frac{b_t - B'(\eta)}{p_t B(\eta)/\eta} \rightarrow -\infty \]

- \( \eta \) is deterministic and converges over time towards 0
Monetary policy: 3 versions

- No MoPo
- No Amplification
- Aggregate risk sharing
## Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th>I Theory of Money Risk (premium) management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate consumption</td>
<td>Alleviate balance sheet constraints</td>
</tr>
<tr>
<td>Substitution effect</td>
<td>Income/wealth effect</td>
</tr>
<tr>
<td>Woodford</td>
<td>Tobin (1982)</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>Both</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td>Financial Frictions</td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Incomplete markets</td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Heterogeneous Agents</td>
</tr>
<tr>
<td>Cut $i$</td>
<td></td>
</tr>
<tr>
<td>Reduces $r$ due to price stickiness</td>
<td></td>
</tr>
<tr>
<td>Consumption $c$ rises</td>
<td></td>
</tr>
</tbody>
</table>
Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th>I Theory of Money Risk (premium) management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate consumption</td>
<td>Alleviate balance sheet constraints</td>
</tr>
<tr>
<td>Substitution effect</td>
<td>Income/wealth effect</td>
</tr>
<tr>
<td>Woodford</td>
<td>Tobin (1982)</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>Both</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td>Financial Frictions</td>
</tr>
<tr>
<td></td>
<td>Incomplete markets</td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Heterogeneous Agents</td>
</tr>
<tr>
<td>Cut $i$</td>
<td>Cut $i$</td>
</tr>
<tr>
<td>Reduces $r$ due to price stickiness</td>
<td>Changes bond prices</td>
</tr>
<tr>
<td>Consumption $c$ rises</td>
<td>Redistributions from low MPC</td>
</tr>
<tr>
<td>consumers</td>
<td>high MPC consumers</td>
</tr>
</tbody>
</table>

Woodford Tobin (1982) BruSan

Perfect capital markets

Both

Financial Frictions

Incomplete markets
Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th>I Theory of Money Risk (premium) management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate consumption</td>
<td>Alleviate balance sheet constraints</td>
</tr>
<tr>
<td>Substitution effect</td>
<td>Income/wealth effect</td>
</tr>
<tr>
<td>Woodford</td>
<td>Tobin (1982)</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>Both</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td>Financial Frictions</td>
</tr>
<tr>
<td>Representaive Agent</td>
<td>Heterogeneous Agents</td>
</tr>
<tr>
<td>Cut ( i )</td>
<td>Cut ( i )</td>
</tr>
<tr>
<td>Reduces ( r ) due to price</td>
<td>Changes bond prices</td>
</tr>
<tr>
<td>stickiness</td>
<td>Redistributions from</td>
</tr>
<tr>
<td>Consumption ( c ) rises</td>
<td>low MPC to high MPC consumers</td>
</tr>
<tr>
<td></td>
<td>Ex-post: Redistributions</td>
</tr>
<tr>
<td></td>
<td>to balance sheet impaired sector</td>
</tr>
<tr>
<td></td>
<td>QE</td>
</tr>
<tr>
<td></td>
<td>QE</td>
</tr>
</tbody>
</table>
## Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th>I Theory of Money Risk (premium) management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate consumption</td>
<td>Alleviate balance sheet constraints</td>
</tr>
<tr>
<td>Substitution effect</td>
<td>Income/wealth effect</td>
</tr>
<tr>
<td>Woodford</td>
<td>Tobin (1982)</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>Both</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td>Financial Frictions</td>
</tr>
<tr>
<td></td>
<td>Incomplete markets</td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Heterogeneous Agents</td>
</tr>
<tr>
<td>Cut $i$</td>
<td>Cut $i$</td>
</tr>
<tr>
<td>Reduces $r$ due to price stickiness</td>
<td>Changes bond prices</td>
</tr>
<tr>
<td>Consumption $c$ rises</td>
<td>Redistributions from</td>
</tr>
<tr>
<td></td>
<td>low MPC to high MPC consumers</td>
</tr>
<tr>
<td>QE</td>
<td>Ex-post: Redistributions</td>
</tr>
<tr>
<td>US: QE1 &amp; QE3: MBS</td>
<td>to balance sheet impaired sector</td>
</tr>
<tr>
<td>Japan 1990: corporate bonds</td>
<td></td>
</tr>
</tbody>
</table>

Brummermeier & Sannikov
Overview

- **No monetary economics**
  - Fixed outside money supply
  - Amplification/endogenous risk through
    - Liquidity spiral asset side of intermediaries’ balance sheet
    - Disinflationary spiral liability side

- **Monetary policy**
  - Wealth shifts by affecting relative price between
    - Long-term bond
    - Short-term money
  - Risk transfers – reduce endogenous aggregate risk

- **Macroprudential policy**
  - Directly affect portfolio positions
MacroPru

- MacroPru complements MoPo
  - Not substitutes

- Good MacroPru enables more aggressive MoPo
  - More redistribution ex-post
  - More risk-transfers/insurance ex-ante
  - Lower $q$
    - reduces cost to repurchase capital after shock
    - Lowers importance of idiosyncratic shocks
MacroPru policy

- Regulator can control
  - Portfolio choice $\psi s, x s$
  - Investment decision $\iota(q)$
  - Consumption decision $c$

- cannot control
  - of intermediaries and households
MacroPru policy

- Regulator can control
  - Portfolio choice $\psi s, x s$
  - De-facto controls $q$ and $p$ within some range
  - De-factor controls wealth share $\eta$
    - Force agents to hold certain assets and generate capital gains

- cannot control
  - Investment decision $\iota(q)$
  - Consumption decision $c$

- In sum, regulator maximizes sum of agents value function
  - Choosing $\psi^b, q, \eta$
MacroPru policy: Welfare frontier

- Stabilize intermediaries net worth and earnings
- Control the value of money to allow HH to insure idiosyncratic risk (investment distortions still exist, otherwise can get 1st best)
Conclusion

- Unified macro “Money and Banking” model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral

- Exogenous risk &
  - Sector specific
  - Idiosyncratic

- Endogenous risk
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable

- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking

- Macro-prudential policies
  - MacroPru + MoPo to achieve superior welfare optimum
Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes
YouTube channel: Markus.economicus