The I Theory of Money
&
On the Optimal Inflation Rate
Markus Brunnermeier & Yuliy Sannikov
“Money and Banking” (in macro-finance)

- Money → store of value/safe asset
“Money and Banking” (in macro-finance)

- Money → store of value/safe asset
- Banking → “diversifier”
  holds risky assets, issues inside money

- Value of capital declines due to fire-sales
- Liquidity spiral
- Flight to safety
  - Value of money rises
  Disinflation spiral a la Fisher
- Demand for money rises
  - less idiosyncratic risk is diversified
- Supply for inside money declines
  - less creation by intermediaries
- Endogenous money multiplier = f(capitalization of critical sector)
- Paradox of Thrift (in risk terms)

- Monetary Policy (redistributive)
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**Amplification/endogenous risk dynamics**
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- Monetary Policy (redistributive)
Some literature

- Roles of money
  - Unit of account
  - Medium of exchange (Clower, Lagos & Wright)
  - Store of value (Samuelson, Bewley, Aiyagari, Scheinkman & Weiss, Kiyotaki & Moore)

- Models without inside money imply inflation in downturns
  - Less money needed to perform fewer transactions

- “Money view” (Friedman & Schwartz)
  - Downturns  ➔  Bank liabilities decrease

- “Credit view”
  - Downturns  ➔  equity capital  ➔  bank cuts assets/credit
  - BGG, Kiyotaki & Moore, He & Krishnamurthy, BruSan2014, Drechsler, Jeanne & Korinek, Savov & Schnabl

- Financial Stability
  - Diamond & Rajan 2010, Curdia & Woodford 2010, Stein 2012
### Monetary Policy Transmission Channel

- **Consumption Boost approach to “Bottleneck approach”**

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- Without intermediaries: Money as store of value = bubble

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Risk tied up with individual capital
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depends on price of capital $q$
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(money) bubbles if $r < g$

Abel et al. vs. Geerolf

$r^m = g$
Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - “On the optimal inflation rate” (inflation target)

- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
  - The “Reversal Interest Rate: The Effective Lower Bound”
Model without intermediaries

- Technologies $\alpha$

- Each household can only operate one firm
  - Physical capital
    \[
    \frac{dk_t}{kt} = (\Phi(t) - \delta)dt + \sigma^a dZ^a_t + \sigma d\tilde{Z}^a_t
    \]
  - Output
    \[
    y_t = Ak_t
    \]

- Demand for money
Adding outside money

- $q_t K_t$ value of physical capital
- $p_t K_t$ value of outside money

Each household can only operate one firm

- Physical capital
  \[
  \frac{dk_t}{k_t} = (\Phi(t_t) - \delta) dt + \sigma^a dZ^a_t + \tilde{\sigma} d\tilde{Z}^a_t
  \]
- Output
  \[
  y_t = A k_t
  \]

Demand for money

Technologies $a$

Outside Money

Money

Net worth

Sector idiosyncratic risk
1. Postulate
   - Price processes \( \frac{dp_t}{p_t} = \mu_t^p dt + \sigma^p dZ_t \), \( \frac{dq_t}{q_t} = \cdots \)
   - Portfolio processes \( \frac{dx_t^a}{x_t^a} \)

2. Derive return processes
   - \( dr^{Ka} = \cdots \)
   - \( dr^M = \cdots \)

3. Optimality conditions & Market clearing conditions

4. Solve “undetermined coefficients” \( (\mu^x(s_t), \sigma^x(s_t)) \)
   - Solving ODE with boundary conditions
   - Solve for constants \( p, q \)

money supply growth rate that is NOT distributed via interest payment
Set \( \mu^{Mi} = 0 \)
Solving

1. Postulate
   - Price processes
     \[ p_t \frac{d}{dt} p_t = \mu_t dt + \sigma_t dZ_t, \]
   - Portfolio processes
     \[ x_t^a \]

2. Derive return processes
   - \[ dr^K_a = (\Phi_i - \delta) dt + \frac{A_i}{q} dt + \tilde{\sigma} d\tilde{Z}_t \]
   - \[ dr^M = (\Phi_i - \delta) dt - (\mu^M - \mu^{Mi}) dt \]
     money supply growth rate that is NOT distributed via interest payment
     Set \( \mu^{Mi} = 0 \)

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Optimality (=) for $E \left[ \int_0^\infty e^{-\rho t} \log c_t \, dt \right]$

- Investment rate, $\iota$

- Portfolio choice, $x^a$

- Consumption, $c_t$
Optimality (=)

- **Investment rate, \( \iota \)**
  - Tobin’s q: \[ \Phi'(\iota) = \frac{1}{q} \] (static problem)
    - For \( \Phi(\iota) = \frac{1}{\kappa} \log(\kappa \iota + 1) \Rightarrow \kappa \iota = q - 1 \)

- **Portfolio choice, \( x^a \)**

- **Consumption, \( c_t \)**
Optimality (=)

- **Investment rate, \( \ell \)**
  - Tobin’s q: \( \Phi'(\ell) = \frac{1}{q} \) (static problem)
    - For \( \Phi(\ell) = \frac{1}{k} \log(\kappa \ell + 1) \Rightarrow \kappa = q - 1 \)

- **Portfolio choice, \( x^a \)**
  - \( E[dr^K - dr^M]/dt = \text{Cov}[dr^K - dr^M, \frac{dnt}{nt}] = x^a (\bar{\sigma})^2 \)
  
  \[
  x^a = E[dr^K - dr^M]/dt = \frac{(A-i)/q + \mu^M}{(\bar{\sigma})^2}
  \]
  - Dividend yield on capital must be \( \rho \)

- **Consumption, \( c_t \)**
Optimality (=)

- **Investment rate, \( \lambda \)**
  - Tobin's q: \( \Phi'(\lambda) = \frac{1}{q} \) (static problem)
    - For \( \Phi(\lambda) = \frac{1}{\kappa} \log(\kappa \lambda + 1) \Rightarrow \kappa \lambda = q - 1 \)

- **Portfolio choice, \( x^a \)**
  - \( E[dr^K - dr^M]/dt = Cov[dr^K - dr^M, \frac{dnt}{nt}] = x^a (\bar{\sigma})^2 \)
  - \( x^a = \frac{E[dr^K - dr^M]/dt}{(\bar{\sigma})^2} = \frac{dr^M + x^a(dr^K - dr^M)}{(A - \lambda)/q + \mu^M} \)
  - Dividend yield on capital must be \( \rho \)

- **Consumption, \( c_t \)**
  - Demand \( \rho N_t = \rho (q + \rho)K_t \)
Optimality (=) & market clearing (=)

- **Investment rate, \( \ell \)**
  - Tobin’s q: \( \Phi'(\ell) = \frac{1}{q} \) (static problem)
    - For \( \Phi(\ell) = \frac{1}{\kappa} \log(\kappa \ell + 1) \Rightarrow \kappa \ell = q - 1 \)

- **Portfolio choice, \( x^a \)**
  - \( E[dr^{Ka} - dr^M]/dt = Cov[dr^{Ka} - dr^M, \frac{dn_t}{n_t}] = x^a(\bar{\sigma})^2 \)
  - \( x^a = \frac{E[dr^{Ka} - dr^M]/dt}{(\bar{\sigma})^2} = \frac{(A-\ell)/q + \mu^M}{(\bar{\sigma})^2} = \frac{q}{q+p} \)
  - Dividend yield on capital must be \( \rho \)

- **Consumption, \( c_t \)**
  - Demand \( \rho N_t = \rho (q + p)K_t = (A - \ell)K_t \) Supply
    - \( q = \left( \frac{q}{q + p} \right) (A - \ell)/\rho = x^a \)
## Equilibrium

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<td>( q_0 = \frac{\kappa A + 1}{\kappa \rho + 1} )</td>
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![Graph showing the relationship between \( p \), \( q \), \( \rho \), and \( \bar{\sigma} \)]
## Welfare analysis

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Roadmap

- Model **without intermediaries**
  - Fixed (outside) money supply
  - Optimal **money growth rate**
    - “On the optimal inflation rate” (inflation target)

- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy

- Intermediaries with market power
  - The “Reversal Interest Rate: The Effective Lower Bound”
Equilibrium

- Collecting the three equations:

\[ q = 1 + \kappa \iota^* \]
\[ \rho(p + q) = A - \iota^* \]
\[ \frac{\sigma^2}{q + p} = A - \iota^* + q\mu \]

- Equilibrium solved in terms of \( \hat{\mu} := x^\kappa \mu \) (monotone transformation)

\[ p = \frac{\sigma(1 + \kappa \rho)}{\sqrt{\rho + \hat{\mu}}} - (1 + \kappa A) \]
\[ q = 1 + \kappa A - \frac{\kappa \rho \sigma}{\sqrt{\rho + \hat{\mu}}} \]
\[ \iota^* = A - \rho \frac{\sigma}{\sqrt{\rho + \hat{\mu}}} \]

Closed form!
Welfare

- Plug in FOC in value function
- Plug in equilibrium
- All households start symmetrically

- Expected Utility of an individual household

$$V = V_0 + \frac{1}{\kappa} \log \left(1 + \kappa A - \frac{\kappa \rho \sigma}{\sqrt{\rho + \mu}}\right) - \delta + \rho - \frac{1}{2}(\rho + \hat{\mu}) + \frac{\log \left(\frac{\sigma}{\sqrt{\rho + \mu}}\right)}{\rho}.$$
Optimal inflation rate

- Money growth $\mu$ affects (steady state) inflation in two ways:
  $\pi = \mu^M - \left( \Phi(i^*(\mu^M)) \right) - \delta \right)$

- Proposition:
  - If $\frac{\sigma}{\sqrt{\rho}} > \frac{2(A\kappa + 1)}{1 + 2\kappa \rho}$, welfare maximizing money growth rate $\mu^* > 0$.
    - Market outcome is not even constrained Pareto efficient
    - Economic growth rate, $g > r^m$, is also higher
  - Growth maximizing $\mu^g \geq \mu^M$, s.t. $p^g = 0$, Tobin (1965)
    $i^* = A - \rho \frac{\sigma}{\sqrt{\rho + \hat{\mu}}}$ increasing in $\hat{\mu}$

- Corollary: No super-neutrality of money
  - Nominal money growth rate affects real economy
    - No price/wage rigidity, no monopolistic competition
Proposition: (comparative static)

- $\mu^M$ does not depend on depreciation rate $\delta$, but inflation does.
- $\mu^M$ is strictly increasing in idiosyncratic risk $\sigma$

“Emerging markets should have higher inflation target”
Main results

- HH portfolio choice
  - Physical capital: w/ idiosyncratic risk + dividend
  - Money: w/o idiosyncratic risk + no dividend (bubble)
    - Tilted inefficiently towards money

- Money supply growth ⇒ inflation ⇒ “tax on money”
- ⇒ lowers real interest rate ⇒ tilts portfolio choice
- ⇒ boosts physical investment ⇒ higher economic growth
- ⇒ raises real interest rate (partially undoes inflation tax)

- Pecuniary externality:
  - individual households do not take this GE effect into account.
  - Planner who can print money and distribute seignorage can improve growth + Pareto welfare.

- Derive optimal money growth rate/inflation rate
Roadmap

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- Model with intermediaries
  - Fixed outside money supply
  - Monetary Policy
  - Macro-prudential policy
Outline of two sector model

- Technologies $b$
  - Households have to
    - Specialize in one subsector for one period
      \[
      \frac{dk_t}{k_t} = \ldots dt + \sigma^b dZ^b_t + \tilde{\sigma} d\tilde{Z}^b_t
      \]
    - Demand for money

- Technologies $a$
  - Sector specific + idiosyncratic risk
    \[
    \frac{dk_t}{k_t} = \ldots dt + \sigma^a dZ^a_t + \tilde{\sigma} d\tilde{Z}^a_t
    \]
Add outside money

- Technologies $b$

- Technologies $a$

Households have to
- Specialize in one subsector for one period
- Demand for money
Add intermediaries

- Technologies $b$

  - Risk can be partially sold off to intermediaries

- Technologies $a$

  - Risk is not contractable (Plagued with moral hazard problems)
Add intermediaries

- Technologies $b$
  - Intermediaries
    - Can hold outside equity & diversify within sector $b$
    - Monitoring

- Technologies $a$
Add intermediaries

- Technologies $b$
  - Intermediaries
    - Can hold outside equity & diversify within sector $b$
    - Monitoring

- Technologies $a$
  - Net worth
  - Money
  - Inside equity
  - Risky Claim
  - Outside Money

$B_1$
Add intermediaries

- Technologies $b$

- Intermediatearies
  - Can hold outside equity & diversify within sector $b$
  - Monitoring
  - Create inside money
  - Maturity/liquidity transformation

- Technologies $a$
Shock impairs assets: 1\textsuperscript{st} of 4 steps

- Technologies $b$
- Technologies $a$
Shrink balance sheet: 2nd of 4 steps

- Technologies $b$
- Technologies $a$

“Paradox of Prudence”
Liquidity spiral: asset price drop: $3^{rd}$ of 4

- Technologies $b$

- Technologies $a$

Switch
Disinflationary spiral: 4\textsuperscript{th} of 4 steps

- Technologies \( b \)
- Technologies \( a \)
... after an adverse shock

- Intermediaries are hit and shrink their balance sheets inducing
  - Asset side: liquidity spiral
  - Liability side: disinflation spiral

- Response of intermediaries to adverse shock leads to endogenous risk
  - Amplification
  - Persistence

- Other sectors can also be undercapitalized
  - Japan 1980: corporate sector
  - US 2000s: household sector
Formal model: capital & output

Technologies

Physical capital $K_t$
- Capital share

$Y_t = \psi_t K_t$

Output goods

Aggregate good (CES)
- Consumed or invested
- Numeraire

$Y_t = \frac{1}{2}(Y_t^b)^{(s-1)/s} + \frac{1}{2}(Y_t^a)^{(s-1)/s} \left(\frac{s}{s-1}\right)$

Price of goods

$b$

$Y_t^b = Ak_t^b$

$a$

$Y_t^a = Ak_t^a$

$P_t^b = \frac{1}{2} \left(\frac{Y_t}{Y_t^b}\right)^{1/s}$

$P_t^a = \frac{1}{2} \left(\frac{Y_t}{Y_t^a}\right)^{1/s}$

Model setup in paper is more general: $Y_t = A(\psi_t)K_t$
Formal model: risk

- When $k_t$ is employed in sector $a$ by agent $j$

$$
dk_t = (\Phi(\lambda_t) - \delta)k_t dt + \sigma^a_k k_t dZ_t^a + \sigma^j k_t d\tilde{Z}_t^a
$$

  - Investment rate (per unit of $k_t$)
  - $\Phi(\lambda_t)$ is increasing and concave, e.g. $\log[(\kappa \lambda_t + 1) / \kappa]$
  - All $dZ$ are independent of each other

- Intermediaries can diversify within sector $b$
  - Face no idiosyncratic risk

- Households cannot become intermediaries or vice versa
Financing constraints

Technologies

Equity issuance
- Special case

\[ \chi_t \geq \chi \]
\[ \chi = 0\% \text{ (no inside equity)} \]

Households’ risk

Intermediaries’ risk

\[ dZ^b \& d\tilde{Z}^b \]
sector & idiosyncratic

\[ dZ^b \]
can diversify
idiosyncratic risk

\[ dZ^a \& d\tilde{Z}^a \]
sector & idiosyncratic

- Inside equity only
Capital/risk shares

- Technologies $b$

  - Inside equity
  - Risky Claim
  - $\psi_t q_t K_t$
  - $1 - \chi_t$  $\chi_t$

- Technologies $a$

  - Fraction $\alpha_t$ of HH

  - Inside Money (deposits)
  - $\chi_t \psi_t q_t K_t$
  - Net worth $N_t$
  - $(1 - \psi_t) q_t K_t$
  - HH Net worth
Formal model: preferences

- All agents have logarithmic utility with discount rate $\rho$

$$E \left[ \int_0^\infty e^{-\rho t} \log c_t \, dt \right]$$

- Implies
  - Consumption $= \rho \times$ net worth
  - Equilibrium Sharpe ratio $\propto$ Covariance with net worth
Solution steps

1. Postulate endogenous processes
   • \[ \frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^{q,a} dZ_t^a + \sigma_t^{q,b} dZ_t^b \]
     - Returns from holding capital
   • \[ \frac{dp_t}{p_t} = \mu_t^p dt + \sigma_t^{p,a} dZ_t^a + \sigma_t^{p,b} dZ_t^b \]

2. Equilibrium conditions
   • Agents’ optimization
     ▪ Internal investment (new capital formation)
     ▪ Optimal portfolio choice Sharpe ratio \( \propto \) Cov. with net worth
     ▪ Optimal consumption \( \rho \) * networth
   • Market clearing conditions

3. Law of motion of state variable
   • wealth (share) distribution \( \eta_t \)

4. Express in ODEs of state variable
Asset returns on technology $b$

- Physical capital: (in technology $b$) also earns dividend yield
  
  - If $dq_t/q_t = \mu_t^q dt + (\sigma_t^q)^T dZ_t$,
  
  - $dk_t/k_t = (\Phi(\iota_t) - \delta)dt + \sigma^b dZ_t^b + \widetilde{\sigma}^j dZ_t^{b,j}$
Asset returns on technology $b$

- **Physical capital**: (in technology $b$) also earns dividend yield

  - $dq_t/q_t = \mu_t^q dt + (\sigma_t^q)^T dZ_t$,
  
  - $dk_t/k_t = (\Phi(\iota_t) - \delta) dt + \sigma^b dZ_t^b + \tilde{\sigma}^j dZ_t^{b,j}$

  - $dr_t^b = \frac{AP_t^b - \iota_t}{q_t} dt + (\Phi(\iota_t) - \delta + \mu_t^q + (\sigma_t^q)^T \iota^1 b) dt + (\sigma_t^q + \sigma^a 1^b)^T dZ_t + \tilde{\sigma}^j dZ_t^{b,j}$

  Dividend yield, Expected capital gains
Asset returns on technology $b$

- Physical capital: (in technology $b$) also earns dividend yield

  - $dq_t/q_t = \mu_t^q dt + (\sigma_t^q)^T dZ_t$
  - $dk_t/k_t = (\Phi(\iota_t) - \delta)dt + \sigma^b dZ^b_t + \tilde{\sigma}^j dZ^{b,j}_t$

  - $dr^b_t = \frac{AP_t^{b-\iota_t}}{q_t}dt + (\Phi(\iota_t) - \delta + \mu_t^q + (\sigma_t^q)^T \sigma^1)^T dZ_t + \tilde{\sigma}^j dZ^{b,j}_t$
  - $dr^a_t = \ldots$ (analogous)

- Return on outside equity held by intermediaries

  - $dr^I_t = dr^b_t - \lambda_t dt$

- Return on inside equity (fraction $\chi_t$) held by $b$-HH

  - $dr^\chi_t = dr^b_t + \frac{1-\chi_t}{\chi_t} \lambda_t dt$
Asset returns on money

- **Money**: fixed supply in baseline model, total value $p_tK_t$
  - Return = capital gains (no dividend/interest in baseline model)
  - If $dp_t/p_t = \mu^p_t \, dt + \sigma^p_t \, dZ_t$,
  - $dK_t/K_t = (\Phi(\iota_t) - \delta) \, dt + \left(1 - \psi_t\right) \sigma^a \, dZ^a_t + \psi_t \sigma^b \, dZ^b_t$
  - $\Phi(\iota_t)$
  - $\sigma^a, \sigma^b$
  - $(\sigma^K_T) dZ_t$
  - $dr^M_t = \left(\Phi(\iota_t) - \delta + \mu^p_t + (\sigma^p_t)^T \sigma^K_t\right) \, dt + (\sigma^p_t + \sigma^K_t) \, dZ_t$

- $\vartheta_t = \frac{p_t}{q_t + p_t}$ fraction of wealth in form of money
Equilibrium is a map

Histories of shocks $\{Z_\tau, 0 \leq \tau \leq t\}$

prices $q_t, p_t, \lambda_t$, allocation $\alpha_t, \chi_t$ & portfolio weights $(x_t, x^a_t, x^b_t)$

wealth distribution

$$\eta_t = \frac{N_t}{(p_t+q_t)K_t} \in (0,1)$$

intermediaries’ wealth share

• All agents maximize utility
  ▪ Choose: portfolio, consumption, technology

• All markets clear
  ▪ Consumption, capital, money, outside equity of $b$
Numerical example: prices

- **Disinflation spiral**
- **Liquidity spiral**
Numerical example: prices

Disinflation spiral

\[ \theta = \frac{p}{p+q} \]
Numerical example: dynamics of $\eta$

$$\sigma_t = \frac{x_t \left( \sigma^b 1^b - \sigma^K_t \right)}{1 - \left( \frac{x_t}{1-\vartheta_t} \right)^{1-\vartheta'(\eta_t)} / \eta_t}$$

fundamental volatility
leverage
elasticity
amplification

Steady state
Numerical example: dynamics of $\eta$
Welfare analysis

- **Challenge:** Heterogeneous agents with idiosyncratic risks
- **Inefficiencies in**
  - Production
  - Investment
  - Risk sharing

\[
\log(\rho \eta)/\rho + U^I(\eta) \\
\log(\rho(1 - \eta))/\rho + U^H(\eta)
\]

Household welfare in autarky
Roadmap

- Model without intermediaries
  - Fixed (outside) money supply
  - Optimal money growth rate
    - “On the optimal inflation rate” (inflation target)

- Model with intermediaries
  - Fixed outside money supply - Amplification/endogenous risk
    - Liquidity spiral asset side of intermediaries’ balance sheet
    - Disinflationary spiral liability side

- Monetary Policy
- Macro-prudential policy

- Intermediaries with market power
  - The “Reversal Interest Rate: The Effective Lower Bound”
Money view

- Restore money supply
  - Replace missing inside money with outside money
- Aim: Reduce deflationary spiral
  - ... but banks extent less credit & diversify less idiosyncratic risk away
  - ... as households have to hold more idiosyncratic risk, money demand rises
  - Undershoots inflation target

Credit view

- Restore credit
- Aim: Switch off deflationary spiral & liquidity spiral
Monetary Policy

- Introduce long-term bond
- Central bank’s actions change money supply/transfer risk
  - Interest rate cuts in downturns raise the value of long-term bonds
  - Change relative price between long-term bond and short-term money
  - Risk transfer (ex-post redistribution)

Macro-prudential policy

1. Leverage upper bounds
2. Affect agents portfolio choice directly
Introducing Long-term Gov. Bond

- Introduce long-term (perpetual) bond
  - No default ... held by intermediaries in equilibrium

\[
\frac{dB_t}{B_t} = \mu_t^B dt + \sigma_t^B dZ_t
\]

Perpetual bonds:
- pay in money (at unit rate)
- endogenous price \( B_t \) (in money)

Value \( b_t K_t \)

Value \( p_t K_t \)

Value \( q_t K_t \)

- Value of long-term bond is endogenous
Redistributive MoPo: Ex-post perspective

- Adverse shock → value of risky claims drops
- Monetary policy
  - Interest rate cut ⇒ long-term bond price
  - Asset purchase ⇒ asset price
  - ⇒ “stealth recapitalization” - redistributive
  - ⇒ risk premia
- Liquidity & Deflationary Spirals are mitigated
Redistributive MoPo: Ex-post perspective

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  - ⇒ risk premia
- Liquidity & Deflationary Spirals are mitigated
Monetary policy and endogenous risk

- Intermediaries’ risk (borrow to scale up)

\[
\sigma_t^\eta = \frac{x_t \left( 1^b \sigma^b - \sigma^K_t \right)}{1 + \left( \frac{x_t \psi_t - \eta_t}{\eta_t} \right) \frac{\vartheta'(\eta_t)}{\vartheta/\eta_t} - \left( x_t + \vartheta t \frac{1 - \eta_t}{\eta_t} \right) \frac{b_t}{p_t} \frac{B'(\eta_t)}{B(\eta_t)/\eta_t}}
\]

- MoPo works through \[ \frac{B'(\eta_t)}{B(\eta_t)/\eta_t} \]
  - with right monetary policy bond price \( B(\eta) \) rises as \( \eta \) drops “stealth recapitalization”
  - Switch off liquidity and disinflationary spiral

- Example:
  Remove amplification s.t. \[ \sigma_t^\eta = x_t \left( 1^b \sigma^b - \sigma^K_t \right) \]
Numerical example with monetary policy

- Prices

$q$ is more stable

$p$ less disinflation
Numerical example with monetary policy

- Drift and volatility of $\eta$

![Graph showing drift and volatility of $\eta$]
Observations

- As interest rates are cut in downturns, bonds held by intermediaries appreciate, this
  - protects intermediaries against shocks
  - increases the supply of assets that can be used as storage (weakens disinflation)

- Ex-post stabilization
  - Liquidity spiral
  - Disinflationary spiral

- Ex-ante
  - Higher leverage
  - (shift in steady state)
Monetary policy ... in the limit

- full risk sharing of all aggregate risk

\[ \sigma_t^\eta = \frac{x_t(1^b \sigma^b - \sigma^K_t)}{1 - \left(\frac{\psi \eta - \vartheta(\eta)}{\eta} + \left(1 - \vartheta\right)\frac{\psi \chi - \eta}{\eta} + \vartheta \frac{1 - \eta}{\eta}\right) \frac{b_t - B'(\eta)}{p_t B(\eta) / \eta} - \infty} \]

- \( \eta \) is deterministic and converges over time towards 0
Monetary policy: 3 versions

- No MoPo
- No Amplification
- Aggregate risk sharing
Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

<table>
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Woodford, Tobin (1982), BruSan, Price stickiness, Perfect capital markets, Representative Agent, Heterogeneous Agents, Cut $i$, Reduces $r$ due to price stickiness, Consumption $c$ rises.
# Monetary Policy Transmission Channel

- **Consumption Boost approach to “Bottleneck approach”**

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## Monetary Policy Transmission Channel

- Consumption Boost approach to “Bottleneck approach”

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- Perfect capital markets
- Both
- Financial Frictions
- Incomplete markets
- Redistributes from low MPC to high MPC consumers
- QE
### Monetary Policy Transmission Channel

- **Consumption Boost approach to “Bottleneck approach”**

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- QE
  - US: QE1 & QE3: MBS
  - Japan 1990: corporate bonds
Overview

- No monetary economics
  - Fixed outside money supply
  - Amplification/endogenous risk through
    - Liquidity spiral: asset side of intermediaries’ balance sheet
    - Disinflationary spiral: liability side

- Monetary policy
  - Wealth shifts by affecting relative price between
    - Long-term bond
    - Short-term money
  - Risk transfers – reduce endogenous aggregate risk

- Macroprudential policy
  - Directly affect portfolio positions
MacroPru

- MacroPru complements MoPo
  - Not substitutes

- Good MacroPru enables more aggressive MoPo
  - More redistribution ex-post
  - More risk-transfers/insurance ex-ante
  - Lower $q$
    - reduces cost to repurchase capital after shock
    - Lowers importance of idiosyncratic shocks
MacroPru policy

- Regulator can control
  - Portfolio choice $\psi s, x s$
  - Investment decision $\nu(q)$
  - Consumption decision $c$
- cannot control

of intermediaries and households
MacroPru policy

- Regulator can control
  - Portfolio choice $\psi$, $x$
- cannot control
  - investment decision $i(q)$
  - consumption decision $c$

  of intermediaries and households

- De-facto controls $q$ and $p$ within some range
- De-factor controls wealth share $\eta$
  - Force agents to hold certain assets and generate capital gains

- In sum, regulator maximizes sum of agents value function
  - Choosing $\psi^b, q, \eta$
MacroPru policy: Welfare frontier

- Stabilize intermediaries net worth and earnings
- Control the value of money to allow HH to insure idiosyncratic risk (investment distortions still exist, otherwise can get 1st best)
Conclusion

- Unified macro “Money and Banking” model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral

- Exogenous risk &
  - Sector specific
  - Idiosyncratic

- Endogenous risk
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable

- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking

- Macro-prudential policies
  - MacroPru + MoPo to achieve superior welfare optimum
Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes
YouTube channel: Markus.economicus