A Welfare Criterion For Models with Distorted Beliefs

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Motivation

Causes of the
"Great Recession"

Rationale for Regulation

Incentive distortions	Belief distortions
• Moral hazard	
	No welfare criterion
• (fire-sale) externality	

An Example

Joe Stiglitz:

With 90% chance it has natural filling

Bob Wilson:

With 90% chance it has artificial filling

Joe and Bob took a bet:

- If it has natural filling, Bob pays Joe $100; otherwise, Joe pays Bob $100.

They had to cut the pillow open to find out its content

- It cost $50 to replace the pillow, which they shared the cost.

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An Example

Expected return from the bet:

$$90\% \times \$100 - 10\% \times \$100 - \frac{\$50}{2} = \$55$$

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Both Joe and Bob found the bet desirable

- The bet is Pareto efficient!

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An Example

- The bet induces a wealth transfer between them, but a perfect pillow is destroyed!
- The bet does not appear efficient, despite being Pareto efficient
  - A negative-sum game!

Welfare Analysis with Conflicting Beliefs

- In the presence of conflicting beliefs, Pareto criterion leads to unappealing outcomes
  - Divergence between ex-ante and ex-post efficiencies, e.g., von Weizsäcker (1962), Dreze (1970), Starr (1973), Harris (1978), Hammond (1981).
  - Spurious unanimity problem, e.g., Mongin (1997) and Gilboa and Schmeidler (2012)

Sources of heterogeneous beliefs

- Subjective beliefs
  - Savage's view: beliefs are part of their preferences under uncertainty.
  - The bet did not help Bob and Joe hedge their state-dependent risk, rather each believed he would win and the other would lose.

- Distorted beliefs
  - Mounting evidence that biases, like overconfidence, representativeness, limited attention, etc., can distort people's beliefs.
  - Then, social planner needs to use a common, objective measure to evaluate agents' welfare on their behalves.
  - Common approach used in the behavioral finance/economics literature.

Challenge and Key Insight

- Whose beliefs should the planner use in welfare analysis?
  - Difficult to discriminate different beliefs.
  - Behavioral economists proposed different ways to recover the objective beliefs, e.g., Bernheim and Rangel (2009) and Koszegi and Rabin (2007).

- We propose a belief-neutral welfare criterion.
  - A set of reasonable beliefs, spanned by convex combinations of agents' beliefs.
    - The objective measure lies between agents' beliefs.
  - A choice x is efficient (inefficient) if the planner finds it efficient (inefficient) by using every reasonable belief as the common measure to evaluate all agents' welfare.
    - Incomplete criterion
  - Useful for spotting negative-sum & positive-sum speculation induced by heterogeneously distorted beliefs.

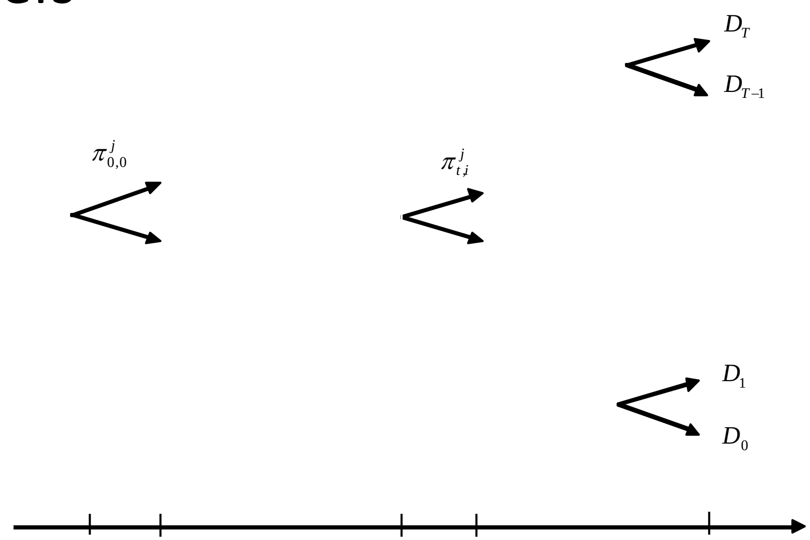
Applications

- Negative-sum speculation in macro and finance models
  - Trading costs in bubble models
  - Over-investment in bubble models
  - Bankruptcy costs in leverage cycle models
  - Excessive risk taking in speculative trading models
  - Consumption-savings distortions in macro models
- Positive-sum speculation
  - Overcoming market breakdown in Lemons models

Belief-Neutral Criterion

- Consider a generic setting with T periods: $t = 0,1,\dots,T$.
  - The state follows a binomial tree.

- N agents holding different beliefs
  - $\Pi^i = \{\pi^i_{t,s}\}, i \in \{1,\dots,N\}$
  - $\pi^i_{t,s} > 0$



- A social choice:
  - $x = \{x^i_T(s_T)\}$

- State-dependent utility
  - $u_i[s_T, x^i_T(s_T)]$
  - Capturing state-dependent preferences and subjective priors

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Belief-Neutral Criterion

- Set of reasonable beliefs:
  - any convex combination of agents' beliefs: $\Pi^h = \sum_i h^i \Pi^i$, where $h^i \geq 0$ and $\sum_i h^i = 1$.
  - The objective measure is somewhere between agents' beliefs.
  - An outside measure implies aggregate beliefs are biased.

- Belief-neutral criterion:
  - An allocation x is belief-neutral inefficient (or efficient) if the planner finds it inefficient (or efficient) by using every Π^h as the common measure to evaluate all agents' welfare.

  - Two ways to implement the criterion
    - Social welfare function
    - Pareto dominance

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Implementation by a Social Welfare Function

- For a given social welfare function
  - Bergsonian social welfare function: $W(u^1, u^2, \dots, u_N) = \sum_i \lambda_i u_i$, where $\{\lambda_i\}$ are non-negative weights
    - Varying the weights gives Pareto frontier
  - Utilitarian social welfare function: $W(u^1, u^2, \dots, u_N) = \sum_i u_i$

- Allocation x belief-neutral superior to y if $\forall \Pi^h$, the planner finds that

$$W(E_0^h[u^1(s_T, x_T^1(s_T))], \dots, E_0^h[u_N(s_T, x_T^N(s_T))]) \geq W(E_0^h[u^1(s_T, y_T^1(s_T))], \dots, E_0^h[u_N(s_T, y_T^N(s_T))])$$

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Implementation by a Social Welfare Function

- Back to the bet between Joe and Bob.
- Suppose that both of them are risk neutral and that the planner uses a utilitarian welfare function:

$$W(u_{Joe}, u_{Bob}) = u_{Joe} + u_{Bob} = w_{Joe} + w_{Bob}$$

  – Social welfare is equivalent to expected social wealth.

- The bet generates a wealth transfer and a pillow being destroyed.
  – The destroyed pillow leads to a negative sum, which is independent of the beliefs used by the planner.

Implementation by Pareto Dominance

- What if Bob and Joe have unequal weights?
  - The bet can transfer wealth from the low-weight person to the other.

- An allocation x is called belief-neutral Pareto efficient (inefficient) if under any measure Π^h there does not exist (exists) another allocation x' such that it improves some agents' expected utilities without reducing anyone's, i.e., $\forall i,\ E_0^h[u_i(s_T, x_T^i(s_T))] \leq E_0^h[u_i(s_T, {x'}_T^i(s_T))]$.
  - Different from standard Pareto dominance, the planner uses a common measure to evaluate all agents' welfare, instead of their own.
  - The standard economic theory: for a given, common belief measure, each allocation on the Pareto frontier maximizes a linear social welfare function with a certain set of Pareto weights.

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Implementation by Pareto Dominance

- Back to the bet between Joe and Bob.

- Suppose that the planner uses Joe's beliefs.
  - The bet leads to an expected gain of $55 to Joe and an expected loss of $105 to Bob.
  - An alternative by transferring $55 to Joe from Bob makes Joe indifferent and improves Bob's welfare by $50.
- Suppose that the planner uses any convex combination of their beliefs, say with weight $h \in (0,1)$ to Joe.
  - A higher h means a larger expected gain to Joe from the bet under the measure.
  - Still, an appropriate transfer from Bob to Joe can make Joe indifferent and save Bob some money.

- Thus, the bet is belief-neutral Pareto inefficient.
  - The belief-neutral inefficiency of the bet does not rely on any particular welfare function.

Link between Two Versions

- Let X denote the set of all feasible allocations. Then, an allocation, $x \in X$, is belief-neutral Pareto efficient (inefficient) if and only if $\forall \Pi^h$, there exists (does not exist) a set of Pareto weights $\{\lambda_i\}$ (with $\lambda_i \geq 0$ for all i and $\sum_i \lambda_i = 1$) such that:

$$x \in argmax_{x \in X} \sum_{i=1}^{N} \lambda_i E_0^h \{u_i(s_T, \hat{x}_T^i(s_T))\}.$$

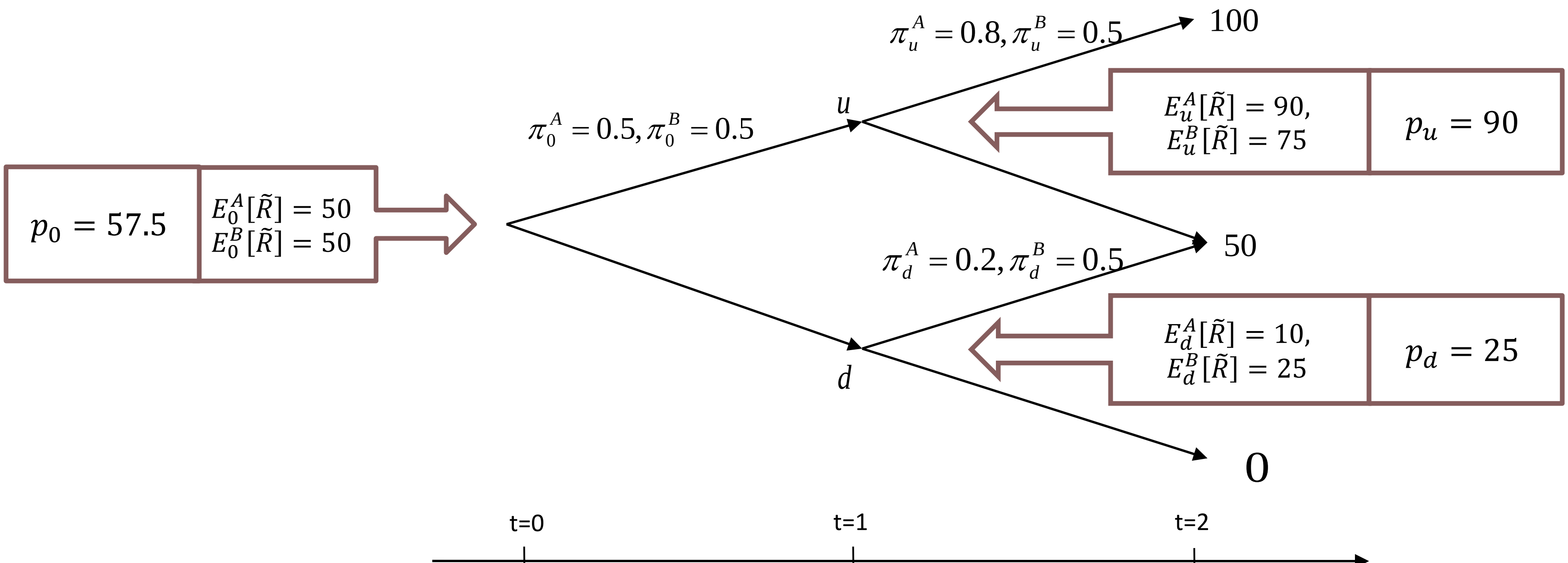
  - The welfare-function-based criterion permits direct comparison of two allocations.
  - The Pareto-efficiency-based criterion categorize all allocations into three sets: 1) those efficient under all reasonable measures, 2) those inefficient under all reasonable measures, and 3) those that are neither uniformly efficient nor uniformly inefficient under all reasonable measures.

Comments on the Criterion

- Separate distorted beliefs from preferences
  - “Anscombe-Aumann objective roulette”
  - Move subjective beliefs into state-dependent beliefs
- Our criterion not applicable to analysis of an individual’s irrational behavior against nature, because nature is not part of the social welfare.
  - Designed to detect negative-sum and positive-sum speculation, which is present in many macro and finance models.

Application 1: Trading Costs in Bubble Models

- Bubble models with heterogeneous beliefs and short-sales constrains, e.g., Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2003), Hong, Scheinkman and Xiong (2006), Hong and Sraer (2011).
- Two risk-neutral agents: A and B.
  - An asset with fixed supply, 1 unit equally divided bw A & B.



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Application 1: Trading Costs in Bubble Models

- Suppose that trading costs κ per share.
  - $\kappa < 15$ so that trading occurs.
  - Trading always occurs on date 1, either from A to B or from B to A.
  - The cost leads to a negative sum, just like the destroyed pillow.
- Trading frenzies during bubbles, e.g., Scheinkman and Xiong (2003), Hong and Stein (2007).
  - Trading cost severely undercut the portfolio performance of retail investors, e.g., Barber and Odean (2000).

- Social welfare function approach
  - Assume linear and symmetric social welfare function:
$$W(u_A, u_B) = u(c_A) + u(c_B) = c_A + c_B.$$
  - At the status quo:
$$E_0^h[W(u_A, u_B)] = E_0^h[\tilde{R}] = 50, \quad \forall \Pi^h = h\Pi^A + (1-h)\Pi^B.$$
  - In the equilibrium:
$$E_0^j[W(u_A, u_B)] = E_0^j[\tilde{R}] - \frac{\kappa}{2} = 50 - \frac{\kappa}{2}, \qquad \forall \Pi^h.$$

- Pareto dominance approach
  - The planner can always find a transfer under any belief measure to make one agent indifferent and the other happier.
  - Belief-neutral inefficient even under any measure

Application 2: Over-investment in Bubble Models

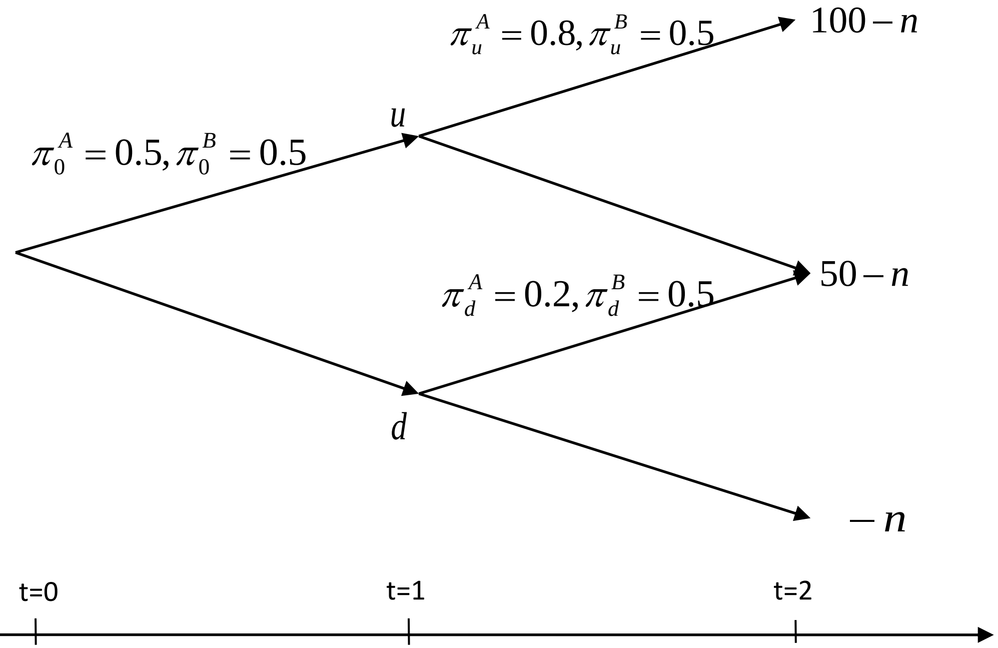
- Investor speculative motives drive over-investment, e.g., Bolton, Scheinkman and Xiong (2006), Gilchrist, Himmelberg, and Huberman (2005).
- Decreasing return to scale and invest n units at $t = 0$.
  - Firm objective: $\max_n \; n \cdot p_0$

Market setting:
$p_0 = 57.5 - n$
$\max_n \; n \cdot (57.5 - n) \;\; \Rightarrow n_* = \frac{57.5}{2}$.

If the planner adopts A's beliefs:
$p_0 = E_0^A[\tilde{R}] = 50 - n$
$\max_n \; n \cdot (50 - n) \;\; \Rightarrow n_* = 25.$

If the planner adopts B's beliefs:
$p_0 = E_0^B[\tilde{R}] = 50 - n$
$\max_n \; n \cdot (50 - n) \;\; \Rightarrow n_* = 25.$

Over-investment from both A and B's beliefs!



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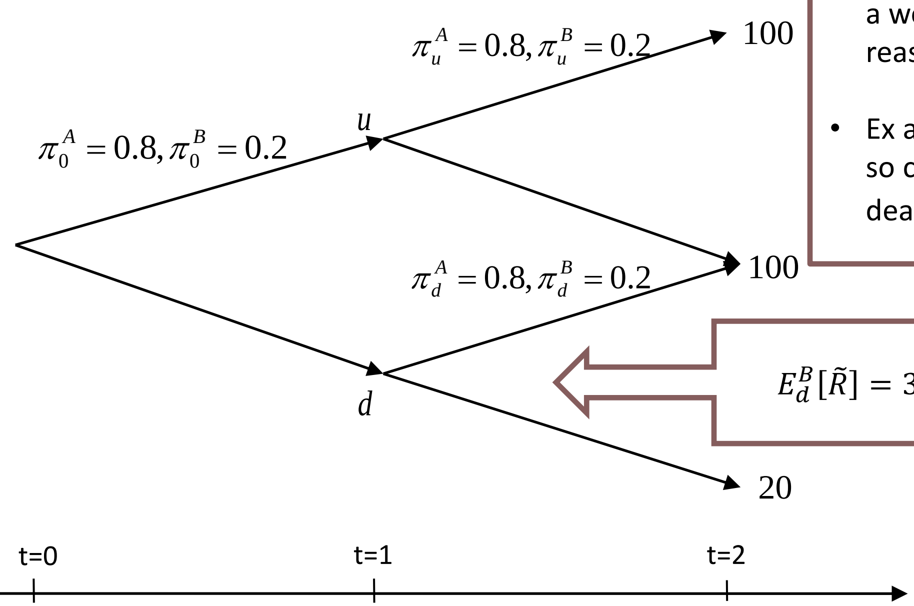
Application 3: Bankruptcy Costs in Leverage Cycle Models

- Cash-constrained optimists tend to use collateralized debt to finance their investments, which fuels initial price boom and later price bust.
  - Geanakoplos (2003, 2009), Fostel and Geanakoplos (2008), Simsek (2010), and He and Xiong (2012).
- A is always more optimistic than B, both risk neutral.
  - At $t = 0$, A is endowed with \$20 and no asset.
  - Owner incurs a cost of $\alpha = \$20$ to liquidate per unit of asset.

- The bankruptcy cost α induces a welfare loss under any reasonable beliefs.
- Ex ante, A believes the asset is so cheap that he has a good deal despite the cost.

$E_0^A[\tilde{R}] = 96.8$
$E_0^B[\tilde{R}] = 48.8$



$E_d^B[\tilde{R}] = 36$

- A uses one-period debt with promise 36.
- $p_0 = 20 + 36 = 56.$
- In state d, A has to promise 36 to rollover his debt, which exposes him to bankruptcy risk if fundamental ends in 20.

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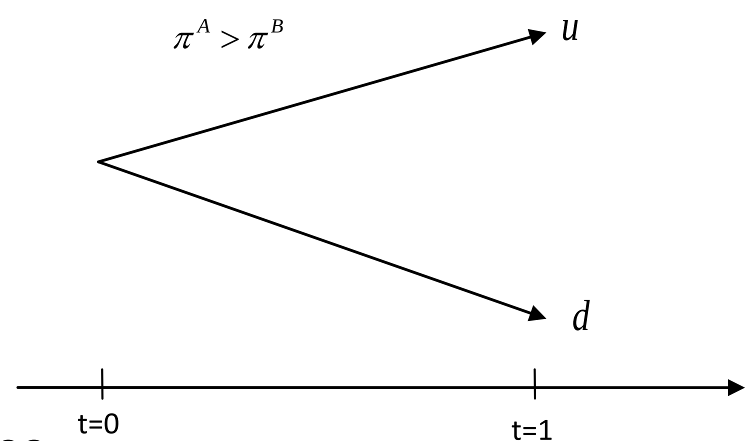
Application 4: Excessive Risk Taking in Speculative Trading Models

- Many general equilibrium models with heterogeneous beliefs:
  - Speculation between optimists and pessimists lead to endogenous risk
  - e.g., Detemple and Murthy (1994), Kurz (1996), Zapatero (1998), Basak (2000), Buraschi and Jiltsov (2006), Jouini and Napp (2007), David (2008), Dumas, Kurshev and Uppal (2009), Xiong and Yan (2010), and Dumas, Lewis, and Osambela (2011)



- Endowment economy with A and B.
  - Each agent endowed with 0.5 dollar;
  - each has a strictly concave utility function $u(c_i)$;
  - Heterogeneous beliefs about $\tilde{f}$

- A contract pays \$1 if $\tilde{f}$ is u and \$0 otherwise.
  - A and B trade the contract at a price of p at t=0.
  - If $\pi^A > \pi^B$, A buys $k^A > 0$ while B buys $k^B = -k^A$.

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Application 4: Excessive Risk Taking in Speculative Trading Models

Social Welfare Function Approach

- Compare the equilibrium allocation

$$x = \{(x_H^i, x_L^i)\}_{i\in\{A,B\}} = \{(0.5 + k^i(1-p), 0.5 - k^i p)\}_{i\in\{A,B\}}$$

with the status quo with no trading $y = \{(y_H^i, y_L^i) \equiv (0.5, 0.5)\}_{i\in\{A,B\}}$

- Suppose the planner uses Utilitarian social welfare function:

$$W[u_A(x), u_B(x)] = u(x^A) + u(x^B).$$

- For any belief $\pi^h = h\pi^A + (1-h)\pi^B, \forall h \in [0,1]$,

$$\begin{aligned} E^h\{W[u_A(x), u_B(x)]\} &= \pi^h\big[u\big(0.5 + k^A(1-p)\big) + u\big(0.5 - k^A(1-p)\big)\big] \\ &\quad + (1-\pi^h)\,[u(0.5 - k^A p) + u(0.5 + k^A p)] \\ &< \pi^h[2u(0.5)] + \big(1-\pi^h\big)[2u(0.5)] \\ &= E^h\{W[u_A(y), u_B(y)]\} \end{aligned}$$

  - The trading makes each agent's consumption more volatile without changing the aggregate wealth, negative-sum game in utility terms.

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Application 4: Excessive Risk Taking in Speculative Trading Models

Pareto Dominance Approach

- Compare the market equilibrium allocation x with the status quo plus a wealth transfer:

$$y(T) = \{(0.5 + T, 0.5 + T), (0.5 - T, 0.5 - T)\}$$

  - The trading makes each agent's consumption variable across the two states and thus his certainty equivalent wealth under any measure less than his expected wealth. As a result, the sum of the two agents' certainty equivalent wealth less than 1 (total wealth).
  - Thus, the planner can always find a suitable transfer to make one agent indifferent with his equilibrium allocation and the other agent happier.

Application 4: Excessive Risk Taking in Speculative Trading Models

Tradeoff between Risk Sharing and Speculation

- Suppose that the agents' endowments are
$$y = \{(y_H^A, y_L^A), (y_H^B, y_L^B)\} = \{(0.5 - e, 0.5), (0.5 + e, 0.5)\}$$
- In the absence of belief disagreements ($\pi^A = \pi^B = \pi$), A and B trade the risky contract to share their endowment risks:
  - $k^{opt} \equiv (k^A = e, k^B = -e)$,
  - $y^{opt} = \{(0.5 - e\pi, 0.5 - e\pi), (0.5 + e\pi, 0.5 + e\pi)\}$

- If $\pi^A \neq \pi^B$, speculative motive causes A and B to trade excessively relative to k^{opt} and their final wealth more volatile than y^{opt}.
  - This outcome is again belief-neutral Pareto dominated by the optimal risk sharing allocation plus appropriate transfer.

- The planner is often unable to implement the optimal risk sharing allocation and might have to either allow or restrict trading
  - The trading equilibrium is belief-neutral Pareto dominated by the status quo if the endowment shock is sufficiently small.

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Application 5: Consumption-Savings Distortions in Macro Models

- In macro models with investment, speculation induced by belief disagreements can also distort savings and thus investments.
  - Speculation not only makes their consumption more volatile but also distorts the aggregate consumption:

- Sims (2008)
  - Two types of agents disagree about future inflation.
  - Inflation optimists prefer to borrow nominal from pessimists.
    - Substitution effect: speculation motivates both types to save
    - Wealth effect: expectations of speculation gains induce both to consume more
    - Depending on their rate of relative risk aversion, substitution effect dominates wealth effect or vice versa, and thus leads to over- or under-investment.
- Our criterion can also detect belief-neutral inefficiency of such distortions.

Application 6: Benefits of Speculation in Lemons Model

- Speculation caused by heterogeneous beliefs can be beneficial in lemons model, a la Akerlof (1970).
- We adopt a simple version of Tirole (2012):
  - A seller has access to a new project with a cost I and a payoff of $I + S$, which is not pledgeable despite its clear gain.
  - He has to finance the project by selling a legacy asset, whose return is either R with probability θ or 0 otherwise.
  - He knows θ, while potential buyers believes θ is uniformed distributed over $[0,1]$.

- The market for the asset freezes if $S < I$.
  - The seller will sell the asset only if $\theta R \leq p + S$.
  - For the buyer to break even, $p = rE\left[\theta \middle| \theta \leq \frac{p+S}{R}\right] \rightarrow p = S$.
  - The seller won't sell if $p < I \leftrightarrow S < I$.

Application 6: Benefits of Speculation in Lemons Model

- Suppose that the asset return in the positive outcome is random, which we denote $\tilde{R}$, and can take two possible values $\{R+1, R-1\}$ with equal probability.
- Suppose that investors hold heterogeneously distorted beliefs regarding the distribution of $\tilde{R}$ and that short sales are prohibited.
  - Then, $p = E^{opt}[\tilde{R}]E\left[\theta \middle| \theta \leq \frac{p+S}{R}\right] \rightarrow p > S$.
  - Thus, market freeze occurs in a smaller region.
- Speculation caused by heterogeneous beliefs overcomes the adverse selection and leads to a positive-sum game.
  - Belief-neutral efficient outcome.

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Conclusion

- A new welfare criterion for behavioral models
- Opens normative analysis for financial regulation
  - Avoid negative-sum speculation and facilitate positive-sum one

- Separate "preferences" from "belief distortions"
  - Don't need to know the truth
  - Incomplete ranking
- Welfare losses
  - Trading costs and over-investment in bubble models
  - Bankruptcy costs in leverage cycle models
  - Excessive risk taking in speculative trading models
  - Consumption-savings distortions in macro models
- Welfare gains
  - Overcoming market breakdown in Lemons models