Banking, Liquidity and Bank Runs in an Infinite Horizon Economy

Mark Gertler and Nobuhiro Kiyotaki
NYU and Princeton University

Motivation

Two complementary approaches to banking crisis

"Macro" (e.g. Gertler and Kiyotaki 2011)

Losses of bank net worth raises the cost of bank credits

Bank runs are excluded

"Micro" (e.g. Diamond and Dybvig 1983)

Maturity mismatch opens up the possibility of runs $\longrightarrow$ inefficient liquidation of assets and loss of banking service

Highly stylized. Runs often unrelated to aggregate conditions

During the recent crisis, both "macro" and "micro" distresses were at work (Gorton, 2010, Bernanke, 2010)

Losses on sub-prime related assets depleted bank capital

Forced a contraction of credits of many financial institutions

Bank credit costs sky-rocketed

Some of the major investment banks and money funds experienced runs

We develop a simple macro model of banking crisis

- Balance sheet financial/accelerator effects
- Banks runs
- Balance sheet conditions affect whether runs are feasible

Two key variables

- Bank leverage ratio (affects maturity mismatch)
- Liquidation prices

Both depend on macroeconomics conditions

Basic Model

Capital is either intermediated by banks or directly held by households

$$K_t^b + K_t^h = \overline{K} = 1$$

$$\begin{array}{c} \textit{date t} \\ K_t^b \text{ capital} \end{array} \Big\} \longrightarrow \begin{array}{c} \textit{date t+1} \\ \left\{ \begin{array}{c} K_t^b \text{ capital} \\ Z_{t+1} K_t^b \text{ output} \end{array} \right. \end{array}$$

$$\begin{array}{c} \textit{date t} \\ \left. \begin{array}{c} K_t^h \text{ capital} \\ f(K_t^h) \text{ goods} \end{array} \right\} \end{array} \longrightarrow \begin{array}{c} \textit{date t+1} \\ \left\{ \begin{array}{c} K_t^h \text{ capital} \\ Z_{t+1} K_t^h \text{ output} \end{array} \right. \end{array}$$

$$f(K_t^h) : \text{ management cost } f' > 0, \; f'' \geq 0$$

Deposit contract

Short term

Non-contingent return R_{t+1} (absent a bank run)

Sequential service constraint (as in Diamond/Dybvig)

In the event of a run, payoff either R_{t+1} or 0, depends on place in line

Bank runs completely unanticipated

Households maximize

$$U_t = E_t \left(\sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right)$$

subject to:

$$C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h$$

$\longrightarrow$

$$E_t(\Lambda_{t,t+1}) R_{t+1} = 1$$

$$E_t(\Lambda_{t,t+1} R_{t+1}^h) = 1$$

$$R_{t+1}^h = \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)}, \text{ and } \Lambda_{t,t+i} = \beta^i \frac{C_t}{C_{t+i}}$$

Many bankers (measure unity)

Each has an i.i.d. survival probability of σ

Banker consumes wealth upon exit

Preferences are linear in "terminal" consumption

$$V_t = E_t \left[\sum_{i=1}^{\infty} \beta^i (1-\sigma)\sigma^{i-1} c_{t+i}^b \right]$$

Each exiting banker replaced by a new banker

Starts with an endowment w^b

Bank balance sheet

$$Q_t k_t^b = d_t + n_t$$

Net worth n_t of surviving bankers

$$n_t = (Z_t + Q_t) k_{t-1}^b - R_t d_{t-1}$$

Net worth of new bankers

$$n_t = w^b$$

Consumption of exiting bankers

$$c_t^b = n_t$$

Agency Problem:

After the banker raises funds, it may divert a fraction of θ of loans at the end of period t

If the bank does not honor its debt in period $t+1$, the creditors shut the bank down

Incentive constraint

$$\theta Q_t k_t^b \leq V_t$$

Bank chooses k_t^b and d_t to maximize

$$V_t = \beta E_t[(1-\sigma)n_{t+1} + \sigma V_{t+1}]$$

subject to $\theta Q_t k_t^b \leq V_t \rightarrow$

$$V_t = \nu_{kt} k_t^b - \nu_t d_t = \underbrace{\left(\frac{\nu_{kt}}{Q_t} - \nu_t\right)}_{\mu_t} Q_t k_t^b + \nu_t n_t \geq \theta Q_t k_t^b$$

$$\frac{Q_t k_t^b}{n_t} \leq \phi_t = \frac{\nu_t}{\theta - \mu_t}$$

$$\nu_t = \beta R_{t+1} E_t[\Omega_{t+1}]$$

$$\mu_t = \beta E_t[(R_{t+1}^b - R_{t+1})\Omega_{t+1}]$$

$$\Omega_{t+1} = 1 - \sigma + \sigma(\nu_{t+1} + \phi_{t+1}\mu_{t+1})$$

$$R_{t+1}^b = \frac{Z_{t+1} + Q_{t+1}}{Q_t}$$

Aggregate leverage constraint

$$Q_t K_t^b = \phi_t N_t$$

Aggregate net worth

$$\begin{aligned} N_t &= \sigma\left[(Z_t + Q_t)\, K_{t-1}^b - R_t D_{t-1}\right] + (1-\sigma) w^b \\ &= \sigma[(R_t^b - R_t)\phi_{t-1} + R_t] N_{t-1} + W^b \end{aligned}$$

Volatility of N_t depends on ϕ_{t-1} and volatility of R_t^b

Bank Runs

Ex ante zero probability of a run

Consider the possibility of a run ex post

At the beginning of period t, depositors decide whether to roll over their deposits

If depositors "run", the bank sells its capital to households who are less efficient in managing capital

A bank run equilibrium exists if:

$$(Z_t + Q_t^*)\, K_{t-1}^b < R_t D_{t-1}$$

$Q_t^* \equiv$ the liquidation price of the bank's assets

Or, a bank run equilibrium exists if

$$\frac{Z_t + Q_t^*}{Q_{t-1}} \equiv R_t^{b*} < R_t \cdot \frac{D_{t-1}}{Q_{t-1} K_{t-1}^b} = R_t \left(1 - \frac{1}{\phi_{t-1}}\right)$$

Whether a bank run equilibrium exists depends on (Q_t^*, ϕ_{t-1}, R_t)

Since Q_t^* is pro-cyclical and and ϕ_{t-1} is counter-cyclical, the likelihood of a bank run is counter-cyclical

Liquidation Price Q_t^*

After a bank run at t :

$$K_{t+i}^h = \overline{K} = 1, \ \forall i$$

Household Euler equation for direct capital holding

$$E_t\{\Lambda_{t,t+1} R_{t+1}^{h*}\} = 1$$

$$R_{t+1}^{h*} = \frac{Z_{t+1} + Q_{t+1}^*}{Q_t^* + f'(1)}$$

or

$$Q_t^* = E_t\left\{\sum_{i=1}^{\infty} \Lambda_{t,t+i}[Z_{t+i} - f'(1)]\right\} - f'(1)$$

where $f'(1)$ is the marginal management cost which as at a maximum at $K_t^h = 1$

Household Liquidity Risks:

Suppose the representative family has a continuum of members of measure unity

With probability π, a member wants to consume c_t^m in emergency

For an individual with emergency consumption needs, period utility is:

$$\ln C_t^h + \kappa \ln c_t^m$$

For someone without:

$$\ln C_t^h$$

Timing of Events:

The family chooses C_t^h and its portfolio before learning of the realization of emergency needs

$$\begin{aligned} & C_t^h + D_t + Q_t K_t^h + f(K_t^h) \\ = \; & R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h + Z_t W^h - \pi \overline{c}_t^m \end{aligned}$$

After choosing D_t, the household divides it evenly amongst its members. A fraction of emergency expenditures must be financed by deposits:

$$c_t^m - \underline{c}^m \leq D_t$$

Those who do not use their deposits return the deposits to the family $\longrightarrow$ The deposit at the end

$$\begin{aligned} D_t' \; &= \; \pi(D_t - c_t^m) + (1-\pi) D_t + \pi \overline{c}_t^m \\ &= \; D_t, \text{ under symmetric equilibrium} \end{aligned}$$

Solution to Household Problem with Liquidity Risks

FONC for D_t

$$E_t(\Lambda_{t,t+1})R_{t+1} + \pi\frac{\chi_t}{1/C_t^h} = 1$$

c_t^m

$$\frac{\kappa}{c_t^m} - \frac{1}{C_t^h} = \chi_t$$

Liquidity constraint

$$c_t^m - \underline{c}^m \leq D_t$$

FONC for K_t^h is the same as in baseline

Table 1: Parameters

Baseline Model		
β	0.99	Discount rate
σ	0.95	Bankers survival probability
θ	0.35	Seizure rate
α	0.1	Household managerial cost
$\bar{K}^h$	0.096	Threshold capital for managerial cost
γ	0.72	Fraction of depositors that can run
ρ	0.95	Serial correlation of productivity shock
Z	0.0161	Steady state productivity
ω^b	0.0019	Bankers endowment
ω^h	0.045	Household endowment
Additional Parameters for Liquidity Model		
κ	62.67	Preference weight on c_m
$\bar{c}^m$	0.01	Threshold for c_m
π	0.03	Probability of a liquidity shock
γ_L	0.67	Fraction of depositors that can run

Table 2: Steady State Values

Steady State for No Bank-Run Equilibrium		
	Baseline	Liquidity
K	1	1
Q	1	1
C^h	0.0541	0.0184
C^m	0	0.0348
C^b	0.0087	0.0088
K^h	0.0594	0.0545
K^b	0.9406	0.9455
ϕ	8	8
R^b	1.0644	1.0624
R^h	1.0404	1.0404
R	1.0404	1.0384

Steady State for Bank-Run Equilibrium		
	Baseline	Liquidity
K	1	1
Q^*	0.6340	1
C^h	0.0520	0.0515
C^m	0	0.01
C^b	0.0019	0.0019
K^h	1	1
K^b	0	0
ϕ		
R^b	1.1016	1.1068
R^h	1.0404	1.0404
R		

Figure 1: A Recession in the Baseline Model: No Bank Run Case

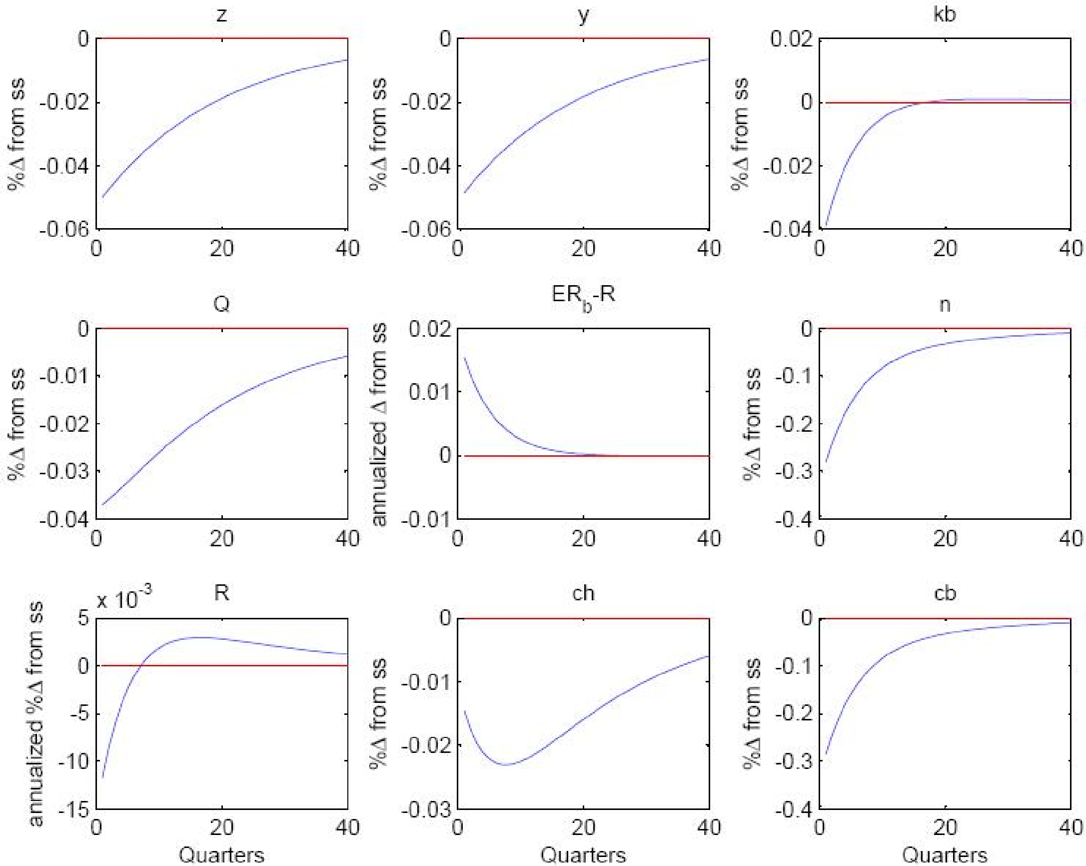


Figure 2: A Recession in the Liquidity Risk Model: No Bank Run Case

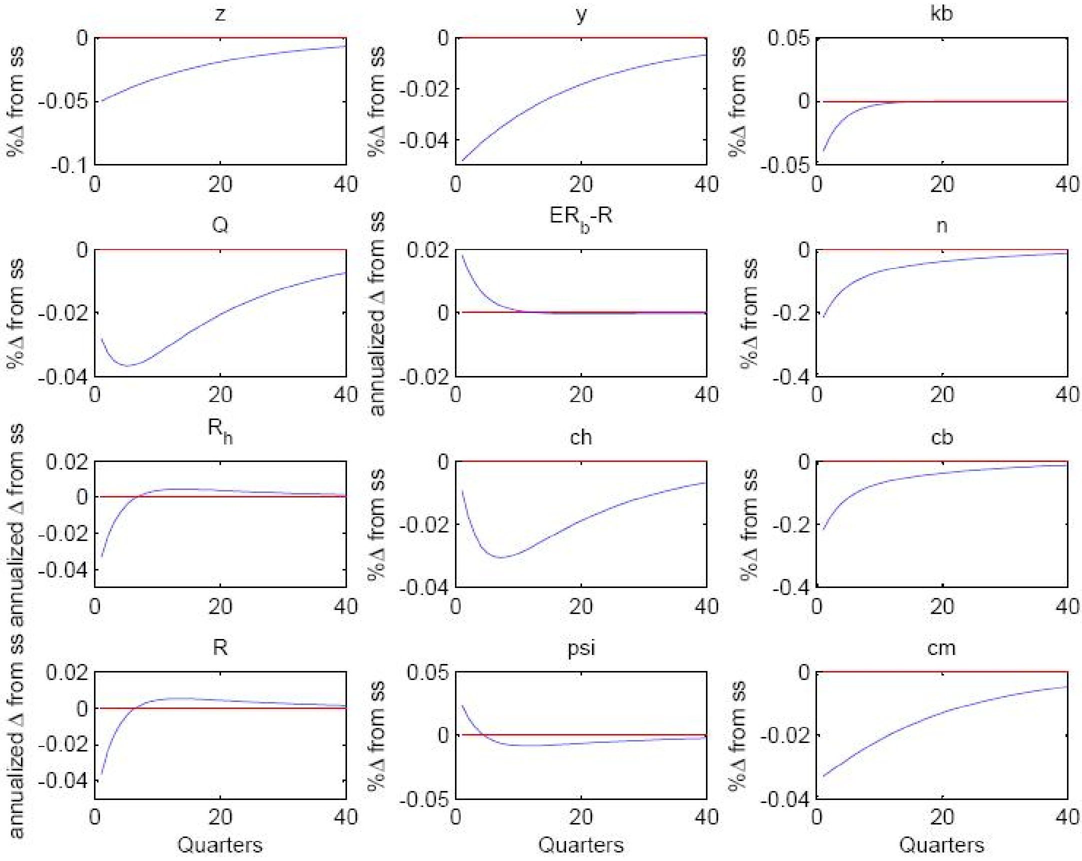


Figure 3: Ex Post Bank Run in the Baseline Model

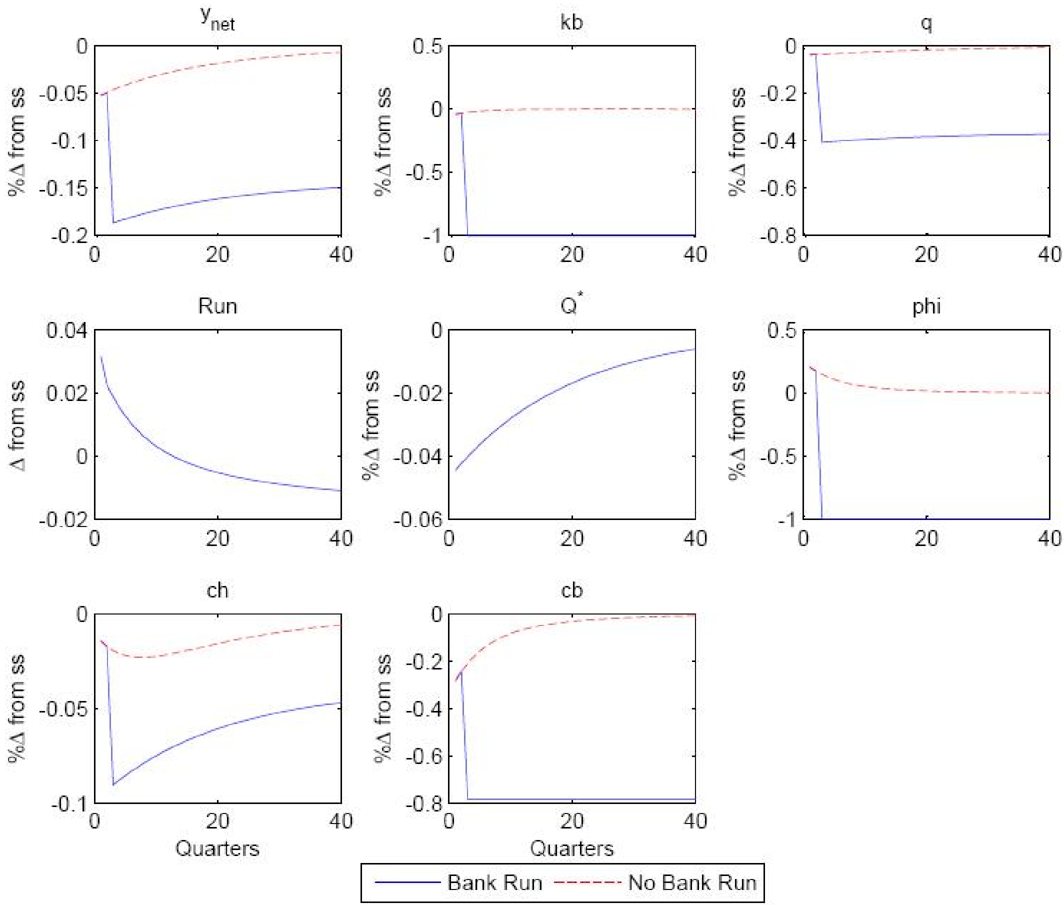
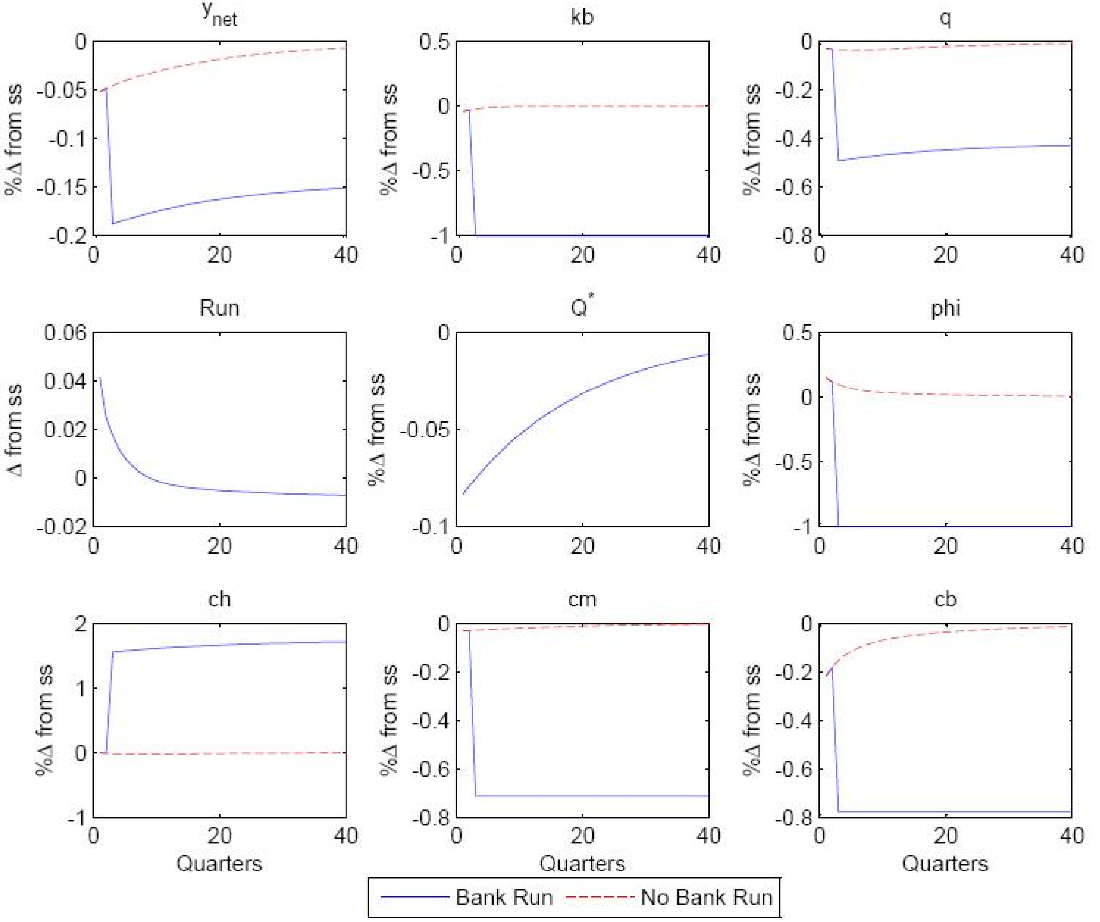


Figure 4: Ex Post Bank Run in the Liquidity Risk Model



Anticipated Bank Runs

Deposit returns

$$R_{t+1} = \begin{cases} \overline{R}_{t+1} \text{ if no bank run} \\ \overline{R}_{t+1} \text{ with prob } \xi_{t+1} \text{ if run} \\ 0 \text{ with prob } 1 - \xi_{t+1} \text{ if run} \end{cases}$$

$$\xi_{t+1} = \frac{\left(Q^*_{t+1} + Z_{t+1}\right) K^b_t}{\overline{R}_{t+1} D_t}$$

Probability of bank run

$$\alpha_t^* = \alpha_t \cdot I(1 - \xi_t)$$

$$I(\cdot) = \begin{cases} 1, \text{ if } \cdot > 0 \\ 0, \text{ otherwise} \end{cases}$$

where α_t follows an exogenous stationary first order process

Household FONC for deposits is

$$1 = \overline{R}_{t+1} E_t[(1 - \alpha_{t+1}^*)\Lambda_{t,t+1} + \alpha_{t+1}^* \Lambda_{t,t+1}^* \xi_{t+1}]$$

where $\Lambda_{t,t+1}^*$ is household's stochastic discount factor conditional on a run

Given $\Lambda_{t,t+1}^* \xi_{t+1} < \Lambda_{t,t+1}$, an increase in α_{t+1}^* raises $\overline{R}_{t+1}$

Leverage constraint

$$\frac{Q_t K_t^b}{N_t} = \phi_t = \frac{\nu_t}{\theta - \mu_t}$$

where

$$\mu_t = \beta E_t \left[(1 - \alpha_{t+1}^*) \Omega_{t+1} (R_{t+1}^b - \overline{R}_{t+1}) \right]$$

Evolution of net worth

$$N_t = \sigma \left[(Z_t + Q_t) \, K_{t-1}^b - \overline{R}_t D_{t-1} \right] + W^b$$

An anticipated increase in α_{t+1}^* is contractionary in two ways

leverage ϕ_t declines since μ_t falls

N_{t+1} decreases even without run since $\overline{R}_{t+1}$ increases

Some Remarks About Policy

A role for deposit insurance as in Diamond/Dybvig

Eliminates bank run equilibrium

But may have moral hazard effects on risk-taking

Can offset with capital requirements

Reduces risk-taking

Reduces the likelihood of a bank run equilibrium

But if bank equity capital is costly to raise, can increase intermediation costs

Alternative: commitment to lender-of-last resort policies

Stabilizing liquidation prices reduces likelihood of bank runs

Examples: lending against good collateral

Purchases a good quality securities (e.g. AMBS)