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Macro, Money and Finance
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Macro-literature on Frictions

1. Net worth effects:
   a. Persistence: Carlstrom & Fuerst
   b. Amplification: Bernanke, Gertler & Gilchrist
   c. Instability: Brunnermeier & Sannikov

2. Volatility effects: impact credit quantity constraints
   a. Margin spirals: Brunnermeier & Pederson
   b. Endogenous constraints: Geanakoplos

3. Demand for liquid assets & Bubbles – “self insurance”
   a. OLG, Aiyagari, Bewley, Krusell-Smith, Holmstrom-Tirole, ...

4. Financial intermediaries & Theory of Money
Demand for Liquid Assets, Bubbles, …

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Demand for Liquid Assets

- So far: Technological and market illiquidity create time *amplification* and *instability*
  - Net worth losses lead depress to price of capital $q_t$, ...
  - Liquidity spirals emerge when price volatility interacts with debt constraints

- Now: Focus on *demand for liquid instruments*
  - No amplification effects: perfect techn. liquidity due to reversibility of investment
    - constant price of capital $q$
    - **Borrowing constraint = collateral constraint**
  - Steps: Introduce (i) idiosyncratic risk, (ii) aggregate risk, (iii) amplification (revisited)
Outline – Demand for Liquid Assets

- Deterministic Fluctuations
  - Overlapping generations
  - Completing markets with liquid asset
- Idiosyncratic Risk
  - Precautionary savings
  - Constrained efficiency
- Aggregate Risk
  - Bounded rationality
- Amplification Revisited
Samuelson (1958) considers an infinite-horizon economy with two-period lived overlapping agents

- Population growth rate $n$

Preferences given by $u(c_t^t, c_{t+1}^t)$

- Pareto optimal allocation satisfies $\frac{u_1}{u_2} = 1 + n$

OLG economies have multiple equilibria that can be Pareto ranked
Assume \( u(c_t, c_{t+1}) = \log c_t + \beta \log c_{t+1} \)
- Endowment \( y_t = e, y_{t+1} = 1 - e \)

Assume complete markets and interest rate \( r \)

Agent’s FOC implies that \( \frac{c_{t+1}}{\beta c_t} = 1 + r \)
- For \( r = n \), this corresponds to the Pareto solution
- For \( r = \frac{1-e}{\beta e} - 1 \), agents will consume their endowment

Autarky solution is clearly Pareto inferior
OLG: Completion with Durable Asset

- Autarky solution is the **unique** equilibrium implemented in a sequential exchange economy
  - Young agents cannot transfer wealth to next period
  - *... more from Chris Sims on this issue on Sunday*

- A durable asset provides a **store of value**
  - Effective store of value reflects *market liquidity*
  - Pareto solution can be attained as a competitive equilibrium in which the price level grows at same rate as the population, i.e. $b_{t+1} = (1 + n)b_t$
  - Old agents trade durable asset for young agents’ consumption goods
Diamond (1965) introduces a capital good and production

- Constant-returns-to-scale production $Y_t = F(K_t, L_t)$

Optimal level of capital is given by the golden rule, i.e. $f'(k^*) = n$

- Here, lowercase letters signify per capita values

Individual (and firm) optimization implies that

- $\frac{u_1}{u_2} = 1 + r = 1 + f'(k)$

- It is possible that $r < n \implies k > k^* \implies$ Pareto inefficient
Diamond recommends issuing government debt at interest rate $r$

Tirole (1985) introduces a rational bubble asset trading at price $b_t$

$$b_{t+1} = \frac{1+r_{t+1}}{1+n} b_t$$

Both solutions *crowd out* investment and increase $r$

- If baseline economy is inefficient, then an appropriately chosen debt issuance or bubble size can achieve Pareto optimum with $r = n$
OLG: Crowding Out vs. Crowding In

- Depending on the framework, government debt and presence of bubbles can have two opposite effects
  - **Crowding out** refers to the decreased investment to increase in supply of capital
  - **Crowding in** refers to increased investment due to improved risk transfer
- Woodford (1990) explores both of these effects
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Precautionary Savings

- Consumption smoothing implies that agents will save in high income states and borrow in low income states
  - If markets are incomplete, agents may not be able to efficiently transfer consumption between these outcomes
- Additional precautionary savings motive arises when agents cannot insure against uncertainty
  - Shape of utility function $u'''$
  - Borrowing constraint $a_t \geq -b$
PCS 1: Prudence

- Utility maximization $E_0[\sum_{t=0}^{\infty} \beta^t u(c_t)]$
  - Budget constraint: $c_t + a_{t+1} = e_t + (1 + r)a_t$
  - Standard Euler equation: $u'(c_t) = \beta (1 + r)E_t[u'(c_{t+1})]$
- If $u''' > 0$, then Jensen’s inequality implies:
  - $\frac{1}{\beta (1+r)} = \frac{E_t[u'(c_{t+1})]}{u'(c_t)} > \frac{u'(E_t[c_{t+1}])}{u'(c_t)}$
  - Marginal value is greater due to uncertainty in $c_{t+1}$
  - Difference is attributed to precautionary savings
- Prudence refers to curvature of $u'$, i.e. $P = -\frac{u'''}{u''}$
PCS 2: Borrowing constraint + Idiosync. Risk

- With *incomplete markets* and *borrowing constraints*, agents engage in precautionary savings in the presence of idiosyncratic income shocks.
- Following Bewley (1977), mean asset holdings $E[a]$ result from individual optimization.
In an exchange economy, aggregate supply of assets must be zero

- Huggett (1993)

Equilibrium interest rate always satisfies $r < \rho$
Aiyagari (1994) combines the previous setup with standard production function $F(K, L)$
- Constant aggregate labor $L$

Demand for capital is given by $f'(k) - \delta = r$
- Efficient level of capital $f'(k^*) - \delta = \rho \Rightarrow k^* < k$
Aiyagari (1995) shows that a tax on capital earnings can address this efficiency problem:
- This decreases the net interest rate received by agents.

Government debt does not work “perfectly”:
- No finite amount of government debt will achieve $r = \rho$.
Constrained Inefficiency

- Bewley-Aiyagari economies result in competitive allocations that are not only Pareto inefficient, but are also \textit{constrained} Pareto inefficient
  - Social planner can achieve a Pareto superior outcome even facing same market incompleteness

- This result can be attributed to \textit{pecuniary externalities}
  - In competitive equilibrium, agents take prices as given whereas a social planner can induce wealth transfers by affecting relative prices
  - Stiglitz (1982), Geanakoplos-Polemarcharkis (1986)
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Krusell, Smith (1998) introduce aggregate risk into the Aiyagari framework

- Aggregate productivity shock that follows a Markov process $z_t$ and $Y_t = z_t F(K_t, L_t)$

- Aggregate capital stock determines equilibrium prices $r_t, w_t$

  - However, the evolution of aggregate stock is affected by the distribution of wealth since poor agents may have a much higher propensity to save
  - Tracking whole distribution is practically impossible
Krusell, Smith assume agents are boundedly rational and approximate the distribution of capital by a finite set of moments $M$

- Regression $R^2$ is relatively high even if $\#M = 1$

This result is strongly dependent on low risk aversion and low persistence of labor shocks

- Weak precautionary savings motive except for poorest agents
- Since wealth-weighted averages are relevant, this has a negligible effect on aggregate quantities
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- Amplification Revisited
Financial instability arises from the fragility of liquidity concepts.

- **Technological liquidity**
  - Reversibility of investment

- **Market liquidity**
  - Specificity of capital
  - Price impact of capital sale

- **Funding liquidity**
  - Maturity structure of debt
    - Can’t roll over short term debt
  - Sensitivity of margins
    - Margin-funding is recalled

- *Liquidity mismatch* determines severity of amplification
Amplification Revisited

- Investment possibility shocks
  - Production possibilities: Scheinkman & Weiss (1986)
  - Investment possibilities: Kiyotaki & Moore (2008)

- Interim liquidity shocks
  - Endogenous shock: Shleifer & Vishny (1997)

- Preference shocks
  - No aggregate risk: Diamond & Dybvig (1983)
  - Aggregate risk: Allen & Gale (1994)
Three period model with $t \in \{0,1,2\}$

Entrepreneurs with initial wealth $A$
- Investment of $I$ returns $RI$ in $t = 2$ with probability $p$
- Interim funding requirement $\rho I$ at $t = 1$ with $\rho \sim G$
- Extreme *technological illiquidity*, as investment is worthless if interim funding is not provided

Moral hazard problem
- Efficiency requires $\rho \leq \rho_1 \equiv pR \Rightarrow$ continuation
- Only $\rho \leq \rho_0 < \rho_1$ of funding can be raised due to manager’s private benefit, i.e. principal-agent conflict
Fund managers choose how aggressively to exploit an arbitrage opportunity

Mispricing can widen in interim period
- Investors question investment and withdraw funds
- Managers must unwind position when mispricing is largest, i.e. most profitable
- Low *market liquidity* due to limited knowledge of opportunity

Fund managers predict this effect, and thus limit arbitrage activity
- Keep buffer of liquid assets to fund withdrawals
Three period model with $t \in \{0,1,2\}$

- Continuum of ex-ante identical agents
  - Endowment of 1 in $t = 0$
  - Idiosyncratic preference shock, i.e. probability $\lambda$ that agent consumes in $t = 1$ and probability $1 - \lambda$ that agent consumes in $t = 2$

- Preference shock is not observable to outsiders
  - Not insurable, i.e. incomplete markets
**DD: Investment**

- Good can be stored without cost
  - Payoff of 1 in any period
- Long term investment project
  - Payoff of $R > 1$ in $t = 2$
  - Salvage value of $r \leq 1$ if liquidated early in $t = 1$
  - Market for claims to long-term project at price $p$
- Trade-off between return and *liquidity*
  - Investment is subject to *technological illiquidity*, i.e. $r \leq 1$
  - Market liquidity is represented by interim price $p$
AG extend DD framework by adding aggregate risk

- Here, $\lambda = \lambda_H$ with probability $\pi$ and $\lambda = \lambda_L < \lambda_H$ with probability $1 - \pi$

Agents observe realization of aggregate state and idiosyncratic preference shock at $t = 1$

- After resolution of uncertainty, agents can trade claims to long-term project at $p_s \in \{p_H, p_L\}$
- Asset’s market liquidity will vary across states

For simplicity, assume $r = 0$