Motivation

- **Aim:** Bridge the gap between
  - Macro/monetary research
  - Finance research

- **Financial sector helps to**
  - overcome financing frictions and
  - channels resources
  - creates money

... but
- **Credit crunch due to**
  - adverse feedback loops & liquidity spirals
  - Non-linear dynamics

- **New insights to monetary and international economics**
- **Price stability**
  - Monetary policy
    - Short-term interest
    - Policy rule (terms structure)

- **Financial stability**
  - Macroprudential policy
    - Reserve requirements
    - Capital/liquidity requirements
    - Collateral policy
    - Margins/haircuts
    - Capital controls

- **Fiscal debt sustainability**
  - Fiscal

Output (gap)
Systemic risk – a broad definition

- Systemic risk build-up during (credit) bubble ... and materializes in a crisis
  - “Volatility Paradox” → contemp. measures inappropriate
- Spillovers/contagion – externalities
  - Direct contractual: domino effect (interconnectedness)
  - Indirect: price effect (fire-sale externalities)
    - credit crunch, liquidity spirals

- Adverse GE response → amplification, persistence
Minsky moment – Wile E. Coyote Effect
Methodology – relation to finance

- **Verbal Reasoning** *(qualitative)*
  - Fisher, Keynes, ...

- **Macro**
  - **Growth theory**
    - *Dynamic (cts. time)*
    - *Deterministic*
  - **Introduce stochastic**
    - *Discrete time*
    - Brock-Mirman, Stokey-Lucas
    - DSGE models
  - **Cts. time macro with financial frictions**

- **Finance**
  - **Portfolio theory**
    - *Static*
    - *Stochastic*
  - **Introduce dynamics**
    - *Continuous time*
    - Options Black Scholes
    - Term structure CIR
    - Agency theory Sannikov
Heterogeneous agents + frictions

- Lending-borrowing/insuring since agents are different

  - Poor-rich
  - Productive
  - Less patient
  - Less risk averse
  - More optimistic

  - Rich-poor
  - Less productive
  - More patient
  - More risk averse
  - More pessimistic

- Limited direct lending due to frictions

- Friction \( \rightarrow p_s \) MRS\(_s\) different even after transactions

- Wealth distribution matters! (net worth of subgroups)

- Financial sector is not a veil
Liquidity Concepts

- Financial instability arises from the fragility of liquidity

Market liquidity
- Specificity of capital
  Price impact of capital sale

Technological liquidity
- Reversibility of investment

Funding liquidity
- Maturity structure of debt
  - Can’t roll over short term debt
- Sensitivity of margins
  - Margin-funding is recalled

Liquidity mismatch determines severity of amplification
Types of Funding Constraints

- Equity constraint
  - “Skin in the game constraint”

+ Debt constraints
  - Costly state verification a la Townsend
    - Borrowing cost increase as net worth drops
  - Collateral/leverage/margin constraints
    - Quantity constraint on borrowing
    - Incomplete contracts a la Hart-Moore
    - Commitment problem
    - Credit rationing a la Stiglitz-Weiss
  - Not binding (precautionary buffer)

Comment: Constraints vs. incomplete markets
Constraints vs. Incomplete Markets

state 1

state 2
Constraints vs. Incomplete Markets

Short-sale constraint

state 1

state 2

constraint
Constraints vs. Incomplete Markets
Constraints & Incomplete Markets

Debt limit can depend on prices/volatility
Macro-literature on Frictions

1. Net worth effects:
   a. Persistence: Carlstrom & Fuerst
   b. Amplification: Bernanke, Gertler & Gilchrist
      “Kocherlakota critique”

2. Volatility effects: impact credit quantity constraints
   a. Instability: Brunnermeier & Sannikov
   b. Margin spirals: Brunnermeier & Pederson
   c. Endogenous constraints: Geanakoplos

3. Demand for liquid assets & Bubbles – “self insurance”
   a. OLG, Aiyagari, Bewley, Krusell-Smith, Holmstrom-Tirole, ...

4. Financial intermediaries & Theory of Money
Amplification & Instability - Overview

  - Perfect (technological) liquidity, but persistence
  - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period
Amplification & Instability - Overview

  - Perfect (technological) liquidity, but persistence
  - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period

  - Technological/market illiquidity
  - KM: Leverage bounded by margins; BGG: Verification cost (CSV)
  - Stronger amplification effects through prices (low net worth reduces leveraged institutions’ demand for assets, lowering prices and further depressing net worth)

- Brunnermeier & Sannikov (2010)
  - Instability, volatility dynamics, volatility paradox, Kocherlakota critique

- Brunnermeier & Pedersen (2009), Geanakoplos
  - Volatility interaction with margins/haircuts (leverage)
Persistence

- Even in standard real business cycle models, temporary adverse shocks can have long-lasting effects
- Due to feedback effects, persistence is much stronger in models with financial frictions
  - Bernanke & Gertler (1989)
  - Carlstrom & Fuerst (1997)
- Negative shocks to net worth exacerbate frictions and lead to lower capital, investment and net worth in future periods
Costly State Verification

- Key friction in previous models is **costly state verification**, i.e. CSV, a la Townsend (1979)
- Borrowers are subject to an idiosyncratic shock
  - Unobservable to lenders, but can be verified at a cost
- Optimal solution is given by a contract that resembles standard debt
CSV: Contracting

- Competitive market for capital
  - Lender’s expected profit is equal to zero
  - Borrower’s optimization is equivalent to minimizing expected verification cost

- Financial contract specifies:
  - Debt repayment for each reported outcome
  - Reported outcomes that should be verified
Incentive compatibility implies that
- Repayment outside of VR is constant
- Repayment outside of VR is weakly greater than inside

Maximizing repayment in VR reduces the size and thus the expected verification cost
Output is produced according to $Y_t = A_t f(K_t)$

Fraction $\eta$ of entrepreneurs and $1 - \eta$ of households

- Only entrepreneurs can create new capital from consumption goods

Individual investment yields $\omega i_t$ of capital

- Shock is given by $\omega \sim G$ with $E[\omega] = 1$
- This implies consumption goods are converted to capital one-to-one in the aggregate
- *No technological illiquidity!*
Households can verify \( \omega \) at cost \( \mu i_t \)

- Optimal contract is debt with audit threshold \( \bar{\omega} \)
- Entrepreneur with net worth \( n_t \) borrows \( i_t - n_t \) and repays \( \min\{\omega_t, \bar{\omega}\} \times i_t \)

Auditing threshold is set by HH breakeven condition

\[
\int_{\omega}^{\bar{\omega}} (\omega - \mu) dG(\omega) + (1 - G(\bar{\omega})) \bar{\omega} \times i_t q_t = i_t - n_t
\]

- Here, \( q_t \) is the price of capital

No positive interest (within period borrowing) and no risk premium (no aggregate investment risk)
**CF: Persistence & Dampening**

- Negative shock in period $t$ decreases $N_t$
  - This increases financial friction and decreases $I_t$

- Decrease in capital supply leads to
  - Lower capital: $K_{t+1}$
  - Lower output: $Y_{t+1}$
  - Lower net worth: $N_{t+1}$
  - Feedback effects in future periods $t + 2, ...$

- Decrease in capital supply also leads to
  - Increased price of capital $q_t$
  - Dampening effect on propagation of net worth shock
Dynamic Amplification

- Bernanke, Gertler and Gilchrist (1999) introduce *technological illiquidity* in the form of nonlinear adjustment costs to capital.

- Negative shock in period $t$ decreases $N_t$
  - This increases financial friction and decreases $I_t$.

- In contrast to the dampening mechanism present in CF, now decrease in capital *demand* (not supply) leads to
  - Decreased price of capital due to adjustment costs
  - *Amplification* effect on propagation of net worth shock.
BGG assume separate investment sector
- This separates entrepreneurs’ capital decisions from adjustment costs

Φ(·) represents *technological illiquidity*
- Increasing and concave with Φ(0) = 0
- $K_{t+1} = \Phi \left( \frac{I_t}{K_t} \right) K_t + (1 - \delta)K_t$

FOC of investment sector
- $\max_{I_t} \{q_t K_{t+1} - I_t\} \Rightarrow q_t = \Phi' \left( \frac{I_t}{K_t} \right)^{-1}$

*jump to KM97*
Entrepreneurs alone can hold capital used in production (of consumption good)

At time $t$, entrepreneurs purchase capital for $t + 1$
- To purchase $k_{t+1}$, an entrepreneur borrows $q_t k_{t+1} - n_t$
- Here, $n_t$ represents entrepreneur net worth

Assume gross return to capital is given by $\omega R_{t+1}^k$
- Here $\omega \sim G$ with $E[\omega] = 1$ and $\omega$ i.i.d.
- $R_{t+1}^k$ is the endogenous aggregate equilibrium return
Shocks to net worth $N_t$ are persistent
- They affect capital holdings, and thus $N_{t+1}$, ...

*Technological illiquidity* for capital “demanders” no introduces amplification effect
- Decrease in capital leads to reduced price of capital from
  $$q_t = \Phi' \left( \frac{I_t}{K_t} \right)^{-1}$$
- Lower price of capital further decreases net worth
Kiyotaki & Moore 97

- Kiyotaki, Moore (1997) adopt a
  - collateral constraint instead of CSV
  - *market illiquidity* – second best use of capital
- Output is produced in two sectors, differ in productivity
- Aggregate capital is fixed, resulting in extreme *technological illiquidity*
  - Investment is completely irreversible
- Durable asset has two roles:
  - Collateral for borrowing
  - Input for production
**KM: Amplification**

- **Static** amplification occurs because fire-sales of capital from productive sector to less productive sector depress asset prices
  - Importance of *market liquidity* of physical capital
- **Dynamic** amplification occurs because a temporary shock translates into a persistent decline in output and asset prices
Two types of infinitely-lived risk neutral agents

Mass $\eta$ of productive agents
- Constant-returns-to-scale production technology yielding $y_{t+1} = ak_t$
- Discount factor $\beta < 1$

Mass $1 - \eta$ of less productive agents
- Decreasing-returns-to-scale production $y_{t+1} = F(k_t)$
- Discount factor $\beta \in (\beta, 1)$
Since productive agents are less patient, they will want to borrow $b_t$ from less productive agents

- However, friction arises in that each productive agent’s technology requires his individual human capital
- Productive agents cannot pre-commit human capital

This results in a collateral constraint

$$Rb_t \leq q_{t+1}k_t$$

- Productive agent will never repay more than the value of his asset holdings, i.e. collateral
Since there is no uncertainty, a productive agent will borrow the maximum quantity and will not consume any of the output

- Budget constraint: \( q_t k_t - b_t \leq (a + q_t)k_{t-1} - Rb_{t-1} \)
- Demand for assets: \( k_t = \frac{1}{q_t} \frac{1}{q_{t+1}} [(a + q_t)k_{t-1} - Rb_{t-1}] \)

Unproductive agents are not borrowing constrained

- \( R = \beta^{-1} \) and asset demand is set by equating margins

- Demand for assets: \( R = \frac{F'(k_t) + q_{t+1}}{q_t} \)

Rewritten to \( \frac{1}{R} F'(k_t) = q_t - \frac{1}{R} q_{t+1} \)
With fixed supply of capital, market clearing requires $\eta K_t + (1 - \eta) K_t = \bar{K}$

- This implies $M(K_t) \equiv \frac{1}{R} F' \left( \frac{\bar{K} - \eta K_t}{1 - \eta} \right) = q_t - \frac{1}{R} q_{t+1}$
- Note that $M(\cdot)$ is increasing

Iterating forward, we obtain: $q_t = \sum_{s=0}^{\infty} \frac{1}{R^s} M(K_{t+s})$
In steady state, productive agents use tradable output $a$ to pay interest on borrowing:

This implies that steady state price $q^*$ must satisfy:

\[ q^* - \frac{1}{R} q^* = a \]

Further, steady state capital $K^*$ must satisfy:

\[ \frac{1}{R} F' \left( \frac{\bar{K} - \eta K^*}{1-\eta} \right) = a \]

This reflects inefficiency since marginal products correspond only to *tradable* output as opposed to total $a + c$, where $c$ is non-tradable fraction.
Log-linearized deviations around steady state:
- Unexpected one-time shock that reduces production of all agents by factor $1 - \Delta$

%\-change in assets for given change in asset price:
\[
\hat{K}_t = -\frac{\xi}{1+\xi} \left( \Delta + \frac{R}{R-1} \hat{q}_t \right), \quad \hat{K}_{t+s} = \frac{\xi}{1+\xi} \hat{K}_{t+s-1}
\]
\[
\frac{1}{\xi} = \frac{d \log M(K)}{d \log K} \bigg|_{K=K^*} \quad \text{(elasticity)}
\]

Reduction in assets comes from two shocks:
- Lost output $\Delta$
- Capital losses on previous assets $\frac{R}{R-1} \hat{q}_t$, amplified by leverage
\[
\frac{\xi}{1+\xi} \text{ terms dampens effect since asset can reallocated}
KM: Productivity Shock

- Change in price for given change in assets:
  - Log-linearize the equation \( q_t = \sum_{s=0}^{\infty} \frac{1}{R^s} M(K_{t+s}) \)
  - This provides: \( \hat{q}_t = \frac{1}{\xi} \frac{R-1}{R} \sum_{s=0}^{\infty} \frac{1}{R^s} \hat{K}_{t+s} \)

- Combining equations:

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>static</th>
<th>dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{K}_t = )</td>
<td>(-\Delta)</td>
<td>(-\frac{1}{(\xi + 1)(R - 1)}\Delta)</td>
</tr>
<tr>
<td>( \hat{q}_t = )</td>
<td>(-\frac{(R - 1)1}{R} \frac{\Delta}{\xi})</td>
<td>(-\frac{1}{R} \frac{1}{\xi} \Delta)</td>
</tr>
</tbody>
</table>

- Static effect results from assuming \( q_{t+1} = q^* \)
“Kocherlakota critique”

- Amplification for negative shocks differs from positive shocks
  - In Kocherlakota (2000) optimal scale of production (positive shock does not lead to expansion)
- Amplification is quantitatively too small
  - Capital share is only 1/3 and hence GDP is too small

- Cordoba and Ripoll (2004)
  - Needs sizeable capital share plus
  - Low intertemporal substitution
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4. Financial intermediaries & Theory of Money
Previous papers only considered log-linearized solutions around steady state.

Brunnermeier & Sannikov (2010) build a continuous time model to study full dynamics:
- Show that financial system exhibits inherent instability due to highly non-linear effects.
- These effects are asymmetric and only arise in the downturn.

Agents choose a *capital cushion*:
- Mitigates moderate shocks near steady state.
- High volatility away from steady state.
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4. Financial intermediaries & Theory of Money
Credit Rationing – Quantity Rationing

- Credit rationing refers to a failure of market clearing in credit
  - In particular, an excess demand for credit that fails to increase market interest rate
  - Pool of loan applicants worsens
  - Stiglitz & Weiss (1981) show how asymmetric information on risk can lead to credit rationing
For collateralized lending, debt constraints are directly linked to the **volatility of collateral**
- Constraints are more binding in volatile environments
- Feedback effect between volatility and constraints

These **margin spirals** force agents to delever in times of crisis
- Collateral runs counterparty bank run
- Multiple equilibria
How are margins set by brokers/exchanges?

- **Value at Risk**: \( \Pr(- (p_{t+1} - p_t) \geq m) = 1\% = \pi \)
BP: Leverage and Margins

- Financing a long position of \( x^+_{j,t} > 0 \) shares at price \( p^+_{j,t} = 100 \):
  - Borrow $90 dollar per share;
  - Margin/haircut: \( m^+_{j,t} = 100 - 90 = 10 \)
  - Capital use: $10 \( x^+_{j,t} \)

- Financing a short position of \( x^-_{j,t} > 0 \) shares:
  - Borrow securities, and lend collateral of 110 dollar per share
  - Short-sell securities at price of 100
  - Margin/haircut: \( m^-_{j,t} = 110 - 100 = 10 \)
  - Capital use: $10 \( x^-_{j,t} \)

- Positions frequently marked to market
  - payment of \( x^+_{j,t} (p^+_{j,t} - p^-_{j,t-1}) \) plus interest
  - margins potentially adjusted – more later on this

- Margins/haircuts must be financed with capital:

\[
\sum_j ( x^+_{j,t} m^+_{j,t} + x^-_{j,t} m^-_{j,t} ) \cdot W_t, \text{ where } x_j = x^+_{j,t} - x^-_{j,t}
\]

with perfect cross-margining: \( M_t ( x^+_{1,t}, ..., x^+_{J,t} ) \cdot W_t \)
**BP: Liquidity Spirals**

- **Borrowers’ balance sheet**
  - **Loss spiral** – net worth drops
    - Net wealth > $\alpha x$
      - for asym. info reasons
    - constant or increasing leverage ratio
  - **Margin/haircut spiral**
    - Higher margins/haircuts
    - No rollover
    - redemptions
    - forces to delever

- **Mark-to-market vs. mark-to-model**
  - worsens loss spiral
  - improves margin spiral

- Both spirals reinforce each other
BP: Margin Spiral – Increased Volatility

\[ v_t = v_{t-1} + \Delta v_t = v_{t-1} + \sigma_t \varepsilon_t \]

\[ \sigma_{t+1} = \sigma + \theta |\Delta v_t| \]

- Selling pressure
  - initial customers
- complementary customers
1. Volatility of collateral increases
   - Permanent price shock is accompanied by higher future volatility (e.g. ARCH)
     - Realization how difficult it is to value structured products
   - Value-at-Risk shoots up
   - Margins/haircuts increase = collateral value declines
   - Funding liquidity dries up
   - Note: all “expert buyers” are hit at the same time, SV 92

2. Adverse selection of collateral
   - As margins/ABCP rate increase, selection of collateral worsens
   - SIVs sell-off high quality assets first (empirical evidence)
   - Remaining collateral is of worse quality
BP: Model Setup

- Time: $t=0,1,2$
- Asset with final asset payoff $v$ follows ARCH process
  - $v_t = v_{t-1} + \Delta v_t = v_{t-1} + \sigma_t \varepsilon_t$, where $v_t := E_t[v]$
  - $\sigma_{t+1} = \sigma + \theta |\Delta v_t|$
- Market illiquidity measure: $\Lambda_t = |v_t - p_t|$
- Agents:
  - *Initial customers* with supply $S(z, v_t - p_t)$ at $t=1,2$
  - *Complementary customers’ demand* $D(z, v_2 - p_2)$ at $t=2$
  - Risk-neutral *dealers* provide *immediacy* and
    - face capital constraint: $xm(\sigma, \Lambda) \leq W(\Lambda) := \max\{0, B + x_0(E[v_1] - \Lambda)\}$
  - *Financiers set margins* cash “price” of stock holding
Margins are set based on Value-at-Risk

*Financiers* do not know whether price move is due to

- *Likely*, movement in fundamental (based on ARCH process)
- *Rare*, Selling/buying pressure by customers who suffered asynchronous endowment shocks.

\[ m_1^+ = \Phi^{-1}(1 - \pi)\sigma_2 = \bar{\sigma} + \bar{\theta} |\Delta p_1| = m_1^- \]

Recall \( \sigma_{t+1} = \sigma + \theta |\Delta v_t| \)
$v_t = v_{t-1} + \Delta v_t = v_{t-1} + \sigma_t \epsilon_t$

$\sigma_{t+1} = \sigma + \theta |\Delta v_t|$

- Selling pressure initial customers
- Complementary customers
1. Margin Spiral – Increased Volatility

\[ x_1 < \frac{W_1}{m_1} = \frac{W_1}{\bar{\sigma} + \bar{\theta} |\Delta p_1|} \]

- \( \gamma = 0.01 \)
- \( \sigma^2 = 16 \)
- \( z_0 = 20 \)
- \( z_1 = 20 \)
- \( v_0 = 140 \)
- \( v_1 = 120 \)
- \( p_0 = 130 \)
- \( k = 10 \)
- \( \theta = 0.3 \)
- \( \eta_1 = 0 \)
- \( W_0 = 700 \)
- \( x_0 = 0 \)
1. Margin Spiral – Increased Volatility

\[ \gamma = 0.025 \quad \sigma^2 = 11 \quad z_0 = 20 \quad z_1 = 20 \quad \nu_0 = 140 \quad \nu_1 = 120 \]
\[ p_0 = 130 \quad k = 5 \quad \sigma = 0.3 \quad \eta_1 = 0 \quad W_0 = 750 \quad x_0 = 0 \]

customers’ supply

\[ x_1 < \frac{W_1}{m_1} = \frac{W_1}{\bar{\sigma} + \bar{\theta} |\Delta p_1|} \]
1. Margin Spiral – Increased Volatility

\[ x_1 < \frac{W_1}{m_1} = \frac{W_1}{\bar{\sigma} + \bar{\theta}|\Delta p_1|} \]

\[ \gamma = 0.025 \quad \sigma^2 = 11 \quad z_0 = 20 \quad z_1 = 20 \quad v_0 = 140 \quad v_1 = 120 \]
\[ p_0 = 130 \quad k = 5 \quad \phi = 0.3 \quad \eta_1 = 0 \quad W_0 = 600 \quad x_0 = 0 \]

customers’ supply
Data Gorton and Metrick (2011)

Haircut Index

“The Run on Repo”
Margins **stable** in tri-party repo market

- contrasts Gorton and Metrick
- no general run on certain collateral

Run (non-renewed financing) only on select **counterparties**

- Bear Stearns (anecdotally)
- Lehman (in the data)

Like 100% haircut… **(counterparty specific!)**
Bilateral and Tri-party Haircuts?

Differences in Median Haircuts

Source: FRBNY Calculations
Dealer maximizes expected profit per capital use
  - Expected profit $E_1[v^j] - p^j = \Lambda^j$
  - Capital use $m^j$

Dealers
  - Invest only in securities with highest ratio $\Lambda^j/m^j$

Hence, illiquidity/margin ratio $\Lambda^j/m^j$ is constant
BP: Commonality & Flight to Quality

- **Commonality**
  - Since funding liquidity is driving common factor

- **Flight to Quality**
  - Quality = Liquidity
    - Assets with lower fund vol. have better liquidity
  - Flight
    - Liquidity differential widens when funding liquidity becomes tight
m^2 = Volatility of Security2 = 2 > 1 = Volatility of Security1 = m^1

\[ \gamma = 0.015 \quad z_0 = 20 \quad z_1 = 20 \quad \nu_0 = 140 \quad \nu_1 = 120 \]
\[ \rho_0 = 130 \quad \sigma_1 = 10 \quad \sigma_2 = 15 \quad \sigma = 0.3 \quad \eta_1 = 2000 \quad x_0 = 0 \]
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