Banking, Liquidity and Bank Runs in an Infinite Horizon Economy

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Recent financial crisis

Slow run on shadow banks from Summer 2007

Loss in subprime loans and related assets → Financial intermediaries loose capital → Spreads between liquid and illiquid assets expand

Fast run after Lehmann failure in September 2008

Securitized assets market freezes

Wholesale and retail funding contracts. Asset prices fall further

"Great Recession"
We develop a simple macro model of banking crisis

Financial accelerator / Credit cycles

Roll-over risk, or "Bank run"

Macroeconomic conditions affect whether runs are feasible

Bank leverage ratio

Liquidation prices

An increase in the likelihood of run contracts the economy severely
Basic Model

Capital is either intermediated by banks or directly held by households

$$K^b_t + K^h_t = \bar{K}$$

$$K^b_t \text{ capital} \to \begin{cases} K^b_t \text{ capital} \\ Z_{t+1}K^b_t \text{ output} \end{cases}$$

$$K^h_t \text{ capital} \left\{ f(K^h_t) \text{ goods} \right\} \to \begin{cases} K^b_t \text{ capital} \\ Z_{t+1}K^b_t \text{ output} \end{cases}$$

$$f(K^h_t) = \frac{\alpha}{2} (K^h_t)^2 : \text{management cost } \alpha > 0$$
\[
Q_t K^b_t = N_t + D_t
\]

\[
Q_t K^h_t
\]

Intermediated Finance

Deposit

Direct Finance
Deposit contract

Short term

Promised rate of return $\bar{R}_{t+1}$ is non-contingent

With run, the returns is the minimum of $\bar{R}_{t+1}$ and total realized bank assets per deposit

In Basic Model, bank run is unanticipated $\rightarrow$

Realized return: $R_{t+1} = \bar{R}_{t+1}$: Promised return
Households maximize

\[ U_t = E_t \left( \sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right) \]

subject to:

\[ C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W_t^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h \]

\[
\rightarrow \\
1 = E_t (\Lambda_{t,t+1}) R_{t+1} \\
1 = E_t \left( \Lambda_{t,t+1} \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)} \right) \\
\Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}}
\]
Many bankers

Each has an i.i.d. survival probability of $\sigma$

Banker consumes wealth upon exit: $c_t^b = n_t$

Preferences are linear in "terminal" consumption

$$V_t = E_t \left[ \sum_{i=1}^{\infty} \beta^i \sigma^{-1}(1 - \sigma)c_{t+i}^b \right]$$

Each exiting banker replaced by a new banker with an endowment $w^b = n_t$

Bank balance sheet

$$Q_t k_t^b = d_t + n_t$$

Net worth $n_t$ of surviving bankers

$$n_t = (Z_t + Q_t)k_{t-1}^b - R_t d_{t-1}$$
$Z_t$ is realized

B/S of Bank

<table>
<thead>
<tr>
<th>Asset: $Q_t k_t^b$</th>
<th>Deposit: $d_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net worth: $n_t$</td>
<td></td>
</tr>
</tbody>
</table>

Incentive constraint:

$$\theta Q_t k_t^b \leq V_t$$

Figure 1: Timing

Date $t$

- Divert $\theta Q_t k_t^b$
- Continue: $V_t$

Date $t+1$

- Repay $R_{t+1}d_t$
- Retain $n_{t+1}$
- Exit or continue
- Bankrupt
Bank chooses $k^b_t$ and $d_t$ to maximize

$$V_t = \beta E_t[(1 - \sigma)n_{t+1} + \sigma V_{t+1}]$$

Bank chooses "leverage multiple" $\phi_t = \frac{Q_t k^b_t}{n_t}$ to maximize

$$\frac{V_t}{n_t} = \psi_t = \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \frac{n_{t+1}}{n_t} \right\}$$

$$= \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - R_{t+1} \right) + R_{t+1} \right] \right\}$$

$$= \mu_t \phi_t + \nu_t$$

subject to $\theta \phi_t \leq \psi_t$. →

$$\frac{Q_t k^b_t}{n_t} = \frac{\psi_t}{\theta} = \frac{\nu_t}{\theta - \mu_t}, \text{ if } \mu_t \in (0, \theta)$$
Aggregate leverage constraint

\[ Q_t K_t^b = \phi_t N_t \]

Aggregate net worth

\[ N_t = \sigma \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + (1 - \sigma) w^b \]

Goods market

\[ C_t^h + (1 - \sigma) \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + f(K_t^h) \]
\[ = Z_t \bar{K} + Z_t W^h + (1 - \sigma) w^b \]
Bank Runs

Ex ante, zero probability of a run

If depositors do not roll over the deposits ("run"), the bank sells its capital to households who are less efficient in managing capital.

In addition to an equilibrium without run, bank run equilibrium exists if:

\[(Z_t + Q_t^*) K_{t-1}^b < R_t D_{t-1}\]

\(Q_t^* \equiv \) the liquidation price of the bank’s assets
After a bank run at $t$:

$$K^h_t = \overline{K},$$

$$N_{t+1} = (1 - \sigma)w^b + \sigma(1 - \sigma)w^b$$

$$N_s = \sigma \left[ (Z_s + Q_s) K^b_{s-1} - R_s D_{s-1} \right] + (1 - \sigma)w^b, \quad \forall \ s \geq t+2$$

Household condition for direct capital holding →

$$Q^*_t = E_t \left\{ \sum_{i=1}^{\infty} \Lambda_{t,t+i} [Z_{t+i} - f'(K^h_{t+i})] \right\} - f'(\overline{K})$$
Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline Model</th>
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<tbody>
<tr>
<td>$\beta$</td>
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<tr>
<td>$\omega^h$</td>
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</table>

- Discount rate
- Bankers survival probability
- Seizure rate
- Household managerial cost
- Serial correlation of productivity shock
- Steady state productivity
- Bankers endowment
- Household endowment
<table>
<thead>
<tr>
<th>Steady State Values</th>
<th>Baseline</th>
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<tr>
<td>$K$</td>
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<tr>
<td>$R$</td>
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</table>
FIGURE 3: A Recession in the Baseline Model; No Bank Run Case
Figure 4: Ex-Post Bank Run in the Baseline Model
Extension: Anticipated Bank Runs

Deposit returns $R_{t+1} = \begin{cases} R_{t+1} & \text{if no bank run} \\ x_{t+1}R_{t+1} & \text{if bank run} \end{cases}$

$x_{t+1} = \text{Min} \left[ 1, \frac{(Q_{t+1} + Z_{t+1}) K_t^b}{R_{t+1}D_t} \right]$  

Household attaches the probability of bank run as

$p_t = 1 - E_t(x_{t+1})$

FONC for deposits is

$1 = \bar{R}_{t+1}[(1 - p_t)E_t(\Lambda_{t,t+1}) + p_tE_t(\Lambda_{t,t+1}^* x_{t+1})]$
Bank’s leverage \( \phi_t = \frac{Q_t k_t^b}{n_t} \) maximizes

\[
\frac{V_t}{n_t} = \psi_t =
\]

\[
\beta(1-p_t)E_t \left\{ (1-\sigma+\sigma \psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - \bar{R}_{t+1} \right) + \bar{R}_{t+1} \right] \right\}
\]

subject to \( \theta \phi_t \leq \psi_t \).

An increase in likelihood of run is contractionary in two ways

leverage \( \phi_t \) declines when the franchise value falls

\( N_{t+1} \) decreases even without run since \( \bar{R}_{t+1} \) increases
Figure 5: Recession with positive probability of a run

![Graphs showing various metrics over quarters with recession and no run recession scenarios.](image-url)
Figure 6: Recession with positive Run Probability and Ex-Post Run

- **p**: The graph shows the change in a variable labeled with a delta (Δ) from the steady state (ss), indicating the impact of recession on the probability variable. The x-axis represents quarters, and the y-axis shows the percentage change from the steady state.

- **y**: The graph represents the impact on the output variable (y), showing the percentage change from the steady state. The change is observed over time, with the x-axis representing quarters.

- **kb**: This graph illustrates the change in capital stock (kb), again with the x-axis showing quarters and the y-axis depicting the percentage change from the steady state.

- **Q**: The graph for Q (a variable that could stand for quantity) shows its percentage change from the steady state over time.

- **φ**: The graph for φ (another variable) depicts its percentage change from the steady state, with time in quarters on the x-axis.

- **n**: This graph shows the change in a variable labeled as n, indicating the percentage change from the steady state.

- **ER^{b,R^d}**: The graph for ER^{b,R^d} (an energy-related variable) illustrates its percentage change from the steady state, with time in quarters.

- **R^{d-R^free}**: The graph for R^{d-R^free} shows the difference between two variables over time, with the x-axis representing quarters.

- **R^{free}**: The graph for R^{free} indicates the percentage change from the steady state.

The graphs are labeled with two lines: **blue** (anticipated Run) and **red** (No Run Recession), illustrating different scenarios. Each graph is annotated with descriptive labels for the variables and their changes.
Figure 7: Credit Spreads and Bank Equity: Model VS Data

Description: The data series for Credit spreads is the Excess Bond Premium as computed by Gilchrist and Zakrasjek (2012); Bank Equity is the S&P500 Financial Index. The model counterparts are the paths of $E(R_p-R_d)$ and $V$ as depicted in Figure 6 normalized so that their steady-state values match the actual values in 2007 Q2.
Some Remarks About Policy

Deposit insurance makes depositors careless $\rightarrow$ Bank will divert the assets

Capital requirement reduces bank risk-taking and likelihood of bank run

  Can increase intermediation cost if capital is costly to raise

Lender-of-last resort stabilizes liquidation price

  May reduce the likelihood of run

But increase the leverage multiple ex ante and the financial accelerator